Basics of Foundation Engineering with Solved Problems

Based on “Principles of Foundation Engineering, 7th Edition”

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Being rich is not about how much you have, but is about how much you can give

Chapter (2)
Subsoil Exploration

Introduction:

The **soil mechanics** course reviewed the fundamental properties of soils and their behavior under stress and strain in idealized conditions. In practice, natural soil deposits are not homogeneous, elastic, or isotropic. In some places, the stratification of soil deposits even may change greatly within a horizontal distance of 15 to 30 m. For **foundation design** and **construction work**, one must know the actual soil stratification at a given site, the laboratory test results of the soil samples obtained from various depths, and the observations made during the construction of other structures built under similar conditions. For most major structures, adequate subsoil exploration at the construction site must be conducted.

Definition:

The process of determining the layers of natural soil deposits that will underlie a proposed structure and their physical properties is generally referred to as **subsurface exploration**.

Purpose of Subsurface Exploration:

The purpose of subsurface exploration is to obtain information that will aid the geotechnical engineer in:

1. Determining the nature of soil at the site and its stratification.
2. Selecting the type and depth of foundation suitable for a given structure.
3. Evaluating the load-bearing capacity of the foundation.
4. Estimating the probable settlement of a structure.
5. Determining potential foundation problems (e.g., expansive soil, collapsible soil, sanitary landfill, etc...).
6. Determining the location of water table.
7. Determining the depth and nature of bedrock, if and when encountered.
8. Performing some in situ field tests, such as permeability tests, van shear test, and standard penetration test.
9. Predicting the lateral earth pressure for structures such as retaining walls, sheet pile, and braced cuts.

Subsurface Exploration Program:

A soil exploration program for a given structure can be divided broadly into three phases:

1. Collection of Preliminary Information:

This step includes obtaining information regarding the **type of structure** to be built and its general use. The following are examples explain the needed information about different types of structures:

- For the construction of **building**:
  - ✓ The approximate column loads and their spacing.
  - ✓ Local building-codes.
  - ✓ Basement requirement.

- For the construction of **bridge:**
  - ✓ The length of their spans.
  - ✓ The loading on piers and abutments.

2. Reconnaissance:

The engineer should always make a visual inspection (field trip) of the site to obtain information about:

- ✓ The general topography of the site, the possible existence of drainage ditches, and other materials present at the site.
- ✓ Evidence of creep of slopes and deep, wide shrinkage cracks at regularly spaced intervals may be indicative of **expansive soil**.
- ✓ Soil stratification from deep cuts, such as those made for the construction of nearby highways and railroads.
- ✓ The type of vegetation at the site, which may indicate the nature of the soil.
- ✓ Groundwater levels, which can be determined by checking nearby wells.
- ✓ The type of construction nearby and the existence of any cracks in walls (**indication for settlement**) or other problems.
- ✓ The nature of the stratification and physical properties of the soil nearby also can be obtained from any available soil-exploration reports on existing structures.

3. Site Investigation:

This phase consists of:

- ✓ Planning (adopting steps for site investigation, and future vision for the site)
- ✓ Making test boreholes.
- ✓ Collecting soil samples at desired intervals for visual observation and laboratory tests.

Determining the number of boring:

There is no hard-and-fast rule exists for determining the number of borings are to be advanced. For most buildings**, at least one boring at each corner and one at the center** should provide a start. Spacing can be increased or decreased, depending on the condition of the subsoil. If various soil strata are more or less uniform and predictable, fewer boreholes are needed than in nonhomogeneous soil strata.
The following table gives some guidelines for borehole spacing between for different types of structures:

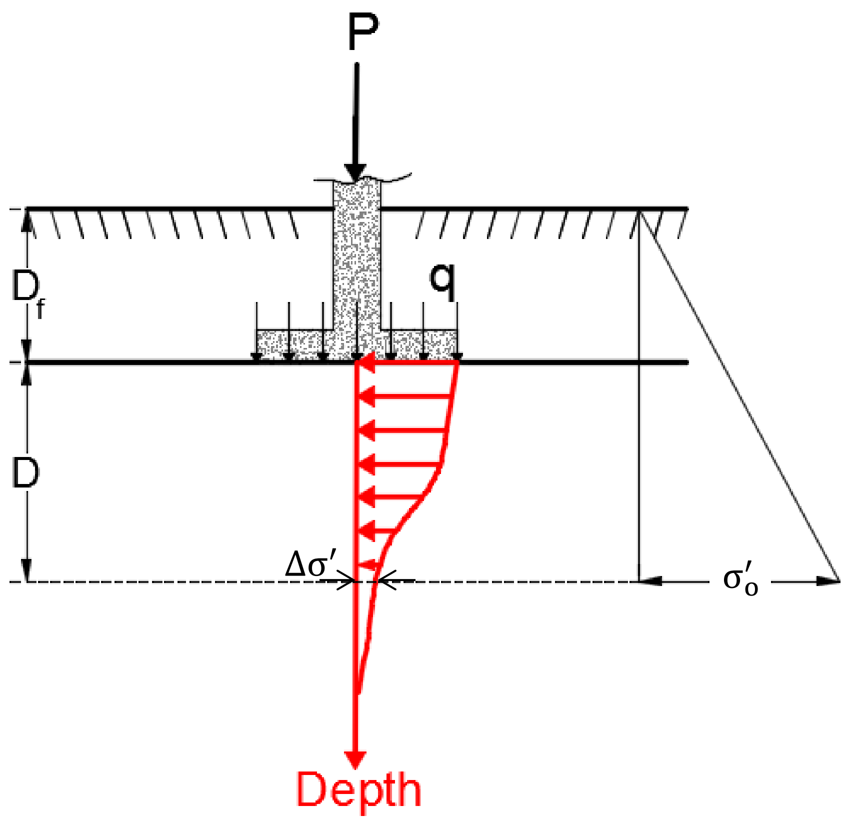
Approximate Spacing of Boreholes	
Type of project	Spacing (m)
Multistory building	10–30
One-story industrial plants	20–60
Highways	250–500
Residential subdivision	250–500
Dams and dikes	40–80

Determining the depth of boring:

The approximate required minimum depth of the borings should be predetermined. The estimated depths can be changed during the drilling operation, depending on the subsoil encountered (e.g., **Rock**).
To determine the approximate required minimum depth of boring, engineers may use the rules established by the American Society of Civil Engineers (ASCE 1972):

1. Determine the net increase in effective stress ($\Delta\sigma'$) under a foundation with depth as shown in the Figure below.

2. Estimate the variation of the vertical effective stress (σ'_o) with depth.
3. Determine the depth ($D = D_1$) at which the effective stress increase **($\Delta\sigma'$) is equal to** $\left(\frac{1}{10}\right)$ **q** (q = estimated net stress on the foundation).
4. Determine the depth ($D = D_2$) at which $(\Delta\sigma'/\sigma'_o) = 0.05$.
5. Determine the depth ($D = D_3$) which is the distance from the **lower face** of the foundation to **bedrock** (if encountered).
6. Choose the **smaller** of the three depths, (D_1, D_2, and D_3), just determined is the approximate required minimum depth of boring.



After determining the value of (D) as explained above the final depth of boring (from the ground surface to the calculated depth) is:

$$D_{boring} = D_f + D$$

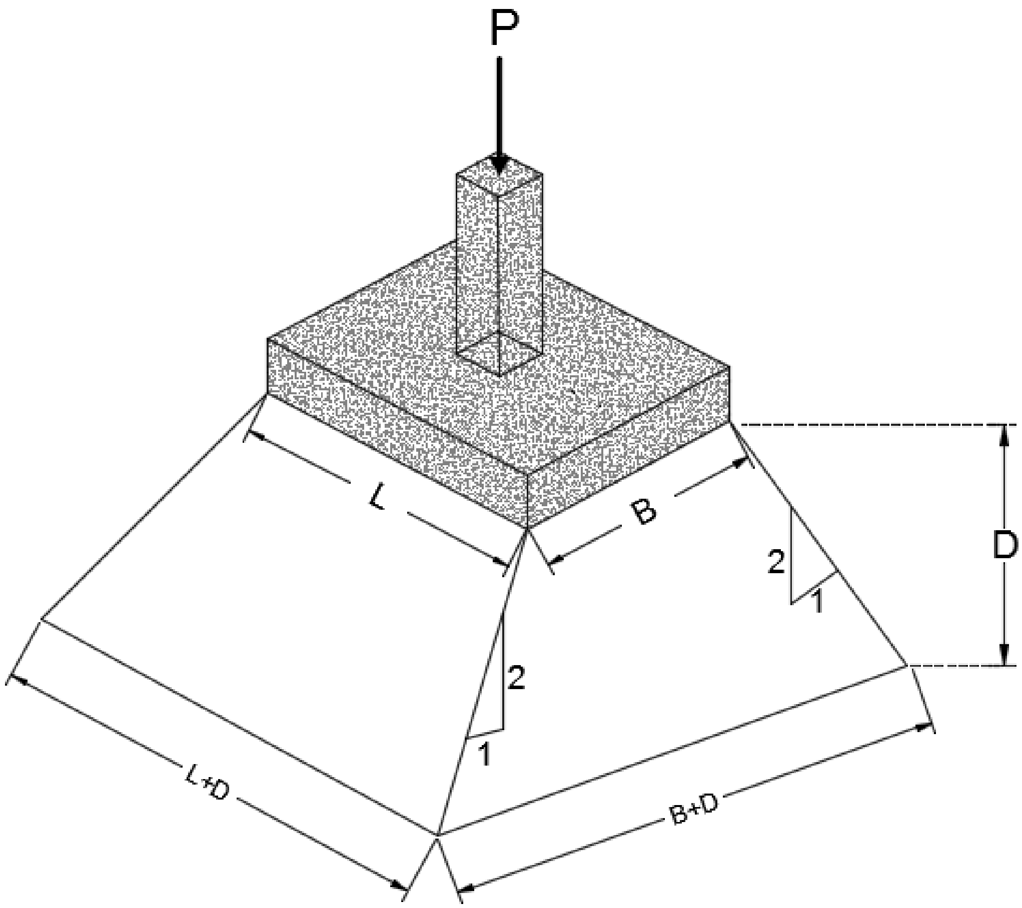
Because the Drilling will starts from the ground surface.

Determining the value of vertical effective stress (σ'_o):
The value of (σ'_o) always calculated from the **ground surface** to the required depth, as we previously discussed in Ch.9 (Soil Mechanics).

Determining the increase in vertical effective stress($\Delta\sigma'$):

The value of ($\Delta\sigma'$) always calculated from the **lower face of the foundation** as we discussed previously in soil mechanics course (Ch.10).

An alternative approximate method can be used rather than (Ch.10) in soil mechanics course, this method is easier and faster than methods in (Ch.10). This method called **(2:1 Method)**. The value of ($\Delta\sigma'$) can be determined using (2:1 method) as following:



According to this method, the value of ($\Delta\sigma'$) at depth (D) is:

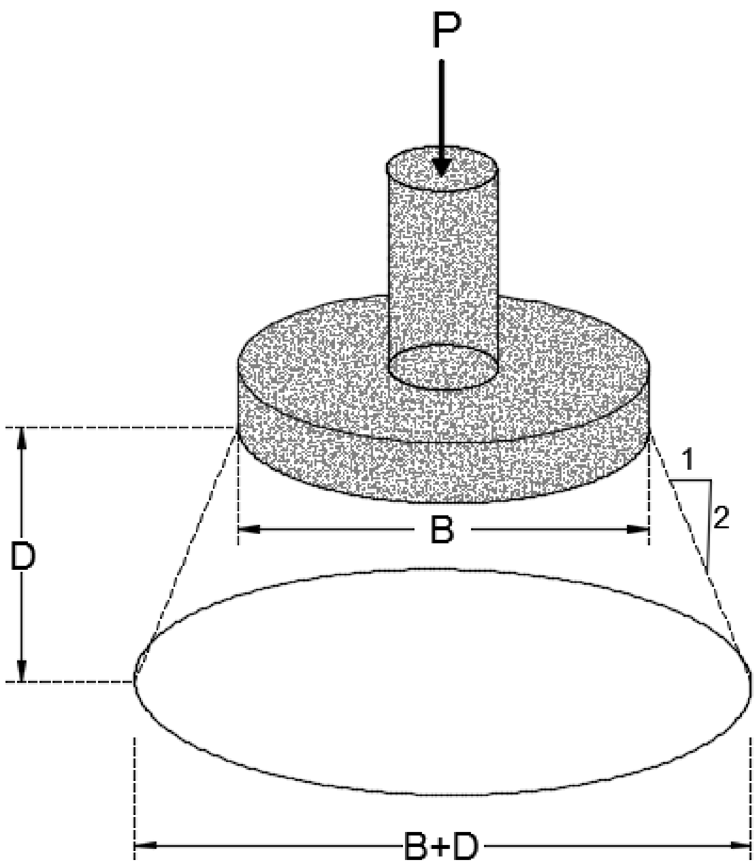
$$\Delta\sigma'_D = \frac{P}{A} = \frac{P}{(B + D) \times (L + D)}$$

$P =$ the load applied on the foundation (KN).

$A =$ the area of the stress distribution at **depth** (**D**).

Note that the above equation is based on the assumption that the stress from the foundation **spreads out** with a **vertical**-to-**horizontal** slope of **2:1.**
Now, the values of (D_1 and D_2) can be calculated easily as will be seen later.

Note: if the foundation is **circular** the value of ($\Delta\sigma'$) at depth (D) can be determined as following:



$$\Delta\sigma'_D = \frac{P}{\text{Area at depth (D)}} = \frac{P}{\frac{\pi}{4} \times (B + D)^2}$$

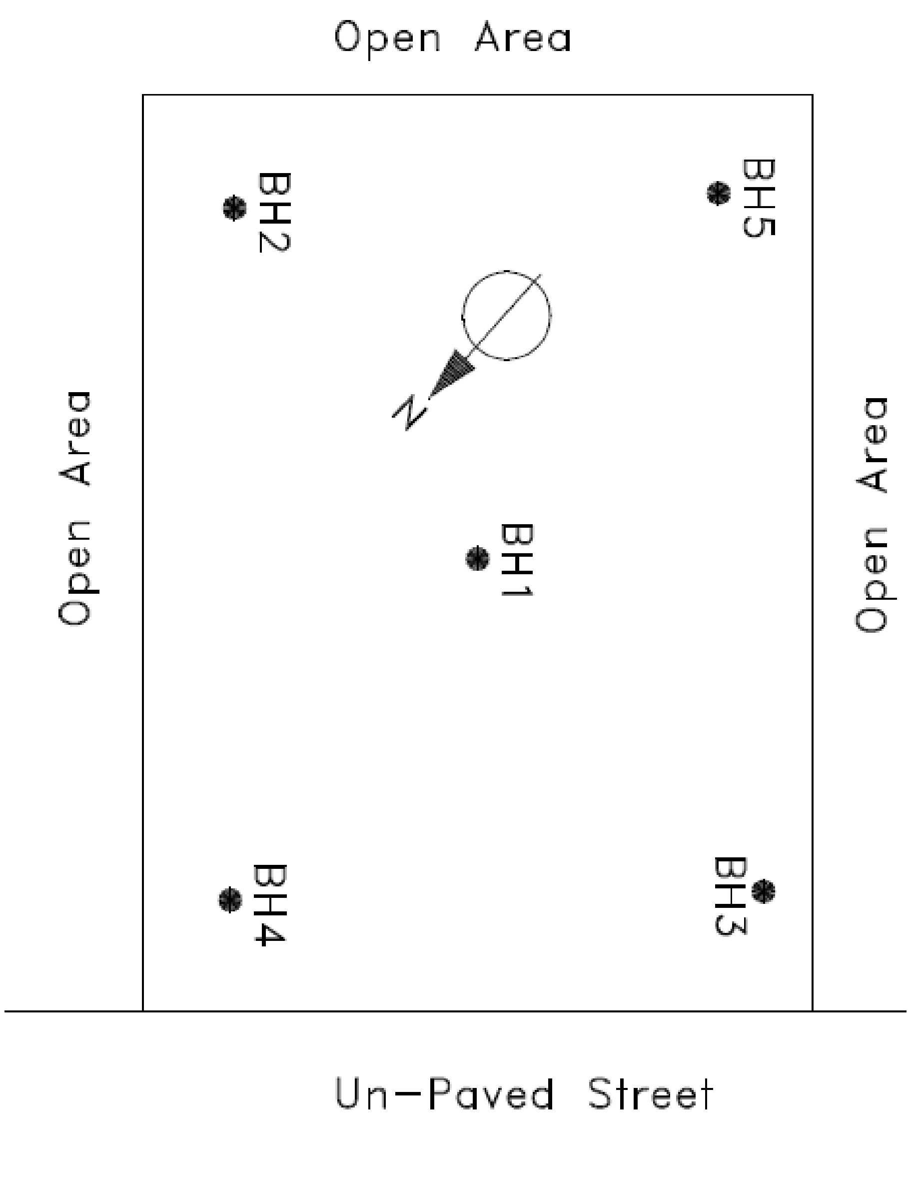
$P =$ the load applied on the foundation (KN).
$B =$ diameter of the foundation(m).

In practice: The number of boreholes and the depth of each borehole will be identified according to the type of project and the subsoil on site, the following is example for a 5 story residential building with dimensions of (40 x 70) m:

✓ The required number of boreholes = 5 boreholes (one at each corner and one at the center) as mentioned previously.

✓ The depth of each borehole for this project is (8-10) m up to a depth of water table.

The following figure shows the distribution of boreholes on the land:



Procedures for Sampling Soil

There are two types of samples:

- **Disturbed Samples:** These types of samples are disturbed but representative, and may be used for the following types of laboratory soil tests:
  - ✓ Grain size analysis.
  - ✓ Determination of liquid and plastic limits.
  - ✓ Specific gravity of soil solids.
  - ✓ Determination of organic content.
  - ✓ Classification of soil.
  - ✓ But disturbed soil samples cannot be used for consolidation, hydraulic conductivity, or shear tests, because these tests must be performed on the same soil of the field without any disturbance (to be representative)

  The major equipment used to obtain disturbed sample is (Split Spoon) which is a steel tube has inner diameter of 34.93 mm and outer diameter of 50.8mm.

- **Undisturbed Samples:** These types of samples are used for the following types of laboratory soil tests:
  - ✓ Consolidation test.
  - ✓ Hydraulic Conductivity test.
  - ✓ Shear Strength tests.

  These samples are more complex and expensive, and it's suitable for **clay**, however in **sand** is very difficult to obtain undisturbed samples. The major equipment used to obtain undisturbed sample is (Thin-Walled Tube).

Degree of Disturbance

If we want to obtain a soil sample from any site, the degree of disturbance for a soil sample is usually expressed as:

$$A_R(\%) = \frac{D_o^2 - D_i^2}{D_i^2} \times 100$$

A_R = area ratio (ratio of disturbed area to total area of soil)

D_o = outside diameter of the sampling tube.
D_i = inside diameter of the sampling tube.
If $(A_R) \leq 10\% \rightarrow$ the sample is **undisturbed**.
If $(A_R) > 10\% \rightarrow$ the sample is **disturbed.**
For a standard split-spoon sampler (which sampler for disturbed samples):

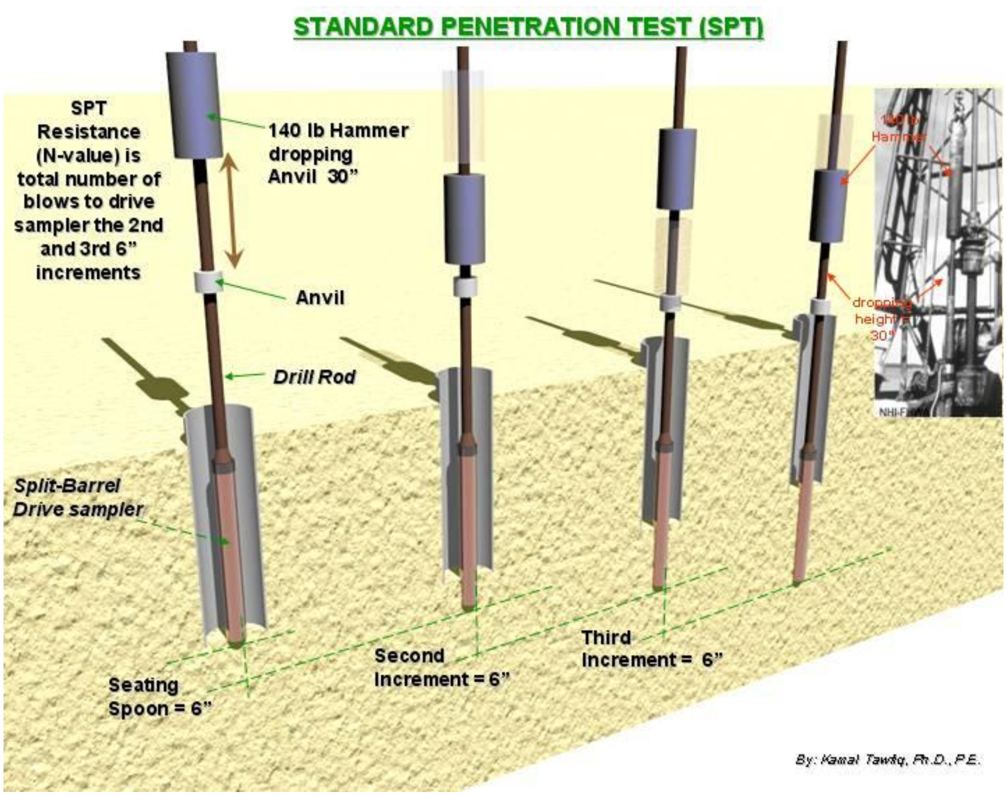
$$A_R(\%) = \frac{(50.8)^2 - (34.93)^2}{(34.93)^2} \times 100 = 111.5\% > 10\% \rightarrow \text{disturbed.}$$

Standard Penetration Test (SPT)

This test is one of the most important soil tests for geotechnical engineers because it’s widely used in calculating different factors as will explained later. This test is performed according the following procedures:

1. Determining the required number and depth of boreholes in the site.
2. The **sampler** used in SPT test is (Standard Split Spoon) which has an inside diameter of 34.39 mm and an outside diameter of 50.8 mm.
3. Using drilling machine, **1.5m** are drilled.
4. The drilling machine is removed and the sampler will lowered to the bottom of the hole.
5. The sampler is driven into the soil by hammer blows to the top of the drill rod, the standard weight of the hammer is 622.72 N (63.48 Kg), and for each blow, the hammer drops a distance of 76.2 cm.
6. The number of blows required for a spoon penetration of three 15 cm intervals are recorded.
7. The first 15 cm drive is considered as seating load and is ignored.
8. The number of blows required for the **last two intervals** are added to give the **Standard Penetration Number (N)** at that depth.
9. The sampler is then withdrawn and the soil sample recovered from the tube is placed in a glass bottle and transported to laboratory.
10. Using the drilling machine to drill another 1.5m and then repeat the above steps for each 1.5 m till reaching the specified depth of borehole.
11. Take the average for (N) value from each 1.5 m to obtain the final Standard Penetration Number.
12. Split Spoon samples are taken at intervals (1.5m) because theses samples are highly disturbed.

Drilling Machine



Correction to N value

There are several factors contribute to the variation of the standard penetration number (N) at a given depth for similar profiles. Among these factors are the SPT hammer efficiency, borehole diameter, sampling method, and rod length.

In the field, the magnitude of hammer efficiency can vary from 30 to 90%, the standard practice now is to express the N-value to an average energy ratio of 60% (N_{60}) (but we assume it 100%), so correcting for field procedures is required as following:

$$N_{60} = \frac{N\eta_H\eta_B\eta_S\eta_R}{60}$$

N= measured penetration number.

N_{60} = standard penetration number, corrected for the field conditions.

η_H = hammer efficiency (%).

η_B = correction for borehole diameter.

η_S = sampler correction.

η_R = correction for rod lenght.

Variations of $\eta_H, \eta_B, \eta_S,$ and η_R are summarized in table 2.5 (page 84).

Note: take $\eta_H = 0.6$ (US safety hammer).

Correlations for N_{60}:

N_{60} can be used for calculating some important parameters such as:

- ✓ Undrained shear strength (C_u) (page 84 in text book).
- ✓ Overconsolidation ratio (OCR) (page 85).
- ✓ Angle of internal friction (ϕ) (page 88).
- ✓ Relative Density (D_r)(page 87).
- ✓ Allowable bearing capacity ($q_{all,net}$) and Settlement (S_e)(Ch. 5 page 263).

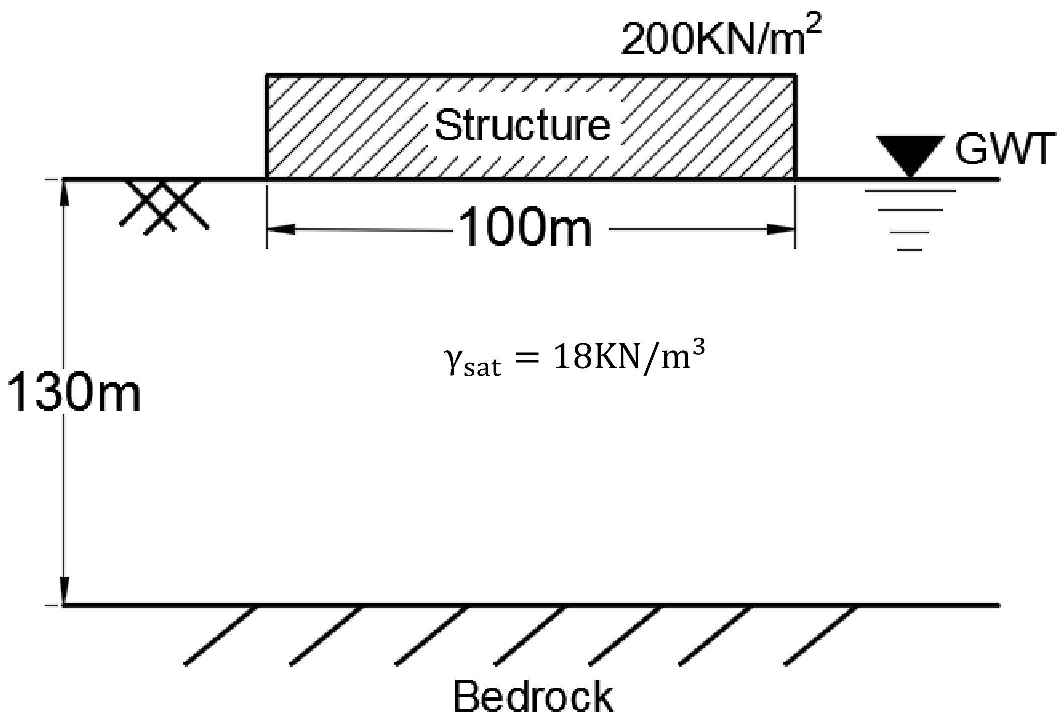
Soil Report

Different soil reports will be discussed on the lecture.

Problems:

1.

Site investigation is to be made for a structure of **100m** length and **70m** width. The soil profile is shown below, if the structure is subjected to **200 KN/m^2** what is the approximate **depth of borehole** (Assume γ_w **=10KN/m^3**).



Solution

Givens:

$q = 200KN/m^2$, structure dimensions $= (70 \times 100)m$

$\rightarrow P = 200 \times (100 \times 70) = 1.4 \times 10^6$ KN.

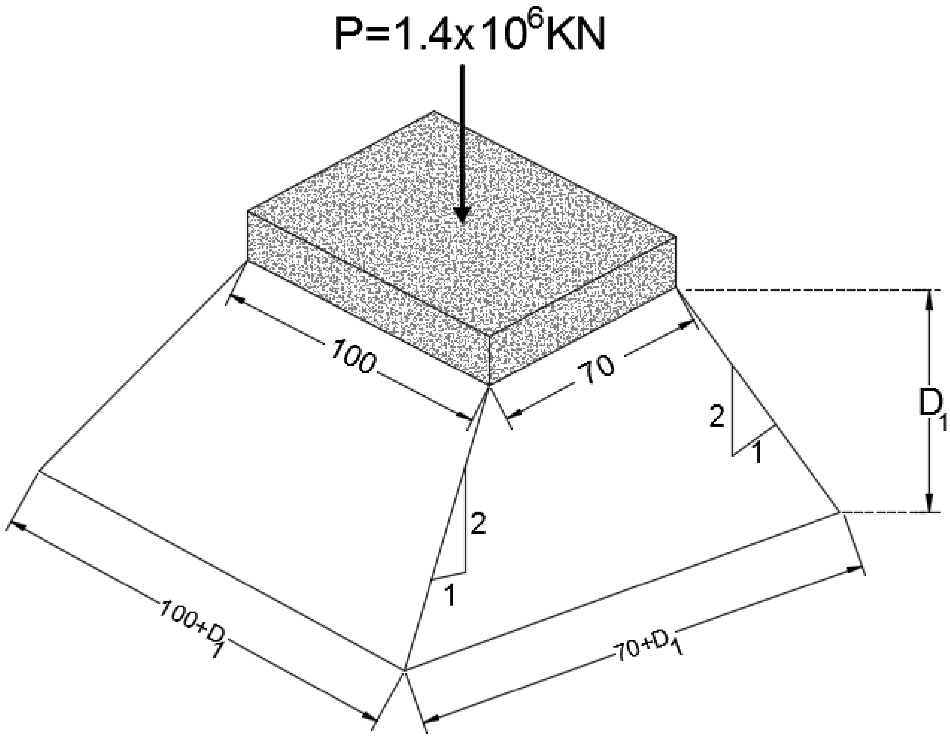
$D_f = 0.0$ (Structure exist on the ground surface) , $\gamma_{sat} = 18KN/m^3$.

$D_3 = 130m$ (distance from the lower face of structure to the bedrock).

1. Calculating the depth (D_1) at which $\Delta\sigma'_{D_1} = \left(\frac{1}{10}\right) \times q$:

$\left(\frac{1}{10}\right) \times q = \left(\frac{1}{10}\right) \times 200 = 20KN/m^2$.

The following figure showing the distribution of stress under the structure at depth (D_1):



The increase in vertical stress ($\Delta\sigma'$) at depth (D_1) is calculated as follows:

$$\Delta\sigma'_{D_1} = \frac{P}{A} = \frac{1.4 \times 10^6}{(100 + D_1) \times (70 + D_1)}$$

$$@\ D_1 \rightarrow \Delta\sigma' = \left(\frac{1}{10}\right) \times q \rightarrow \frac{1.4\times10^6}{(100+D_1)\times(70+D_1)} = 20 \rightarrow D_1 = 180 \text{ m.}$$

2. Calculating the depth (D_2) at which $\left(\frac{\Delta\sigma'}{\sigma'_o}\right) = \mathbf{0.05}$:

The effective stress(σ'_o) at depth D_2 is calculated as following:

$$\sigma'_{o,D_2} = (\gamma_{sat} - \gamma_w) \times D_2$$

$$\rightarrow \sigma'_{o,D_2} = (18 - 10) \times D_2 \rightarrow \sigma'_{o,D_2} = 8D_2.$$

The increase in vertical stress ($\Delta\sigma'$) at depth (D_2) is calculated as follows:

$$\Delta\sigma'_{D_2} = \frac{P}{A} = \frac{1.4 \times 10^6}{(100 + D_2) \times (70 + D_2)}$$

$$@\ D_2 \rightarrow \left(\frac{\Delta\sigma'}{\sigma'_o}\right) = 0.05 \rightarrow \frac{1.4\times10^6}{(100+D_2)\times(70+D_2)} = 0.05 \times (8D_2) \rightarrow D_2 = 101.4 \text{ m}$$

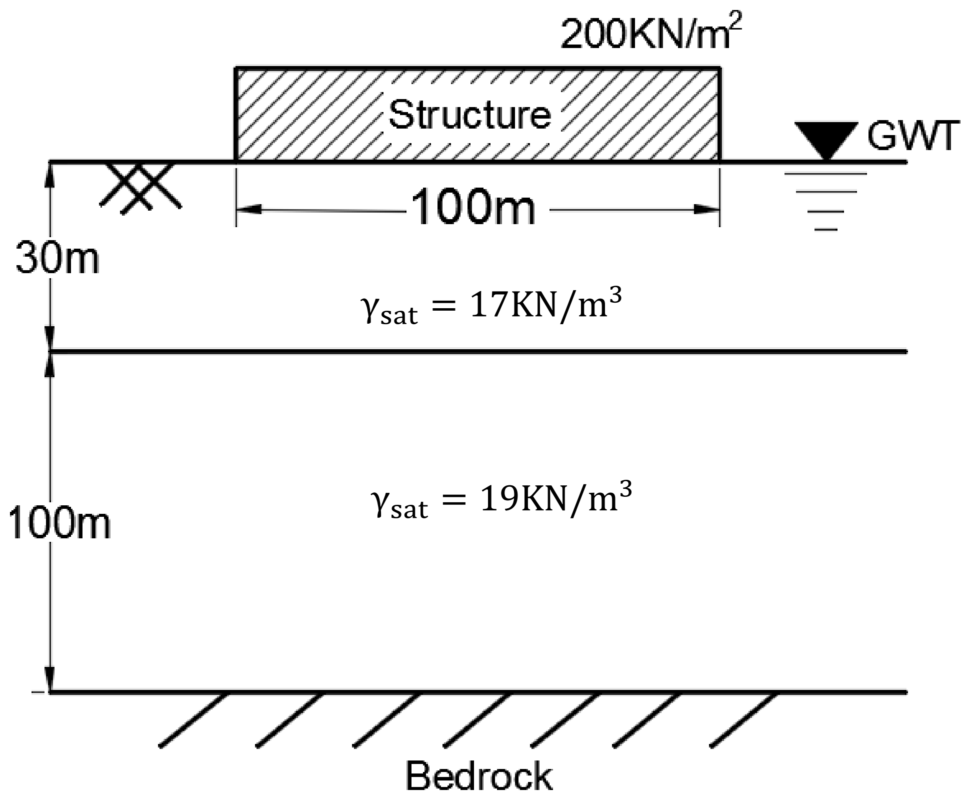
So, the value of (D) is the smallest value of D_1, D_2, and $D_3 \rightarrow D = D_2 = 101.4$ m.

$$\rightarrow D_{boring} = D_f + D \rightarrow D_{boring} = 0.0 + 101.4 = 101.4 \text{ m} ✓.$$

2. (Mid 2005)

Site investigation is to be made for a structure of **100m** length and **70m** width. The soil profile is shown below. Knowing that the structure exerts a uniform pressure of **200 KN/m^2** on the surface of the soil, and the load transports in the soil by **2V:1H** slope.

<u>What is the approximate depth of borehole?</u> (Assume $\boldsymbol{\gamma_w}$ **=10KN/m^3**).



Solution

Givens:

$q = 200\text{KN/m}^2$, structure dimensions $= (70 \times 100)\text{m}$

$\rightarrow P = 200 \times (100 \times 70) = 1.4 \times 10^6$ KN.

$D_f = 0.0$ (Structure exist on the ground surface).

$D_3 = 130\text{m}$ (distance from the lower face of structure to the bedrock).

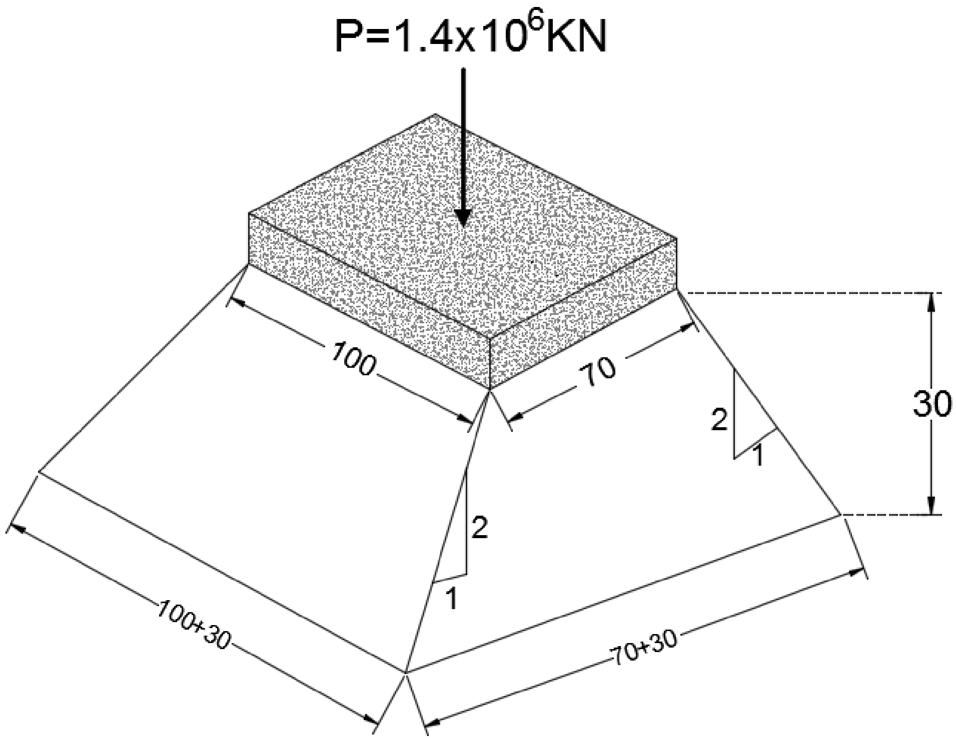
1. Check if ($\mathbf{D_1}$<30m or $\mathbf{D_1}$>30m):

@ depth D=30 m if $\Delta\sigma' < \left(\frac{1}{10}\right) \times q \rightarrow D_1 < 30\text{m}, \text{else} D_1 > 30\text{m} \rightarrow\rightarrow$

Because the magnitude of $(\Delta\sigma')$ **decreased** with depth.

$\left(\frac{1}{10}\right) \times \mathrm{q} = \left(\frac{1}{10}\right) \times 200 = 20\mathrm{KN/m^2}.$

The following figure showing the distribution of stress under the structure at depth (30m):



The increase in vertical stress ($\Delta\sigma'$) at depth (30m) is calculated as follows:

$$\Delta\sigma'_{30\mathrm{m}} = \frac{\mathrm{P}}{\mathrm{A}} = \frac{1.4 \times 10^6}{(100 + 30) \times (70 + 30)} = 107.7\ \mathrm{KN/m^2}.$$

$$\rightarrow \Delta\sigma'_{30\mathrm{m}} > \left(\frac{1}{10}\right) \times \mathrm{q} \rightarrow \mathrm{D_1} > 30\mathrm{m}.$$

2. Calculating the depth (D_1) at which $\Delta\sigma'_{D_1} = \left(\frac{1}{10}\right) \times q$:

$\left(\frac{1}{10}\right) \times \mathrm{q} = \left(\frac{1}{10}\right) \times 200 = 20\mathrm{KN/m^2}.$

The increase in vertical stress ($\Delta\sigma'$) at depth ($\mathrm{D_1}$) is calculated as follows:

$$\Delta\sigma'_{\mathrm{D_1}} = \frac{\mathrm{P}}{\mathrm{A}} = \frac{1.4 \times 10^6}{(100 + \mathrm{D_1}) \times (70 + \mathrm{D_1})}$$

$$@\ \mathrm{D_1} \rightarrow \Delta\sigma' = \left(\frac{1}{10}\right) \times \mathrm{q} \rightarrow \frac{1.4\times10^6}{(100+\mathrm{D_1})\times(70+\mathrm{D_1})} = 20 \rightarrow \mathrm{D_1} = 180\ \mathrm{m}.$$

3. Check if (D_2<30m or D_2>30m):

@ depth D=30 m if $\left(\frac{\Delta\sigma'}{\sigma'_o}\right) < 0.05 \rightarrow D_2 < 30\text{m}, \text{else} D_2 > 30\text{m} \rightarrow\rightarrow$

Because the magnitude of $\left(\frac{\Delta\sigma'}{\sigma'_o}\right)$ **decreased** with depth.

$\Delta\sigma'_{30m} = 107.7\ \text{KN/m}^2$(as calculated above).

The effective stress at depth (30m) is calculated as follows:

$\sigma'_{o,30m} = (\gamma_{sat} - \gamma_w) \times 30$

$\rightarrow \sigma'_{o,30m} = (17 - 10) \times 30 \rightarrow \sigma'_{o,30m} = 210\text{KN/m}^2$

$$\rightarrow \left(\frac{\Delta\sigma'}{\sigma'_o}\right) = \left(\frac{107.7}{210}\right) = 0.51 > 0.05 \rightarrow D_2 > 30\text{m}.$$

4. Calculating the depth (D_2) at which $\left(\frac{\Delta\sigma'}{\sigma'_o}\right) = \mathbf{0.05}$:

Let $D_2 = 30 + X$ (X: distance from layer (2) to reach (D_2).

The effective stress(σ'_o) at depth D_2 is calculated as following:

$\sigma'_{o,D_2} = (17 - 10) \times 30 + (19 - 10) \times X$

$\rightarrow \sigma'_{o,D_2} = 210 + 9X.$

The increase in vertical stress ($\Delta\sigma'$) at depth (D_2) is calculated as follows:

$$\Delta\sigma'_{D_2} = \frac{P}{A} = \frac{1.4 \times 10^6}{(100 + D_2) \times (70 + D_2)}\ \text{, but } D_2 = 30 + X \rightarrow\rightarrow$$

$$\Delta\sigma'_{D_2} = \frac{1.4 \times 10^6}{(130 + X) \times (100 + X)}$$

$$@\ D_2 \rightarrow \left(\frac{\Delta\sigma'}{\sigma'_o}\right) = 0.05 \rightarrow \frac{1.4\times10^6}{(130+X)\times(100+X)} = 0.05 \times (210 + 9X)$$

$\rightarrow X = 69\text{ m} \rightarrow D_2 = 69 + 30 = 99\text{ m}$

So, the value of (D) is the smallest value of D_1, D_2, and $D_3 \rightarrow D = D_2 = 99$ m.

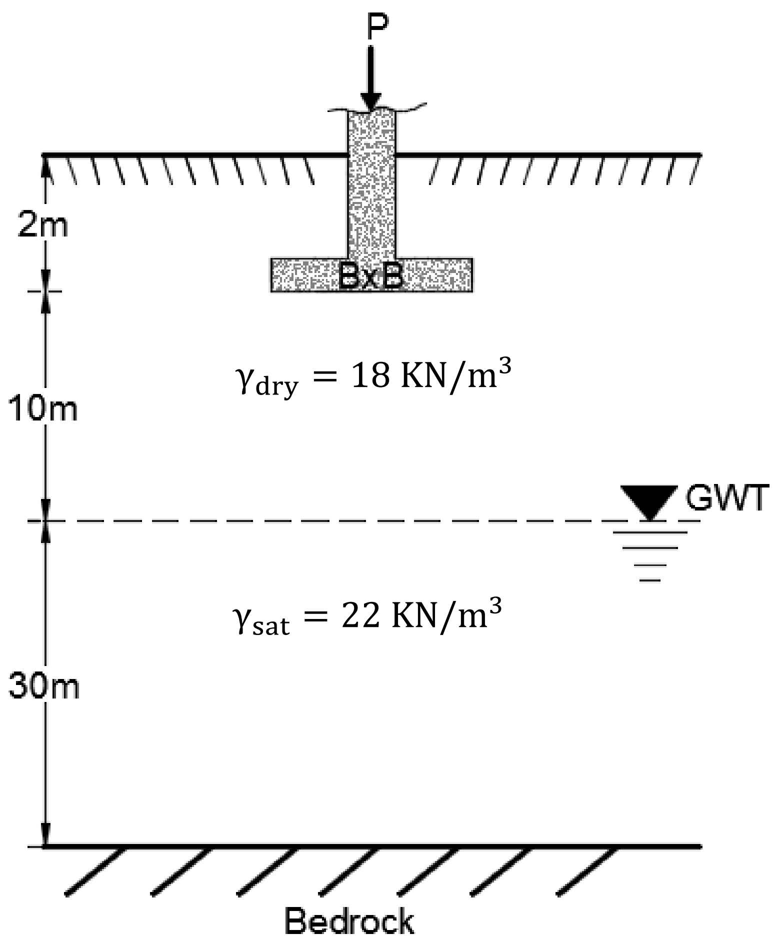
$\rightarrow D_{boring} = D_f + D \rightarrow D_{boring} = 0.0 + 99 = 99\text{ m}$ ✓.

3. (Mid 2013)

For the soil profile shown below, if $\mathbf{D_1=10m}$ and $\mathbf{D_2=2D_1}$.

A- Determine the dimensions of the foundation to achieve the required depth of borehole.

B- Calculate the load of column which should be applied on the foundation to meet the required depth of boring.



Solution

Givens:

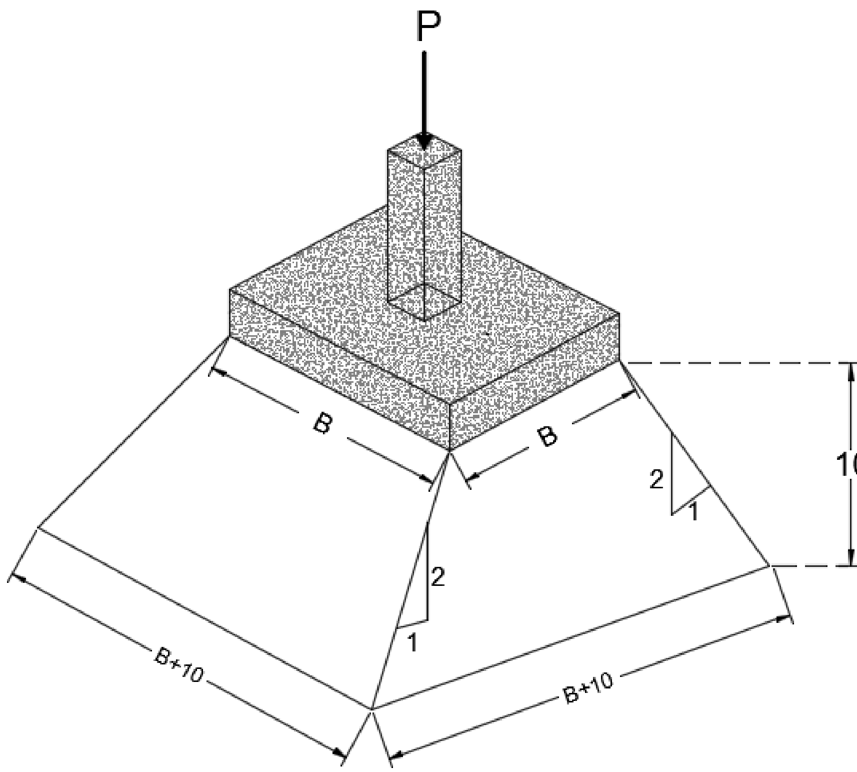
$D_1 = 10m \quad , \quad D_2 = 2D_1 \ \rightarrow \ D_2 = 2 \times 10 = 20m \quad , \quad D_f = 2m$

$D_3 = 40m$ (distance from the lower face of foundation to the bedrock)

A. (B=??)

$$@D_1 \rightarrow \Delta\sigma'_{D_1} = \left(\frac{1}{10}\right) \times q$$

The following figure showing the distribution of stress under the structure at depth (D_1=10m):



The increase in vertical stress ($\Delta\sigma'$) at depth ($D_1 = 10$m) is calculated as follows:

$$\Delta\sigma'_{D_1} = \frac{P}{A} = \frac{P}{(B+10)\times(B+10)} \rightarrow \boxed{\Delta\sigma'_{D_1} = \frac{P}{(B+10)^2}} \longrightarrow \text{Eq.1}$$

$$q = \frac{P}{A} = \frac{P}{(B\times B)} \rightarrow \boxed{\left(\frac{1}{10}\right) q = \frac{P}{10(B\times B)}} \longrightarrow \text{Eq.2}$$

By equal 1&2→ $\frac{P}{(B+10)^2} = \frac{P}{10B^2} \rightarrow B = 4.62\text{m}\checkmark.$

B. (P=??)

$D_2 = 2D_1 \rightarrow D_2 = 2\times 10 = 20\text{m}\ ,\ B = 4.62\text{m}$

$@D_2 \rightarrow \Delta\sigma'_{D_2} = 0.05\times\sigma'_{o,D_2}$

$$\Delta\sigma'_{D_2} = \frac{P}{A} = \frac{P}{(B+20)\times(B+20)} \rightarrow \boxed{\Delta\sigma'_{D_2} = \frac{P}{(4.62+20)^2}} \longrightarrow \text{Eq.1}$$

The effective stress(σ'_o) at depth (D_2=20m) is calculated as following:

σ'_o is calculated from the **ground surface**

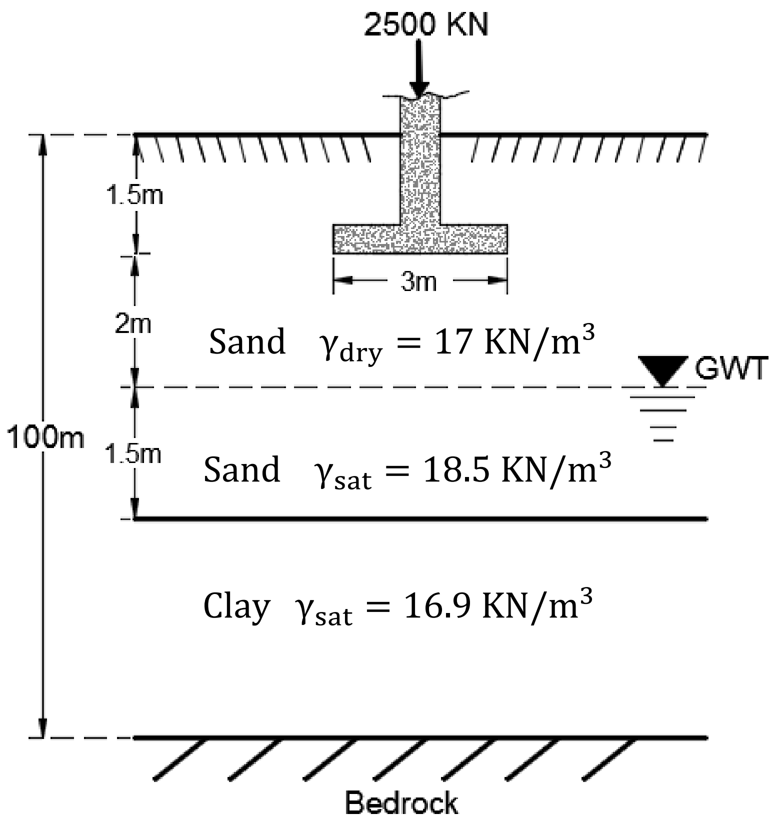
$$\sigma'_{o,D_2} = 18 \times 2 + 18 \times 10 + (22 - 10) \times 10 = 336 KN/m^2$$

$$@D_2 \rightarrow \Delta\sigma'_{D_2} = 0.05 \times \sigma'_{o,D_2} \rightarrow \frac{P}{(4.62 + 20)^2} = 0.05 \times 336$$

$$\rightarrow P = 10{,}183.2\ KN \checkmark.$$

4.

Site investigation is to be made for **2500 KN** load carried on **(3.0 m x 2.0 m)** footing. The foundation will be built on layered soil as shown in the figure below, estimate the depth of bore hole. (Assume γ_w= **10KN/m^3**).



Solution

Givens:

$P = 2500\ KN$, foundation dimensions $= (3 \times 2)m$

$$q = \frac{P}{A} = \frac{2500}{3 \times 2} = 416.67\ KN/m^2 \ ,\ D_f = 1.5m$$

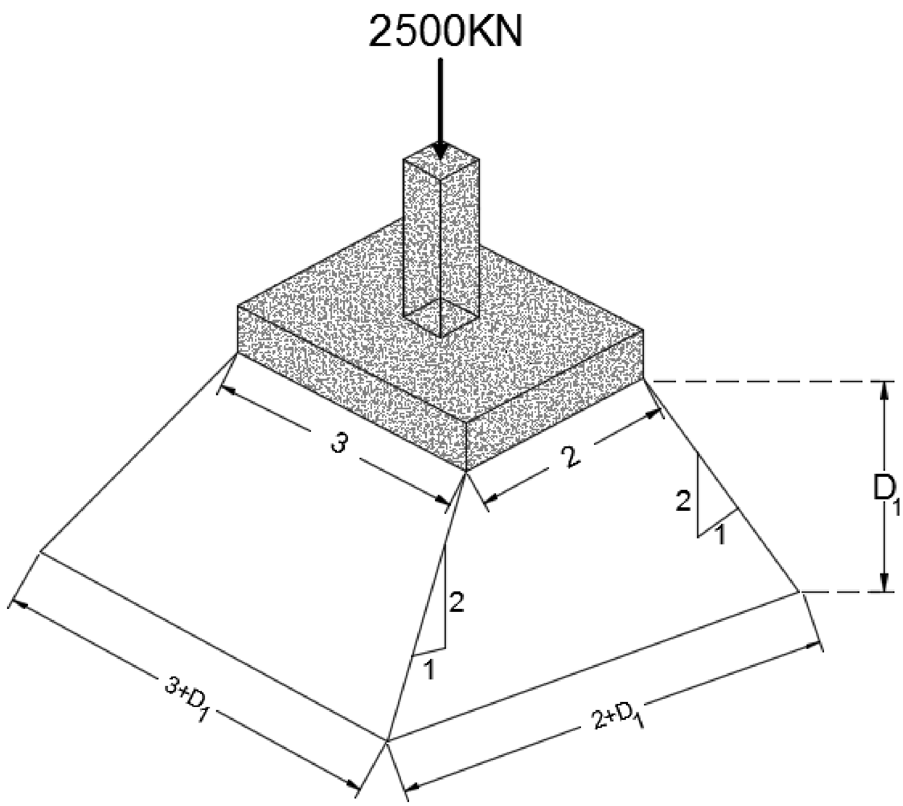
$$D_3 = 100 - 1.5 = 98.5m$$

Without check, it's certainly the values of D_1 & $D_2 > 3.5$m, but if you don't sure you should do the check at every change in soil profile (like problem 2).

1. Calculating the depth (D_1) at which $\Delta\sigma'_{D_1} = \left(\frac{1}{10}\right) \times q$:

$\left(\frac{1}{10}\right) \times q = \left(\frac{1}{10}\right) \times 416.67 = 41.67 KN/m^2$.

The following figure showing the distribution of stress under the foundation at depth (D_1):



The increase in vertical stress ($\Delta\sigma'$) at depth (D_1) is calculated as follows:

$$\Delta\sigma'_{D_1} = \frac{P}{A} = \frac{2500}{(3 + D_1) \times (2 + D_1)}$$

$$@\ D_1 \rightarrow \Delta\sigma' = \left(\frac{1}{10}\right) \times q \rightarrow \frac{2500}{(3+D_1)\times(2+D_1)} = 41.67 \rightarrow D_1 = 5.26\ m.$$

3. Calculating the depth (D_2) at which $\left(\frac{\Delta\sigma'}{\sigma'_o}\right) = 0.05$:

Let $D_2 = 3.5 + X$ (X: distance from the clay layer to reach(D_2).

The effective stress(σ'_o) at depth D_2 is calculated as following:

$$\sigma'_{o,D_2} = 17 \times 1.5 + 17 \times 2 + (18.5 - 10) \times 1.5 + (16.9 - 10) \times X$$

$\rightarrow \sigma'_{o,D_2} = 72.25 + 6.9X \rightarrow \sigma'_{o,D_2} = 72.25 + 6.9 \times (D_2 - 3.5)$

$\rightarrow \sigma'_{o,D_2} = 48.1 + 6.9D_2$

The increase in vertical stress ($\Delta\sigma'$) at depth (D_2) is calculated as follows:

$$\Delta\sigma'_{D_2} = \frac{P}{A} = \frac{2500}{(3 + D_2) \times (2 + D_2)}$$

$@\ D_2 \rightarrow \left(\frac{\Delta\sigma'}{\sigma'_o}\right) = 0.05 \rightarrow \frac{2500}{(3+D_2)\times(2+D_2)} = 0.05 \times (48.1 + 6.9D_2)$

$\rightarrow D_2 = 15.47\ \text{m}$

So, the value of (D) is the smallest value of D_1, D_2, and $D_3 \rightarrow D = D_1 = 5.26$ m.

$\rightarrow D_{boring} = D_f + D \rightarrow D_{boring} = 1.5 + 15.26 = 6.76\ \text{m}$ ✓.

Chapter (3)
Ultimate Bearing Capacity of Shallow Foundations

Introduction

To perform satisfactorily, shallow foundations must have two main characteristics:

1. They have to be safe against **overall shear failure** in the soil that supports them.
2. They cannot undergo excessive displacement, or excessive settlement.

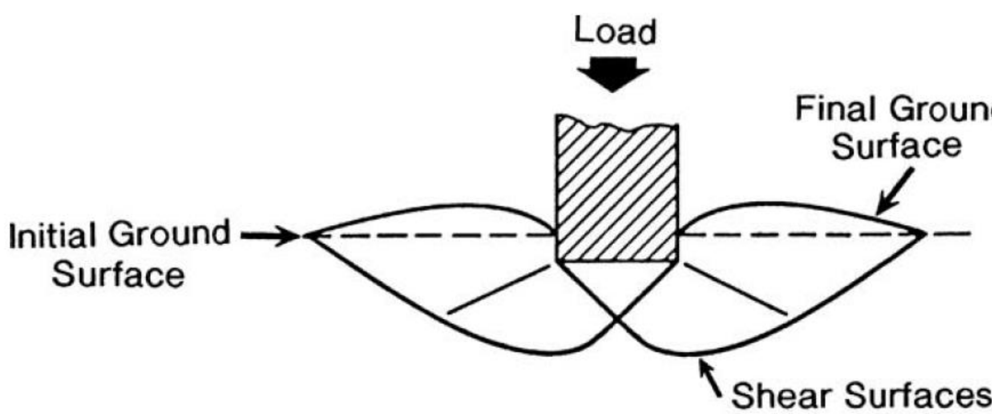
Note: The term excessive settlement is relative, because the degree of settlement allowed for a structure depends on several considerations.

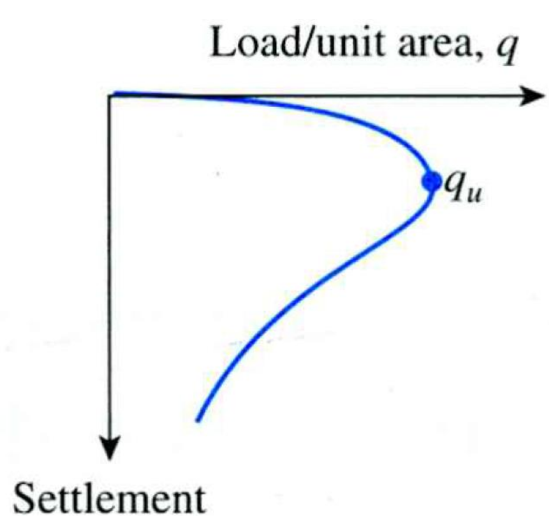
Types of Shear Failure

Shear Failure: Also called "Bearing capacity failure" and it's occur when the shear stresses in the soil exceed the shear strength of the soil.

There are three types of shear failure in the soil:

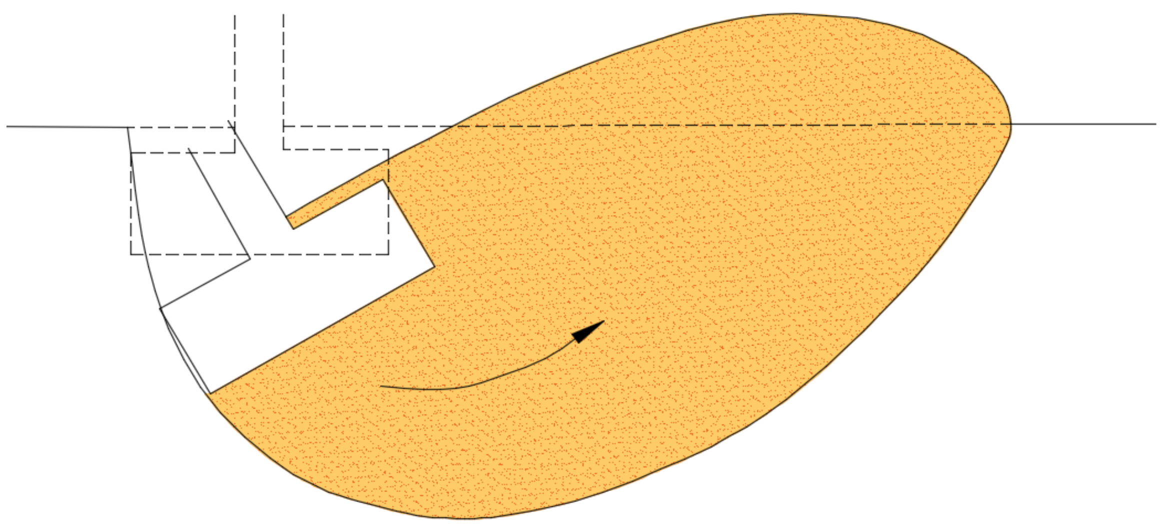
1. General Shear Failure



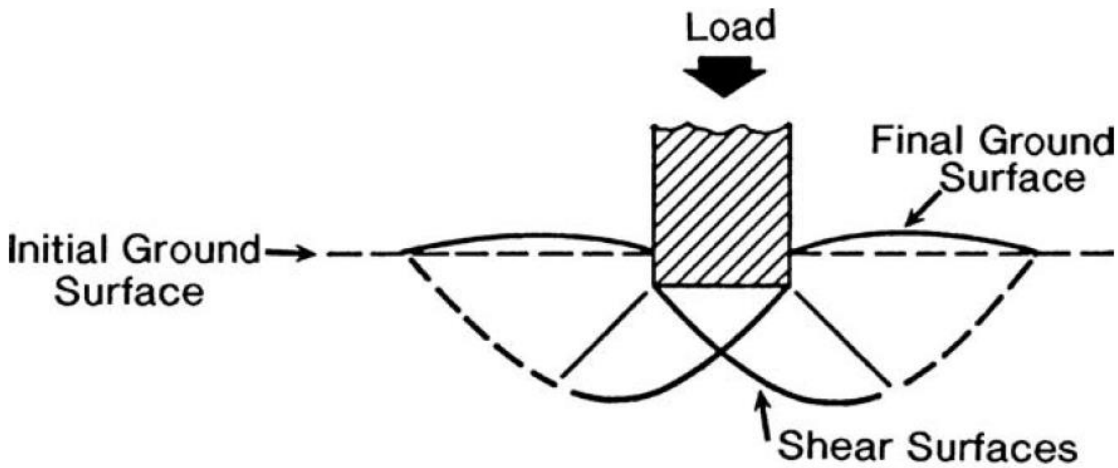


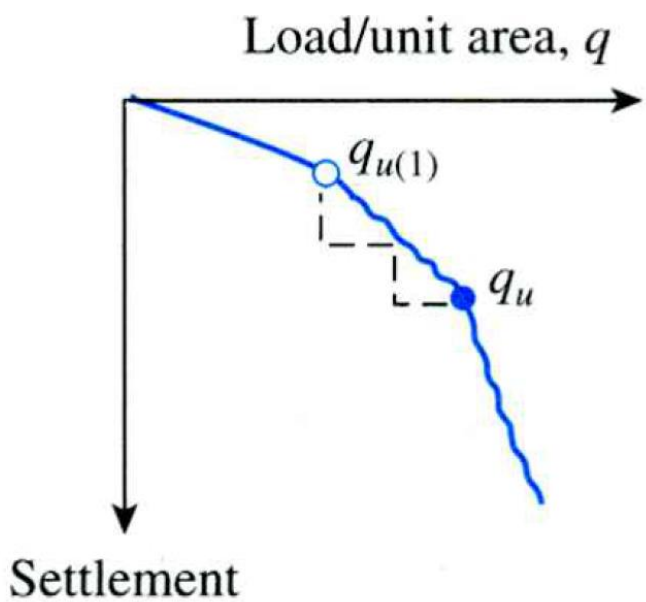
The following are some characteristics of general shear failure:

✓ Occurs over dense sand or stiff cohesive soil.
✓ Involves total rupture of the underlying soil.
✓ There is a continuous shear failure of the soil from below the footing to the ground surface (solid lines on the figure above).
✓ When the (load / unit area) plotted versus settlement of the footing, there is a distinct load at which the foundation fails (Q_u)
✓ The value of (Q_u) divided by the area of the footing is considered to be the ultimate bearing capacity of the footing(q_u).
✓ For general shear failure, the ultimate bearing capacity has been defined as the bearing stress that causes **a sudden** catastrophic failure of the foundation.
✓ As shown in the above figure, a general shear failure ruptures occur and pushed up the soil on both sides of the footing (In laboratory).
✓ However, for actual failures on the field, the soil is often pushed up on **only one side** of the footing with **subsequent tilting** of the structure as shown in figure below:



2. **Local Shear Failure:**





The following are some characteristics of local shear failure:

✓ Occurs over sand or clayey soil of medium compaction.
✓ Involves rupture of the soil only immediately below the footing.
✓ There is soil bulging (انتفاخ او بروز) on both sides of the footing, but the bulging is not as significant as in general shear. That's because the underlying soil compacted less than the soil in general shear.
✓ The failure surface of the soil will **gradually** (not sudden) extend outward from the foundation (not the ground surface) as shown by **solid lines** in the above figure.

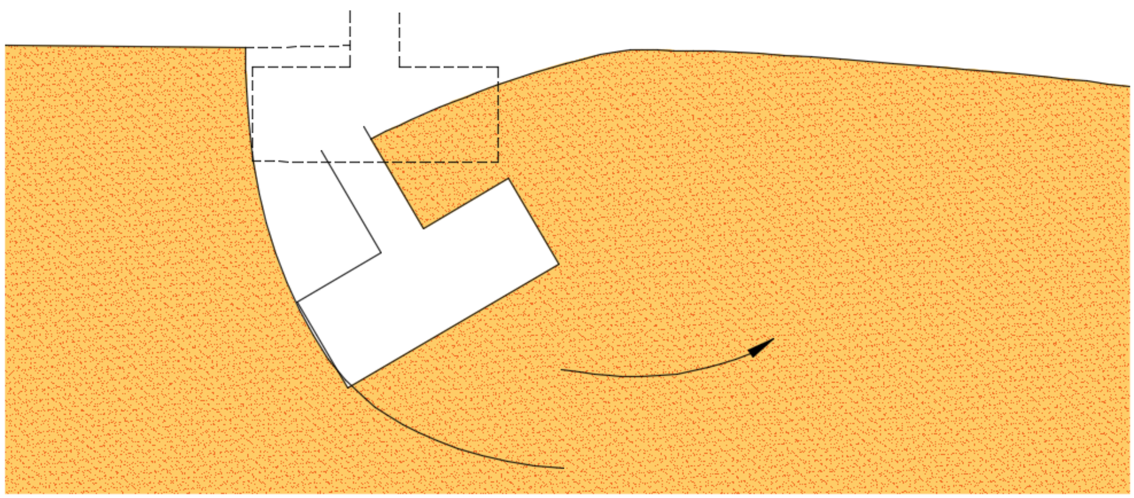
✓ So, local shear failure can be considered as a transitional phase between general shear and punching shear.

✓ Because of the transitional nature of local shear failure, the ultimate bearing capacity could be defined as the firs failure load ($q_{u,1}$) which occur at the point which have the first measure nonlinearity in the load/unit area-settlement curve (open circle), or at the point where the settlement starts rabidly increase (q_u) (closed circle).

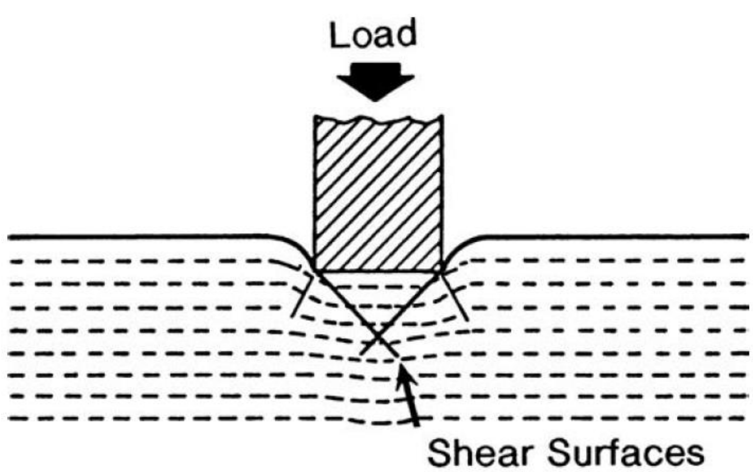
✓ This value of (q_u) is the required (load/unit area) to extends the failure surface to the ground surface (dashed lines in the above figure).

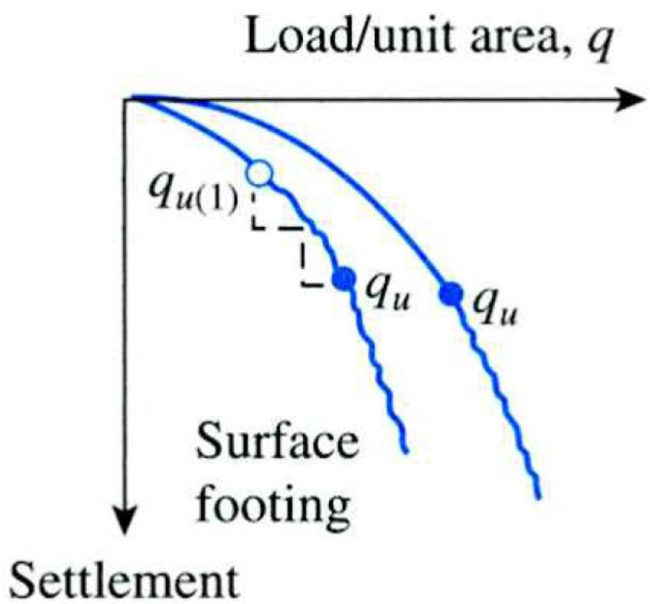
✓ In this type of failure, the value of (q_u) it's not the peak value so, this failure called (Local Shear Failure).

✓ The actual local shear failure in field is proceed as shown in the following figure:



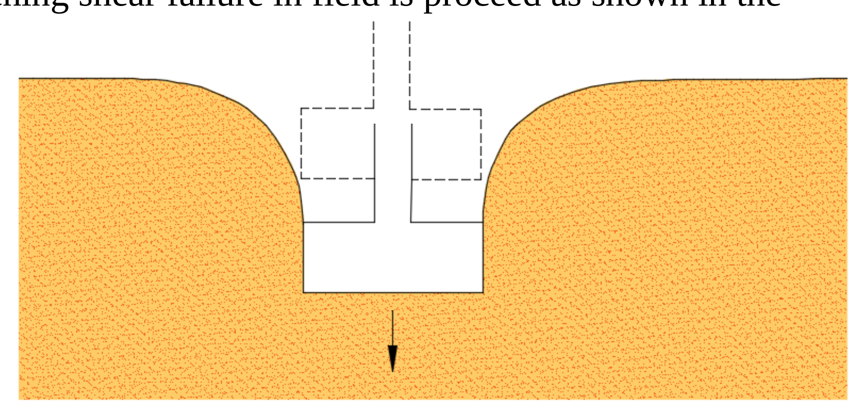
3. Punching Shear Failure:





The following are some characteristics of punching shear failure:

- ✓ Occurs over fairly loose soil.
- ✓ Punching shear failure does not develop the distinct shear surfaces associated with a general shear failure.
- ✓ The soil outside the loaded area remains relatively uninvolved and there is a minimal movement of soil on both sides of the footing.
- ✓ The process of deformation of the footing involves compression of the soil directly below the footing as well as the vertical shearing of soil around the footing perimeter.
- ✓ As shown in figure above, the (q)-settlement curve does not have a dramatic break (تغير مفاجئ), and the bearing capacity is often defined as the first measure nonlinearity in the (q)-settlement curve($q_{u,1}$).
- ✓ Beyond the ultimate failure (load/unit area) ($q_{u,1}$), the (load/unit area)-settlement curve will be steep and practically linear.
- ✓ The actual punching shear failure in field is proceed as shown in the following figure:



Ultimate Bearing Capacity (q_u)

It's the **minimum load per unit area** of the foundation that causes shear failure in the underlying soil.
Or, it's the **maximum load per unit area** of the foundation can be resisted by the underlying soil without occurs of shear failure (if this load is exceeded, the shear failure will occur in the underlying soil).

Allowable Bearing Capacity (q_{all})

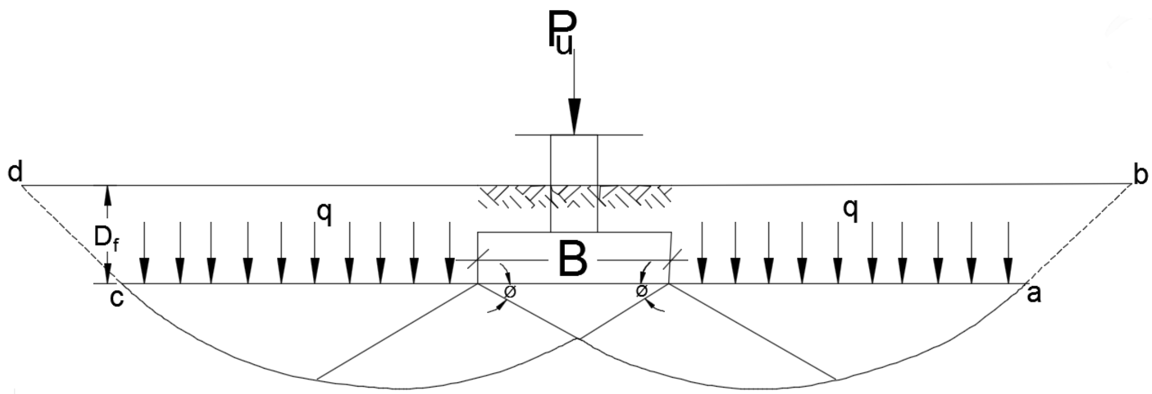
It's the load per unit area of the foundation can be resisted by the underlying soil without any unsafe movement occurs (shear failure) and if this load is exceeded, the shear failure **will not** occur in the underlying soil **till** reaching the ultimate load.

Terzaghi's Bearing Capacity Theory

Terzaghi was the first to present a comprehensive theory for evaluation of the ultimate bearing capacity of rough shallow foundation. **This theory is based on the following assumptions:**

1. The foundation is considered to be sallow if ($D_f \leq B$).
2. The foundation is considered to be strip or continuous if ($\frac{B}{L} \rightarrow 0.0$).
(Width to length ratio is very small and goes to zero), and the derivation of the equation is to a strip footing.
3. The effect of soil above the bottom of the foundation may be assumed to be replaced by an equivalent surcharge ($q = \gamma \times D_f$). So, the shearing resistance of this soil along the failure surfaces is neglected (Lines ab and cd in the below figure)
4. The failure surface of the soil is similar to **general shear failure** (i.e. equation is derived for general shear failure) as shown in figure below.

Note:
1. In recent studies, investigators have suggested that, foundations are considered to be shallow if [$D_f \leq (3 \rightarrow 4)B$], otherwise, the foundation is deep.
2. Always the value of (q) is the **effective stress** at the bottom of the foundation.



Terzaghi's Bearing Capacity Equations

As mentioned previously, the equation was derived for a **strip footing** and **general shear failure,** this equation is:

$$q_u = cN_c + qN_q + 0.5B\gamma N_\gamma \text{ (for continuous or strip footing)}$$

Where

q_u = Ultimate bearing capacity of the **underlying** soil (KN/m^2)
c = Cohesion of **undelying** soil (KN/m^2)
q = **Efeective** stress at the bottom of the foundation (KN/m^2)
N_c, N_q, N_γ = Bearing capacity factors (nondimensional)and are functions **only** of the **underlying** soil friction angle, ϕ, →→
The variations of bearing capacity factors and underlying soil friction angle are given in **(Table 3.1, P.139) for general shear failure.**

The above equation (for strip footing) was modified to be useful for both **square** and **circular footings** as following:

For square footing:
$$q_u = 1.3cN_c + qN_q + 0.4B\gamma N_\gamma$$
B = The dimension of each side of the foundation .

For circular footing:
$$q_u = 1.3cN_c + qN_q + 0.3B\gamma N_\gamma$$
B = The diameter of the foundation .

Note:

These two equations are also for **general shear failure,** and all factors in the two equations (except, B,) are the same as explained for strip footing.

Now for **local shear failure** the above three equations were modified to be useful for local shear failure as following:

$$q_u = \frac{2}{3}cN'_c + qN'_q + 0.5B\gamma N'_\gamma \quad \text{(for continuous or strip footing)}$$

$$q_u = 0.867cN'_c + qN'_q + 0.4B\gamma N'_\gamma \quad \text{(for square footing)}$$

$$q_u = 0.867cN'_c + qN'_q + 0.3B\gamma N'_\gamma \quad \text{(for circular footing)}$$

N'_c, N'_q, N'_γ = Modified bearing capacity factors and could be determined by the following **two methods:**

1. **(Table 3.2 P.140)** variations of modified bearing capacity factors and **underlying** soil friction angle.
2. **[(Table 3.1 P.139)(if you don't have Table 3.2)],** variation of bearing capacity factors and **underlying** soil friction angle, **but you must** do the following **modification** for the underlying soil friction angle:

$$\tan\phi\ \text{(General Shear)} = \frac{2}{3} \times \tan\phi\ \text{(Local Shear)} \rightarrow\rightarrow$$

$$\phi_{\text{modified,general}} = \tan^{-1}\left(\frac{2}{3}\tan\phi_{\text{local}}\right)$$

For example: Assume we have local shear failure and the value of $\phi = 30^\circ$

1. By using (**Table 3.2**) $N'_c, N'_q, N'_\gamma = 18.99, 8.31, \text{and } 4.9$ respectively

2. By using (**Table 3.1**) $\rightarrow \phi_{\text{modified,general}} = \tan^{-1}\left(\frac{2}{3}\tan 30^\circ\right) = 21.05^\circ \rightarrow$

$(N_c, N_q, N_\gamma)_{21.05,\text{table } 3.1} \cong (N'_c, N'_q, N'_\gamma)_{30,\text{table } 3.2} = 18.92, 8.26, \text{and } 4.31$ respectively

General Bearing Capacity Equation (Meyerhof Equation)

Terzagi's equations shortcomings:

- They don't deal with rectangular foundations ($0 < \frac{B}{L} < 1$).
- The equations do not take into account the shearing resistance along the failure surface in soil above the bottom of the foundation (as mentioned previously).
- The inclination of the load on the foundation is not considered (if exist).

To account for all these shortcomings, Meyerhof suggested the following form of the **general** bearing capacity equation:

$$q_u = cN_cF_{cs}F_{cd}F_{ci} + qN_qF_{qs}F_{qd}F_{qi} + 0.5B\gamma N_\gamma F_{\gamma s}F_{\gamma d}F_{\gamma i}$$

Where
c = Cohesion of the underlying soil
q = Effective stress at the level of the bottom of the foundation.
γ = unit weight of the underlying soil
B = Width of footing (= diameter for a circular foundation).
N_c, N_q, N_γ = Bearing capacity factors(will be discussed later).
$F_{cs}, F_{qs}, F_{\gamma s}$ = Shape factors (will be discussed later).
$F_{cd}, F_{qd}, F_{\gamma d}$ = Depth factors (will be discussed later).
$F_{ci}, F_{qi}, F_{\gamma i}$ = Inclination factors (will be discussed later).

Notes:
1. This equation is valid for both general and local shear failure.
2. This equation is similar to original equation for ultimate bearing capacity (Terzaghi's equation) which derived for continuous foundation, but the shape, depth, and load inclination factors are added to this equation (Terzaghi's equation) to be suitable for any case may exist.

Bearing Capacity Factors:
The angle $\alpha = \phi$ (according Terzaghi theory in the last figure "above") was replaced by $\alpha = 45 + \frac{\phi}{2}$. So, the bearing capacity factor will be change.
The variations of bearing capacity factors (N_c, N_q, N_γ) and **underlying** soil friction angle (ϕ) are given in **(Table 3.3, P.144).**

Shape Factors:

$$F_{cs} = 1 + \left(\frac{B}{L}\right)\left(\frac{N_q}{N_c}\right)$$

$$F_{qs} = 1 + \left(\frac{B}{L}\right)\tan\phi$$

$$F_{\gamma s} = 1 - 0.4\left(\frac{B}{L}\right)$$

Notes:

1. If the foundation is continuous or strip $\rightarrow \frac{B}{L} = 0.0$
2. If the foundation is circular$\rightarrow$ $B = L = \text{diameter} \rightarrow \frac{B}{L} = 1$

Depth Factors:

➢ **For $\frac{D_f}{B} \leq 1$**

1. **For $\phi = 0.0$**

$$F_{cd} = 1 + 0.4\left(\frac{D_f}{B}\right)$$

$$F_{qd} = 1$$

$$F_{\gamma d} = 1$$

2. **For $\phi > 0.0$**

$$F_{cd} = F_{qd} - \frac{1 - F_{qd}}{N_c \tan\phi}$$

$$F_{qd} = 1 + 2\tan\phi\,(1 - \sin\phi)^2\left(\frac{D_f}{B}\right)$$

$$F_{\gamma d} = 1$$

➢ **For $\frac{D_f}{B} > 1$**

1. **For $\phi = 0.0$**

$$F_{cd} = 1 + 0.4\,\underbrace{\tan^{-1}\left(\frac{D_f}{B}\right)}_{\textbf{radians}}$$

$$F_{qd} = 1$$

$$F_{\gamma d} = 1$$

2. **For $\phi > 0.0$**

$$F_{cd} = F_{qd} - \frac{1 - F_{qd}}{N_c \tan\phi}$$

$$F_{qd} = 1 + 2\tan\phi\,(1-\sin\phi)^2 \underbrace{\tan^{-1}\left(\frac{D_f}{B}\right)}_{\textbf{radians}}$$

$$F_{\gamma d} = 1$$

Important Notes:

1. If the value of (B) or (D_f)is required, you should do the following:
   - ✓ Assume $\left(\frac{D_f}{B} \le 1\right)$ and calculate depth factors in term of (B) or (D_f).
   - ✓ Substitute in the general equation, then calculate (B) or (D_f).
   - ✓ After calculated the required value, you must check your assumption→ $\left(\frac{D_f}{B} \le 1\right)$.
   - ✓ If the assumption is true, the calculated value is the final required value.
   - ✓ If the assumption is wrong, you must calculate depth factors in case of $\left(\frac{D_f}{B} > 1\right)$ and then calculate (B) or (D_f) to get the true value.
2. For both cases $\left(\frac{D_f}{B} \le 1\right)$ and $\left(\frac{D_f}{B} > 1\right)$ if $\phi > 0 \rightarrow$ calculate F_{qd} firstly, because F_{cd} depends on F_{qd}.

Inclination Factors:

$$F_{ci} = F_{qi} = \left(1 - \frac{\beta^{\circ}}{90}\right)^2$$

$$F_{\gamma i} = \left(1 - \frac{\beta^{\circ}}{\phi^{\circ}}\right)$$

β° = Inclination of the load on the foundation with respect to the **vertical**

Note:

If $\beta^{\circ} = \phi \rightarrow F_{\gamma i} = 0.0$, so you **don't need** to calculate $F_{\gamma s}$ and $F_{\gamma d}$, because the last term in Meyerhof equation will be zero.

Factor of Safety

From previous two equations (Terzaghi and Meyerhof), we calculate the value of **ultimate** bearing capacity (q_u) which the maximum value the soil can bear it (i.e. if the bearing stress from foundation exceeds the ultimate bearing capacity of the soil, shear failure in soil will be occur), so we must design a foundation for a bearing capacity less than the ultimate bearing capacity to prevent shear failure in the soil. This bearing capacity is **"Allowable Bearing Capacity"** and we design for it (i.e. the applied stress from foundation must not exceeds the allowable bearing capacity of soil).

$$q_{all,gross} = \frac{q_{u,gross}}{FS} \rightarrow\rightarrow \text{Applied stress} \leq q_{all,gross} = \frac{q_{u,gross}}{FS}$$

$q_{all,gross}$ = Gross allowable bearing capacity

$q_{u,gross}$ = Gross ultimate bearing capacity (Terzaghi or Meyerhof equations)

FS = Factor of safety for bearing capacity ≥ 3

However, practicing engineers prefer to use the "**net** allowable bearing capacity" such that:

$$q_{all,net} = \frac{q_{u,net}}{FS}$$

$q_{u,net}$ = Net ultimate bearing capacity, and it's the difference between the gross ultimate bearing capacity (upward as soil reaction) and the weight of the soil and foundation at the foundation level (downward), to get the net pressure from the soil that support the foundation.

$$q_{u,net} = q_{u,gross} - \gamma_c h_c - \gamma_s h_s$$

Since the unit weight of concrete and soil are convergent, then

$$q_{u,net} = q_{u,gross} - q \rightarrow q_{all,net} = \frac{q_{u,gross} - q}{FS}$$

q = Effective stress at the level of foundation level.

If we deal with loads (Q)

$$q_{u,gross} = \frac{Q_{u,gross}}{Area} \xrightarrow{\div FS} q_{all,gross} = \frac{Q_{all,gross}}{Area}$$

Modification of Bearing Capacity Equations for Water Table

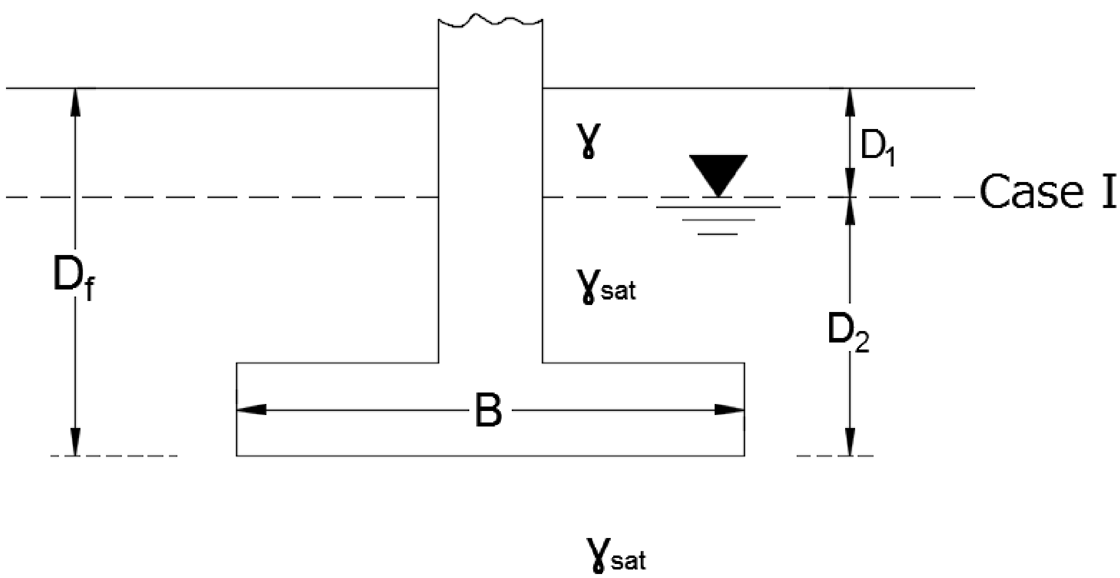
Terzaghi and Meyerhof equations give the ultimate bearing capacity based on the assumption that the water table is located well below the foundation. However, if the water table is close to the foundation, the bearing capacity will decreases due to the effect of water table, so, some modification of the bearing capacity equations (Terzaghi and Meyerhof) will be necessary.

The values which will be modified are:

1. (q for soil above the foundation) in the second term of equations.
2. (γ for the underlying soil) in the third (last) term of equations .

There are three cases according to location of water table:

Case I. The water table is located so that $0 \le D_1 \le D_f$ as shown in the following figure:



✓ The factor ,q, (second term) in the bearing capacity equations will takes the following form: (For the soil above the foundation)

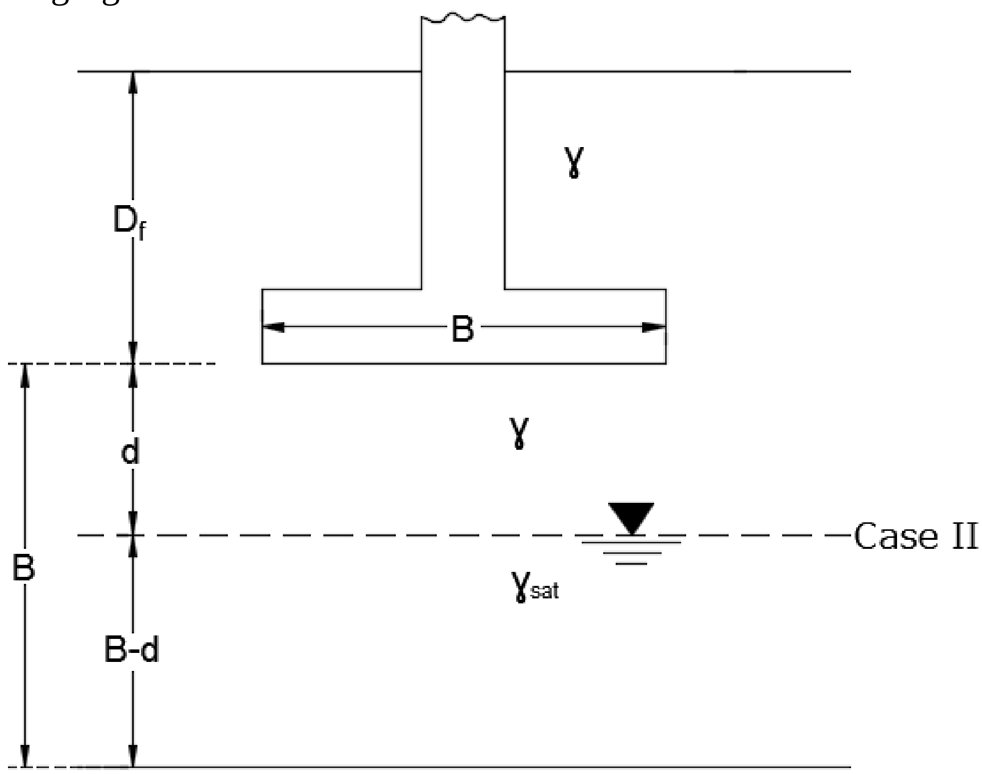
$q =$ effective stress at the level of the bottom of the foundation

$\rightarrow q = D_1 \times \gamma + D_2 \times (\gamma_{sat} - \gamma_w)$

✓ The factor , γ, (third term) in the bearing capacity equations will takes the following form: (For the soil under the foundation)

$\gamma =$ effective unit weight for soil below the foundation $\rightarrow \gamma' = \gamma_{sat} - \gamma_w$

Case II. The water table is located so that $0 \le d \le B$ as shown in the following figure:



✓ The factor ,q, (second term) in the bearing capacity equations will takes the following form: (For the soil above the foundation)

q = effective stress at the level of the bottom of the foundation

$\rightarrow q = D_f \times \gamma$

✓ The factor , γ, (third term) in the bearing capacity equations will takes the following form: (For the soil under the foundation)

γ = effective unit weight for soil below the foundation **at depth d = B**

I.e. calculate the effective stress for the soil below the foundation from ($d = 0$ to $d = B$), and then divide this value by depth ($d = B$) to get the representative effective unit weight ($\bar{\gamma}$) for this depth.

$$\sigma'_{0\rightarrow B} = d \times \gamma + (B-d) \times (\gamma_{sat} - \gamma_w) \rightarrow \sigma'_{0\rightarrow B} = d \times \gamma + (B-d) \times \gamma'$$

$$\xrightarrow{\div B} \frac{\sigma'_{0\rightarrow B}}{B} = \frac{d \times \gamma + B \times \gamma' - d \times \gamma'}{B} \rightarrow \bar{\gamma} = \gamma' + \frac{d \times (\gamma - \gamma')}{B}$$

Case III. The water table is located so that $d \ge B$, in this case the water table is assumed have no effect on the ultimate bearing capacity.

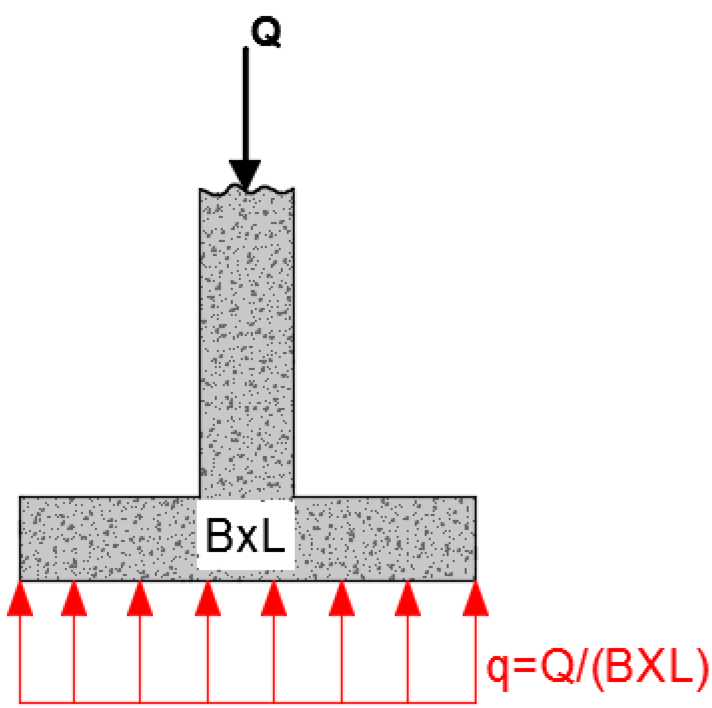
Eccentrically Loaded Foundation

If the load applied on the foundation is in the center of the foundation without eccentricity, the bearing capacity of the soil will be uniform at any point under the foundation (as shown in figure below) because there is no any moments on the foundation, and the general equation for stress under the foundation is:

$$\text{Stress} = \frac{Q}{A} \pm \frac{M_x\, y}{I_x} \pm \frac{M_y\, X}{I_y}$$

In this case, the load is in the center of the foundation and there are no moments so,

$$\text{Stress} = \frac{Q}{A} \text{ (uniform at any point below the foundation)}$$

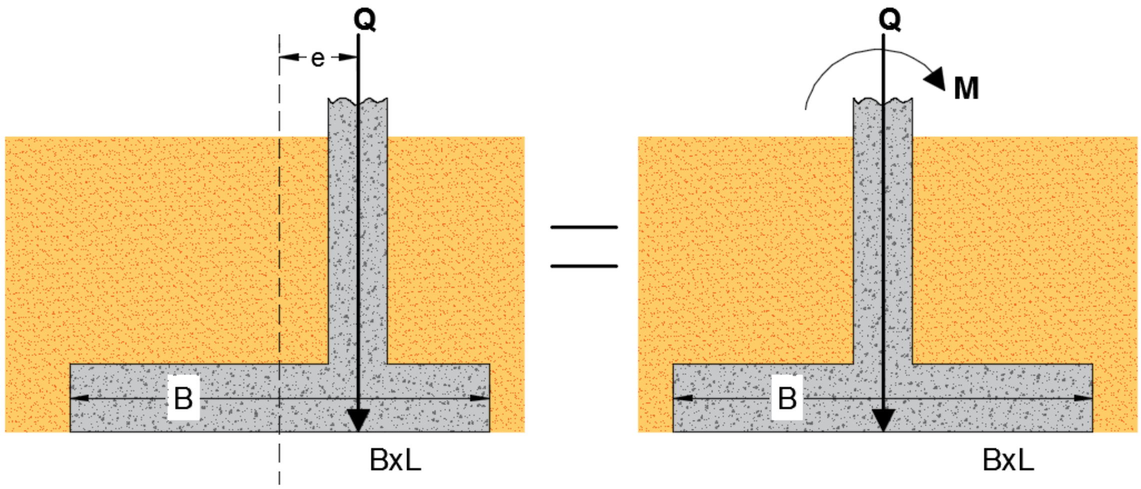


However, in several cases, as with the base of a retaining wall or neighbor footing, the loads does not exist in the center, so foundations are subjected to moments in addition to the vertical load (as shown in the below figure). In such cases, the distribution of pressure by the foundation on the soil is not uniform because there is a moment applied on the foundation and the stress under the foundation will be calculated from the general relation:

$$\text{Stress} = \frac{Q}{A} \pm \frac{M_x\, y}{I_x} \pm \frac{M_y\, X}{I_y} \text{ (in case of two way eccentricity)}$$

But, in this section we deal with (one way eccentricity), the equation will be:

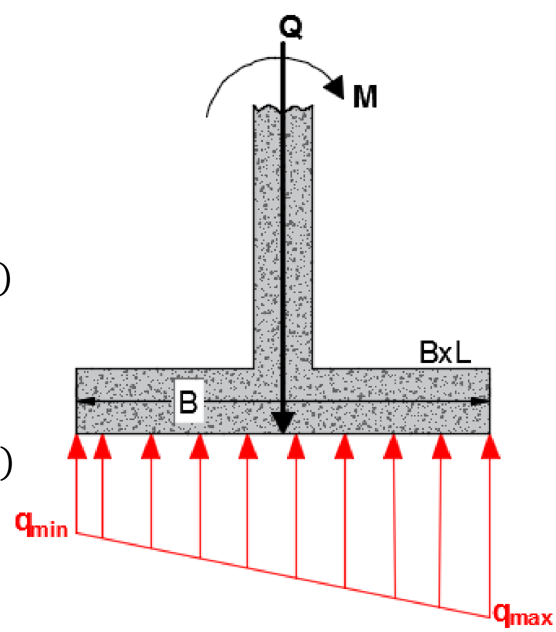
$$\text{Stress} = \frac{Q}{A} \pm \frac{M\, c}{I}$$



Since the pressure under the foundation is not uniform, there are maximum and minimum pressures (under the two edges of the foundation) and we concerned about calculating these two pressures.

General equation for calculating maximum and minimum pressure:

Assume the eccentricity is in direction of (B)



$$\text{Stress} = q = \frac{Q}{A} \pm \frac{M\, c}{I}$$

$$A = B \times L$$

$$M = Q \times e$$

$$c = \frac{B}{2} \text{ (maximum distance from the center)}$$

$$I = \frac{B^3 \times L}{12}$$

(I is about the axis that resists the moment)

Substitute in the equation, the equation will be:

$$q = \frac{Q}{B \times L} \pm \frac{Q \times e \times B}{\frac{2\,B^3 \times L}{12}} \rightarrow q = \frac{Q}{B \times L} \pm \frac{6eQ}{B^2L} \rightarrow q = \frac{Q}{B \times L}\left(1 \pm \frac{6e}{B}\right)$$

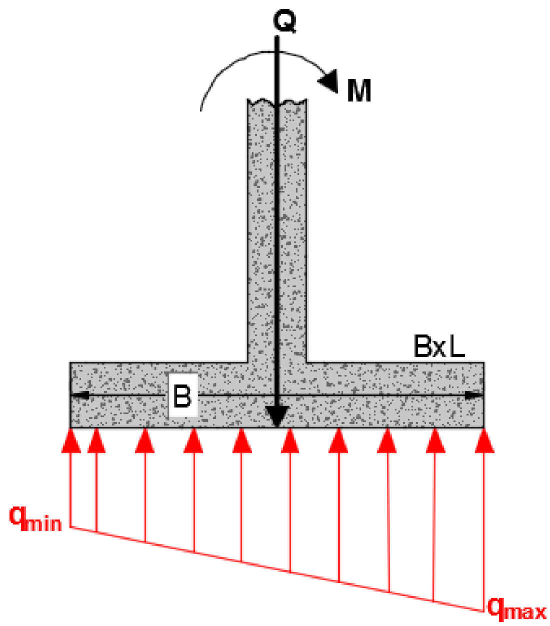
$$q = \frac{Q}{B \times L}\left(1 \pm \frac{6e}{B}\right) \text{ General Equation}$$

Now, there are three cases for calculating maximum and minimum pressures according to the values of (e and $\frac{B}{6}$) **to maintain minimum pressure always**≥ 0

Case I. (For e $< \frac{B}{6}$):

$$q_{max} = \frac{Q}{B \times L}\left(1 + \frac{6e}{B}\right)$$

$$q_{min} = \frac{Q}{B \times L}\left(1 - \frac{6e}{B}\right)$$



Note that when $e < \frac{B}{6}$ the value of q_{min} Will be positive (i.e. compression).

If eccentricity in (L) direction:
(For $e < \frac{L}{6}$):

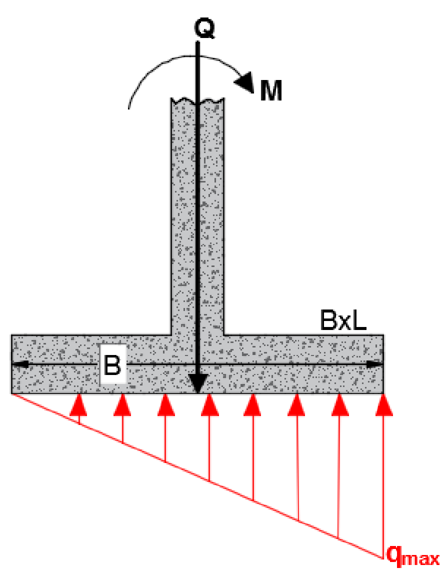
$$q_{max} = \frac{Q}{B \times L}\left(1 + \frac{6e}{L}\right)$$

$$q_{min} = \frac{Q}{B \times L}\left(1 - \frac{6e}{L}\right)$$

Case II. (For e $= \frac{B}{6}$):

$$q_{max} = \frac{Q}{B \times L}\left(1 + \frac{6e}{B}\right)$$

$$q_{min} = \frac{Q}{B \times L}(1 - 1) = 0.0$$



Note that when $e = \frac{B}{6}$ the value of q_{min} will be zero (i.e. no compression and no tension) and this case is the critical case and it is accepted.

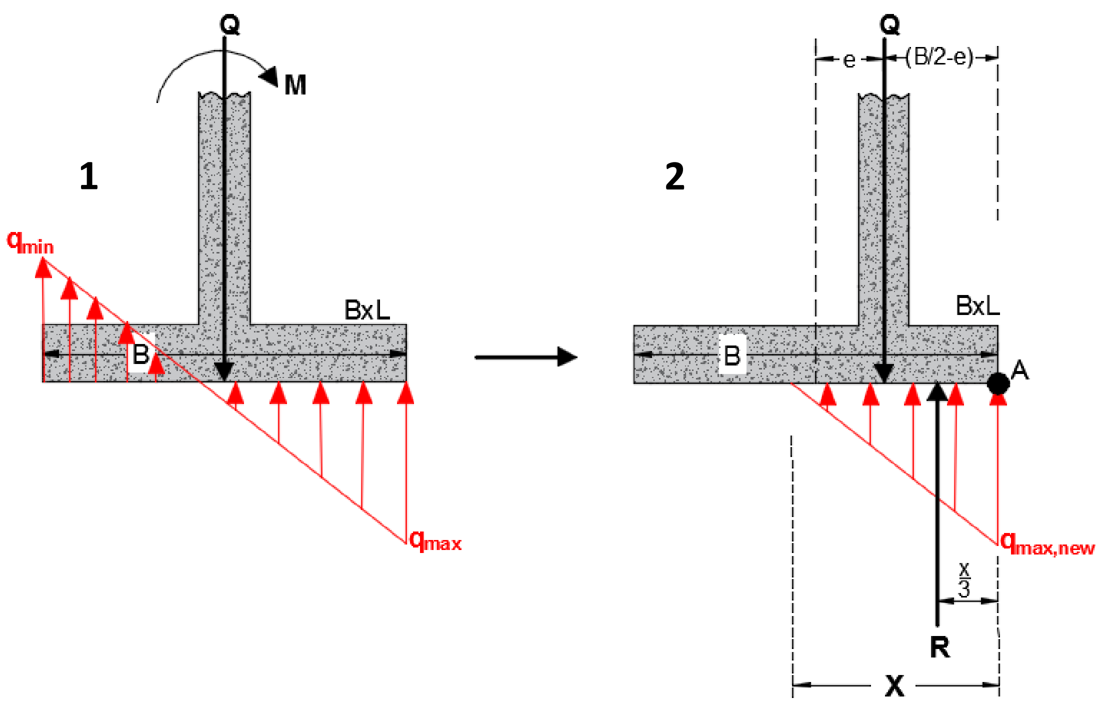
If eccentricity in (L) direction:

(For $e = \frac{L}{6}$):

$$q_{max} = \frac{Q}{B \times L}\left(1 + \frac{6e}{L}\right)$$

$$q_{min} = \frac{Q}{B \times L}(1 - 1) = 0.0$$

Case III. (For $e > \frac{B}{6}$):



As shown in the above figure (1) the value of (q_{min}) is negative (i.e. tension in soil), but we know that soil can't resist any tension, thus, negative pressure must be prevented by making ($q_{min} = 0$) at distance (x) from point (A) as shown in the above figure (2), and determine the new value of (q_{max})by static equilibrium as following:

$$R = \text{area of triangle} \times L = \frac{1}{2} \times q_{max,new} \times X \times L \rightarrow\rightarrow (1)$$

$$\sum F_y = 0.0 \rightarrow R = Q \rightarrow\rightarrow (2)$$

$$\sum M_{@A} = 0.0$$

$$\rightarrow Q \times \left(\frac{B}{2} - e\right) = R \times \frac{X}{3} \ \ (\text{but from Eq.}\,2 \rightarrow R = Q) \rightarrow X = 3\left(\frac{B}{2} - e\right)$$

Substitute by X in Eq. (1) →

$$R = Q = \frac{1}{2} \times q_{max,new} \times 3\left(\frac{B}{2} - e\right) \times L \rightarrow q_{max,new} = \frac{4Q}{3L(B - 2e)}$$

If eccentricity in (L) direction:

(For $e > \frac{L}{6}$):

$$q_{max,new} = \frac{4Q}{3B(L - 2e)}$$

Note:

All the above equations are derived for rectangular or square footing, but if the foundation is circular you should use the original equation for calculating the stress:

$$q = \frac{Q}{A} \pm \frac{M\,c}{I}$$

Where

$A = \frac{\pi}{4}D^2$ (D is the diameter of the circular foundation)

$$c = \frac{D}{2}$$

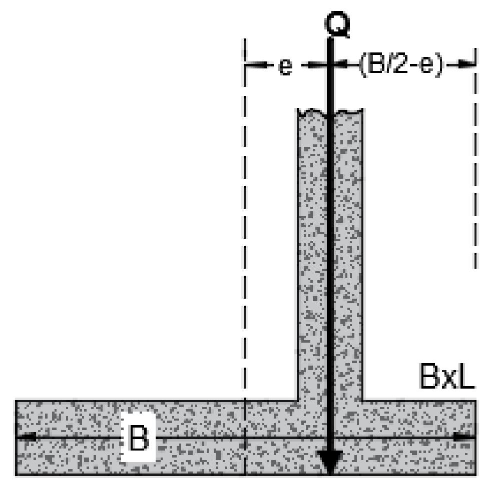
$$I = \frac{\pi}{64}D^4$$

And then calculate q_{max} and q_{min}

Ultimate Bearing Capacity under Eccentric Loading—One-Way Eccentricity

Effective Area Method:

As we discussed previously, if the load does not exist in the center of the foundation, or if the foundation located to moment in addition to the vertical loads, the stress distribution under the foundation is not uniform. So, to calculate the ultimate (uniform) bearing capacity under the foundation, new area should be determined to make the applied load in the center of this area and to develop uniform pressure under this new area. This new area is called **Effective area. The following is how to calculateq_u for this case:**

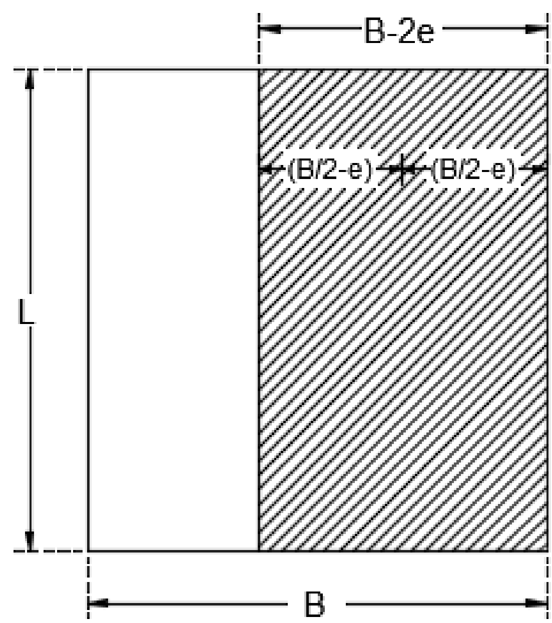


1. Determine the effective dimensions of the foundation:

$\text{Effective width} = B' = B - 2e$
$\text{Effective Length} = L' = L$
$B'_{used} = \min(B', L')$
$L'_{used} = \max(B', L')$

If the eccentricity were in the direction of (L) of the foundation:

$\text{Effective width} = B' = B$
$\text{Effective Length} = L' = L - 2e$
$B'_{used} = \min(B', L')$
$L'_{used} = \max(B', L')$



2. If we want to use terzaghi's equation for example, for square footing:
$q_u = 1.3cN_c + qN_q + 0.4B\gamma N_\gamma$

The value of B (in last term) will be B'_{used} because the pressure is uniform for this value of width and the pressure does not uniform for width B. Other factors in the equation will not change.

3. If we want to use Meyerhof Equation:

$$q_u = cN_cF_{cs}F_{cd}F_{ci} + qN_qF_{qs}F_{qd}F_{qi} + 0.5B\gamma N_\gamma F_{\gamma s}F_{\gamma d}F_{\gamma i}$$

The value of B (in last term) will be B'_{used} to get uniform pressure on this width.

In calculating of shape factors ($F_{cs}, F_{qs}, F_{\gamma s}$) use B'_{used} and L'_{used} because we concerned about the shape of the footing that make the pressure uniform.

In calculating of depth factors ($F_{cd}, F_{qd}, F_{\gamma d}$) use the original value (B) and **don't** replace it by B'_{used} due to the following two reasons:

✓ Depth factors are used to consider the depth of the foundation and thereby the depth of soil applied on the **original** dimensions of the foundation.

✓ In equations of depth factors, as the value of (B) decrease the depth factors will increase and then the value of (q_u) will increase, so for more safety we use the larger value of width (B) to decreases depth factors and thereby decrease (q_u) which less than (q_u) if we use B'_{used}(i.e. more safe).

4. If there is a water table (Case II), we need the following equation to calculate (γ) in the last term of equations (Terzaghi and Meyerhof):

$$\bar{\gamma} = \gamma' + \frac{d \times (\gamma - \gamma')}{B}$$

The value of B used in this equation should be the original value (B) because we calculate the effective unit weight $(\bar{\gamma})$ for depth (B) below the foundation.

Safety Consideration

Calculate the gross ultimate load:

$$Q_u = q_u \times \underbrace{(L'_{used} \times B'_{used})}_{A'} \qquad (A' = \text{effective area})$$

The factor of safety against bearing capacity is: $FS = \frac{Q_u}{Q_{all}} \geq 3$

Maximum Applied Load $\leq Q_{all} = \frac{Q_u}{F.S}$

The factor of safety against q_{max} is: $FS = \frac{q_u}{q_{max}} \geq 3$

The value of q_{all} should be equal or more than q_{max}: $q_{all} \geq q_{max}$

The value of q_{min} should be equal or more than zero: $q_{min} \geq 0.0$

Important Notes (before solving any problem)

1. The soil above the bottom of the foundation **are used only** to calculate the term (q) in the second term of bearing capacity equations (Terzaghi and Meyerhof) and all other factors are calculated for the **underlying** soil.

2. Always the value of (q) is the effective stress at the level of the bottom of the foundation.

3. For the underlying soil, if the value of (c=cohesion=0.0) you don't have to calculate factors in the first term in equations (N_c in terzaghi's equations) and (N_c, F_{cs}, F_{cd}, F_{ci} in Meyerhof equation).

4. For the underlying soil, if the value of ($\phi = 0.0$) you don't have to calculate factors in the last term in equations (N_γ in terzaghi's equations) and (N_γ, $F_{\gamma s}, F_{\gamma d}, F_{\gamma i}$ in Meyerhof equation).

5. If the load applied on the foundation is inclined with an angle ($\beta = \phi$) $\rightarrow$ The value of ($F_{\gamma i}$)will be zero, so you don't have to calculate factors in the last term of Meyerhof equation (N_γ, $F_{\gamma s}, F_{\gamma d}$).

6. Always if we want to calculate the eccentricity, it's calculated as following:

$$e = \frac{\text{Overall Moment}}{\text{Vertical Loads}}$$

7. If the foundation is square, strip or circular, you may calculate (q_u) from terzaghi or Meyerhof equations (should be specified in the problem).

8. But, if the foundation is **rectangular**, you **must** calculate (q_u) from Meyerhof general equation.

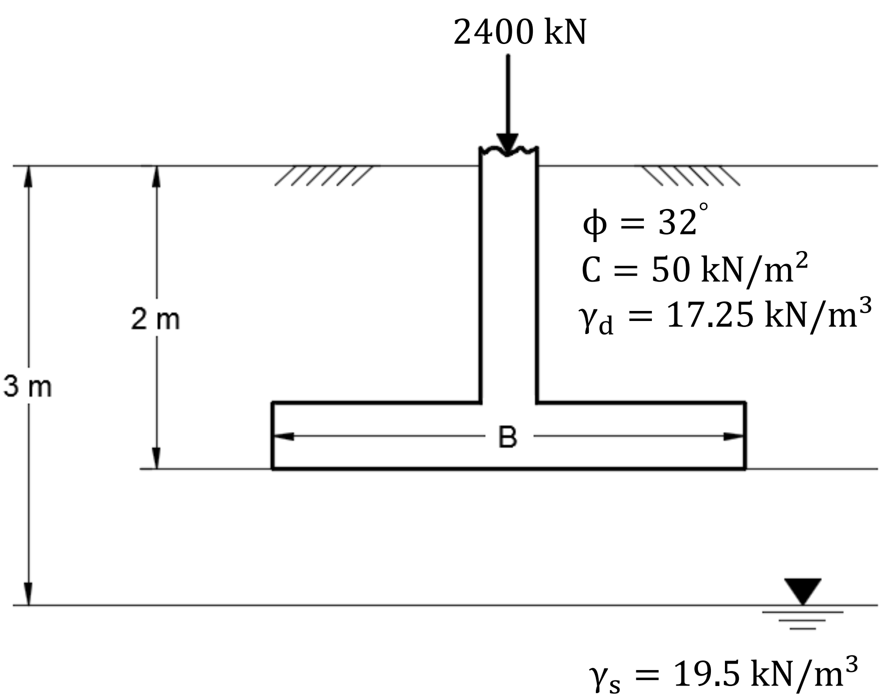
9. If the foundation width (B) is required, and there exist water table below the foundation at distance (d), you should assume $d \leq B$, and calculate B, then make a check for your assumption.

Problems

1.

The square footing shown below must be designed to carry a 2400 KN load. Use Terzaghi's bearing capacity formula and factor of safety = 3.
Determine the foundation dimension B in the following two cases:

1. The water table is at 1m below the foundation (as shown).
2. The water table rises to the ground surface.



Solution

1.

$q_u = 1.3cN_c + qN_q + 0.4B\gamma N_\gamma$

$q_u = q_{all} \times FS \qquad (q_{all} = \frac{Q_{all}}{Area} \ , \ FS = 3)$

Applied load $\leq Q_{all} \rightarrow Q_{all} = 2400kN$

$q_{all} = \frac{Q_{all}}{Area} = \frac{2400}{B^2}$, $FS = 3 \rightarrow\rightarrow q_u = \frac{3 \times 2400}{B^2}$

$c = 50\ kN/m^2$

$q(\text{effective stress}) = \gamma \times D_f = 17.25 \times 2 = 34.5\ kN/m^2$

Since the width of the foundation is not known, assume $d \leq B$

$\gamma = \bar{\gamma} = \gamma' + \dfrac{d \times (\gamma - \gamma')}{B}$

$\gamma' = \gamma_{sat} - \gamma_w = 19.5 - 10 = 9.5 kN/m^3$, $d = 3 - 2 = 1m$

$\rightarrow \bar{\gamma} = 9.5 + \dfrac{1 \times (17.25 - 9.5)}{B} \rightarrow \bar{\gamma} = 9.5 + \dfrac{7.75}{B}$

Assume general shear failure

Note:

Always we design for general shear failure (soil have a high compaction ratio) except if we can't reach high compaction, we design for local shear (medium compaction).

For $\phi = 32^{\circ} \rightarrow N_c = 44.04,\ N_q = 28.52,\ N_\gamma = 26.87$ (**Table 3.1**)

Now substitute from all above factors on terzaghi equation:

$$\frac{7200}{B^2} = 1.3 \times 50 \times 44.04 + 34.5 \times 28.52 + 0.4 \times B \times \left(9.5 + \frac{7.75}{B}\right) \times 26.87$$

$$\frac{7200}{B^2} = 3923.837 + 102.106\,B$$

Multiply both sides by $(B^2) \rightarrow\ 102.106\,B^3 + 3923.837B^2 - 7200 = 0.0$

$\rightarrow B = 1.33m$ ✓.

2.

All factors remain unchanged except q and γ:

$q(\text{effective stress}) = (19.5 - 10) \times 2 = 19\ kN/m^2$

$\gamma = \gamma' = 19.5 - 10 = 9.5\ kN/m^3$

Substitute in terzaghi equation:

$$\frac{7200}{B^2} = 1.3 \times 50 \times 44.04 + 19 \times 28.52 + 0.4 \times B \times 9.5 \times 26.87$$

$$\frac{7200}{B^2} = 3404.48 + 102.106B$$

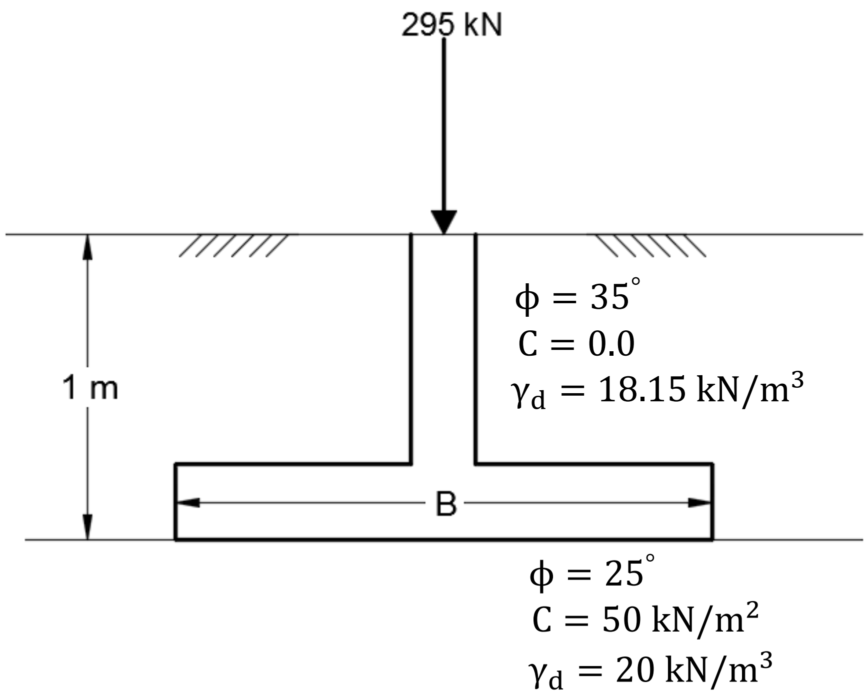
Multiply both sides by $(B^2) \rightarrow 102.106\,B^3 + 3404.48\,B^2 - 7200 = 0.0$

$\rightarrow B = 1.42m$ ✓.

Note that as the water table elevation increase the required width (B) will also increase to maintain the factor of safety (3).

2.

Determine the size of square footing to carry net allowable load of 295 KN. FS=3. Use Terzaghi equation assuming general shear failure.



Solution

$Q_{all,net} = 295\ kN$ and we know $q_{all,net} = \frac{Q_{all,net}}{Area} \rightarrow q_{all,net} = \frac{295}{B^2}$

Also, $q_{all,net} = \frac{q_u - q}{FS}$

$q(\text{effective stress}) = \gamma \times D_f = 18.15 \times 1 = 18.15\ kN/m^2$, $FS = 3$

$$\rightarrow \frac{295}{B^2} = \frac{q_u - 18.15}{3} \rightarrow q_u = \frac{885}{B^2} + 18.15 \rightarrow\rightarrow (1)$$

$q_u = 1.3cN_c + qN_q + 0.4B\gamma N_\gamma$

$c = 50\ kN/m^2$

$q(\text{effective stress}) = 18.15\ kN/m^2$

$\gamma = 20\ kN/m^3$ (for underlying soil)

For $\phi = 25^{\circ} \rightarrow N_c = 25.13,\ N_q = 12.72,\ N_\gamma = 8.34$ (**Table 3.1**)

Substitute from all above factor in Terzaghi equation:

$q_u = 1.3 \times 50 \times 25.13 + 18.15 \times 12.72 + 0.4 \times B \times 20 \times 8.34$

$\rightarrow q_u = 1864.318 + 66.72B$

Substitute from Eq. (1):

$$\frac{885}{B^2} + 18.15 = 1864.318 + 66.72B$$

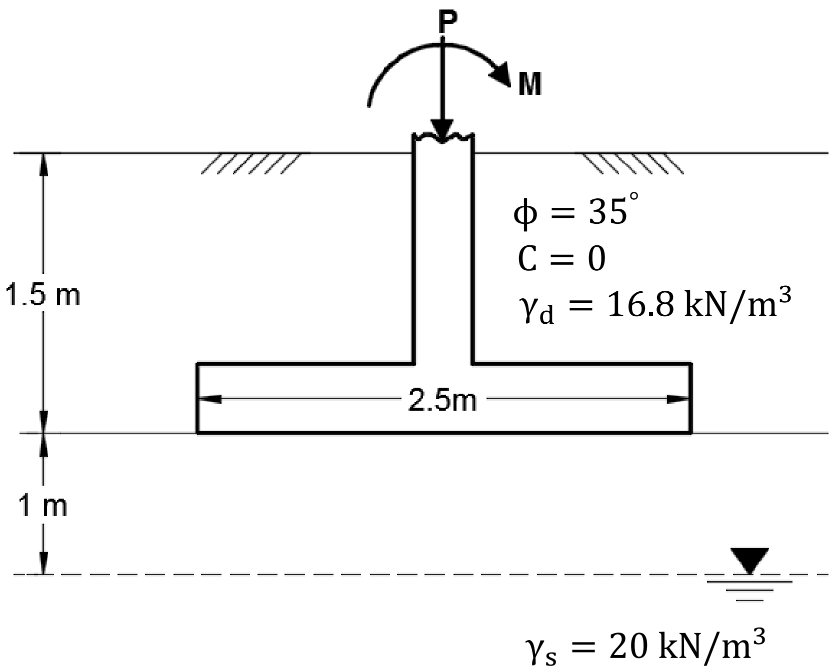
Multiply both side by B^2:

$66.72\ B^3 + 1846.168B^2 - 885 = 0.0$

$\rightarrow B = 0.68\ m$✓.

3.

For the square footing (2.5m x 2.5m) shown in the figure below, determine the allowable resisting moment (M), if the allowable load P = 800 KN and F.S = 3. (**Using Meyerhof Equation).**



Solution

$M = Q \times e = 800e$

$q_u = cN_cF_{cs}F_{cd}F_{ci} + qN_qF_{qs}F_{qd}F_{qi} + 0.5B\gamma N_\gamma F_{\gamma s}F_{\gamma d}F_{\gamma i}$

The first term in the equation will be zero because($c = 0$), so the equation will be:

$q_u = qN_qF_{qs}F_{qd}F_{qi} + 0.5B\gamma N_\gamma F_{\gamma s}F_{\gamma d}F_{\gamma i}$

$q(\text{effective stress}) = \gamma \times D_f = 16.8 \times 1.5 = 25.2\ kN/m^2$

Calculating the new area that maintains q_u uniform:

$B' = B - 2e \rightarrow B' = 2.5 - 2e \quad , \quad L' = 2.5$

$B'_{used} = \min(B', L') = 2.5 - 2e \quad , \quad L'_{used} = 2.5\ m$

$q_u = q_{all} \times FS \qquad \left(q_{all} = \frac{Q_{all}}{A'} \rightarrow A' = B'_{used} \times L'_{used} \quad , \quad FS = 3\right)$

$\text{Applied load} \leq Q_{all} \rightarrow Q_{all} = 800kN$

$$q_{all} = \frac{800}{(2.5 - 2e) \times 2.5} = \frac{320}{2.5 - 2e}, \rightarrow\rightarrow q_u = 3 \times \frac{320}{2.5 - 2e} = \frac{960}{2.5 - 2e}$$

$d = 1m \le B = 2.5m \rightarrow$ water table will effect on $q_u \rightarrow\rightarrow$

$\gamma = \bar{\gamma} = \gamma' + \dfrac{d \times (\gamma - \gamma')}{B}$ (Use B not B'_{used} as we explained previously)

$\gamma' = \gamma_{sat} - \gamma_w = 20 - 10 = 10 \text{ kN/m}^3$, $d = 1m$, $\gamma = 16.8 \text{ kN/m}^3 \rightarrow$

$$\bar{\gamma} = 10 + \frac{1 \times (16.8 - 10)}{2.5} = 12.72 \text{ kN/m}^3$$

Bearing Capacity Factors:

For $\phi = 35^{\circ} \rightarrow N_c = 46.12,\ N_q = 33.3,\ N_\gamma = 48.03$ (**Table 3.3**)

Shape Factors:

As we explained previously, use B'_{used} and L'_{used}

$F_{cs} = 1 + \left(\dfrac{B'_{used}}{L'_{used}}\right)\left(\dfrac{N_q}{N_c}\right)$ does not required (because $c = 0.0$)

$$F_{qs} = 1 + \left(\frac{B'_{used}}{L'_{used}}\right)\tan\phi = 1 + \left(\frac{2.5 - 2e}{2.5}\right) \times \tan 35 = 1.7 - 0.56\, e$$

$$F_{\gamma s} = 1 - 0.4\left(\frac{B'_{used}}{L'_{used}}\right) = 1 - 0.4 \times \left(\frac{2.5 - 2e}{2.5}\right) = 0.6 + 0.32\, e$$

Depth Factors:

As we explained previously, use B not B'_{used}

$\dfrac{D_f}{B} = \dfrac{1.5}{2.5} = 0.6 < 1$ and $\phi = 35 > 0.0 \rightarrow\rightarrow\rightarrow$

$F_{cd} = F_{qd} - \dfrac{1 - F_{qd}}{N_c \tan\phi}$ does not required (because $c = 0.0$)

$$F_{qd} = 1 + 2\tan\phi\,(1 - \sin\phi)^2 \left(\frac{D_f}{B}\right)$$

$$= 1 + 2\tan 35 \times (1 - \sin 35)^2 \times 0.6 = 1.152$$

$$F_{\gamma d} = 1$$

Inclination Factors:

The load on the foundation is not inclined, so all inclination factors are (1).

Now substitute from all above factors in Meyerhof equation:

$$\frac{960}{2.5 - 2e} = 25.2 \times 33.3 \times (1.7 - 0.56\, e) \times 1.152$$
$$+ 0.5 \times (2.5 - 2e) \times 12.72 \times 48.03 \times (0.6 + 0.32\, e)$$

$$\frac{960}{2.5-2e} = 2101.6 - 663.54\,e - 195.5e^2$$

Multiply both sides by $(2.5-2e) \rightarrow\rightarrow$

$960 = 5254 - 4203.2e - 1658.85e + 1327.08e^2 - 488.75e^2 + 391e^3$

$\rightarrow 391e^3 + 838.33e^2 - 5862.05e + 4294 = 0.0$

Solve for e$\rightarrow$ $e = -5.33$ or $e = 2.29$ or $e = 0.89$

Now, the value of (e) must be less than $\frac{B}{2}$ and must be positive value

$$\frac{B}{2} = \frac{2.5}{2} = 1.25 < 2.29 \rightarrow \text{reject the value of } e = 2.29 \text{ and negative value}$$

$\rightarrow e = 0.89\ \text{m}$

Before calculate the value of moment, we check for q_{max}:

$$q_{all} = \frac{320}{2.5-2e} = \frac{320}{2.5-2\times 0.89} = 444.44\ \text{kN/m}^2$$

To calculate q_{max} we firstly should check the value of $(e = 0.89\text{m})$

$$\frac{B}{6} = \frac{2.5}{6} = 0.416\text{m} \rightarrow e = 0.89 > \frac{B}{6} = 0.416 \rightarrow\rightarrow$$

$$q_{max} = q_{max,new} = \frac{4Q}{3L(B-2e)}$$

$$q_{max,new} = \frac{4\times 800}{3\times 2.5\times(2.5-2\times 0.89)} = 592.6\ \text{kN/m}^2 > q_{all} = 444.44$$

Now, we calculate the adequate value of "e" (that makes $q_{all} = q_{max}$)

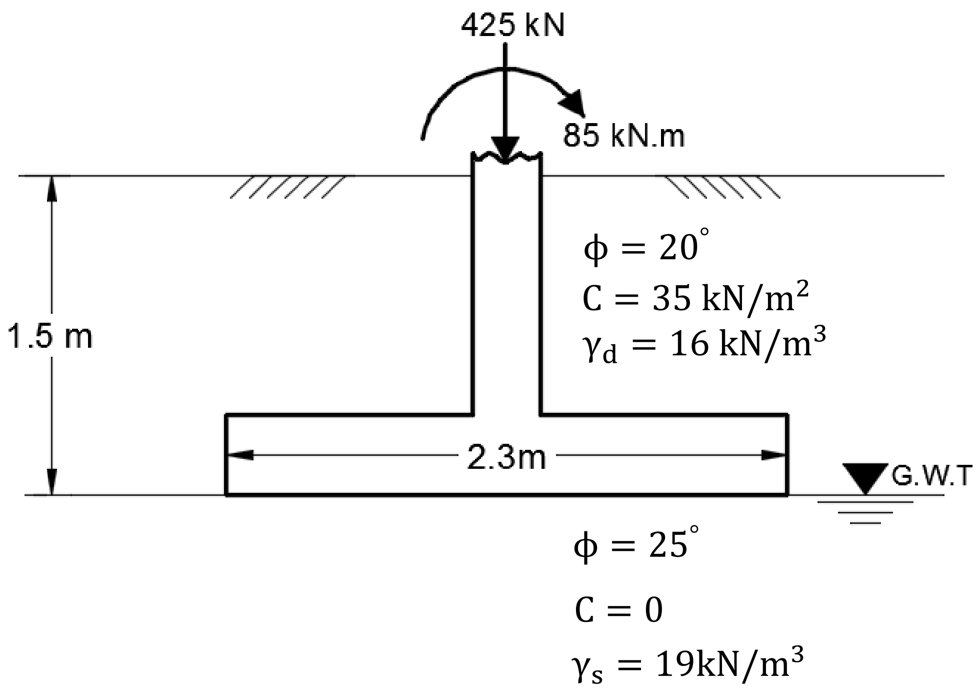
$$444.44 = \frac{4\times 800}{3\times 2.5\times(2.5-2\times e)} \rightarrow e = 0.77\text{m}$$

$M = Q\times e = 800\times 0.77 = 616\ \text{kN.m}$ ✓.

Note that the only variable in this problem is e, so we calculate the value of e that insure that the maximum pressure q_{max} does not exceed the allowable pressure q_{all}.

4.

For the soil profile is given below, determine the allowable bearing capacity of the isolated rectangular footing (2m x 2.3m) that subjected to a vertical load (425 kN) and moment (85 kN.m), FS=3.



Solution

$$q_u = q_{all} \times FS \rightarrow \quad q_{all} = \frac{q_u}{FS} \rightarrow q_{all} = \frac{q_u}{3}$$

$$q_u = cN_cF_{cs}F_{cd}F_{ci} + qN_qF_{qs}F_{qd}F_{qi} + 0.5B\gamma N_\gamma F_{\gamma s}F_{\gamma d}F_{\gamma i}$$

Note that the value of (c) for the soil under the foundation equal zero, so the first term in the equation will be terminated (because we calculate the bearing capacity for soil below the foundation) and the equation will be:

$$q_u = qN_qF_{qs}F_{qd}F_{qi} + 0.5B\gamma N_\gamma F_{\gamma s}F_{\gamma d}F_{\gamma i}$$

$$q(\text{effective stress}) = \gamma \times D_f = 16 \times 1.5 = 24\ kN/m^2$$

Calculating the new area that maintains q_u uniform:

Note that the eccentricity in the direction of (L=2.3)

$$e = \frac{M}{Q} = \frac{85}{425} = 0.2m$$

$B' = B = 2\ m \rightarrow\ ,\ \ L' = L - 2e \rightarrow L' = 2.3 - 2 \times 0.2 = 1.9m$

$B'_{used} = \min(B', L') = 1.9\ m\ \ ,\ \ L'_{used} = 2\ m$

Effective Area $(A') = 1.9 \times 2 = 3.8\ m^2$

Water table is at the bottom of the foundation $\rightarrow \gamma = \gamma' = \gamma_s - \gamma_w$

$\rightarrow \gamma = \gamma' = 19 - 10 = 9\ kN/m^3$

Bearing Capacity Factors:

For $\phi = 25^{\circ} \rightarrow N_c = 20.72,\ N_q = 10.66,\ N_\gamma = 10.88$ (**Table 3.3**)

Shape Factors:

As we explained previously, use B'_{used} and L'_{used}

$$F_{cs} = 1 + \left(\frac{B'_{used}}{L'_{used}}\right)\left(\frac{N_q}{N_c}\right)\ \ \text{does not required (because } c = 0.0)$$

$$F_{qs} = 1 + \left(\frac{B'_{used}}{L'_{used}}\right)\tan\phi = 1 + \left(\frac{1.9}{2}\right) \times \tan 25 = 1.443$$

$$F_{\gamma s} = 1 - 0.4\left(\frac{B'_{used}}{L'_{used}}\right) = 1 - 0.4 \times \left(\frac{1.9}{2}\right) = 0.62$$

Depth Factors:

As we explained previously, use B not B'_{used}

$$\frac{D_f}{B} = \frac{1.5}{2} = 0.75 < 1\ \ \text{and}\ \ \phi = 25 > 0.0 \rightarrow\rightarrow\rightarrow$$

$$F_{cd} = F_{qd} - \frac{1 - F_{qd}}{N_c \tan\phi}\ \ \ \text{does not required (because } c = 0.0)$$

$$F_{qd} = 1 + 2\tan\phi\,(1 - \sin\phi)^2\left(\frac{D_f}{B}\right)$$

$$= 1 + 2\tan 25 \times (1 - \sin 25)^2 \times 0.75 = 1.233$$

$$F_{\gamma d} = 1$$

Inclination Factors:

The load on the foundation is not inclined, so all inclination factors are (1).

Now substitute from all above factors in Meyerhof equation:

$$q_u = 24 \times 10.66 \times 1.443 \times 1.233 + 0.5 \times 1.9 \times 9 \times 10.88 \times 0.62 \times 1$$

$$\rightarrow q_u = 512.87\ kN/m^2$$

$$q_{all} = \frac{q_u}{3} = \frac{512.87}{3} = 170.95\ \text{kN/m}^2 ✓.$$

Now, we check for $q_{max} \rightarrow q_{max} \leq q_{all}$

Now, to calculate q_{max} we firstly should check the value of $(e = 0.2\text{m})$

$$\frac{L}{6} = \frac{2.3}{6} = 0.38\text{m} \rightarrow e = 0.2 < \frac{B}{6} = 0.38 \rightarrow\rightarrow$$

$$q_{max} = \frac{Q}{B \times L}\left(1 + \frac{6e}{L}\right)$$

$$q_{max} = \frac{425}{2 \times 2.3}\left(1 + \frac{6 \times 0.2}{2.3}\right) = 140.6\text{kN/m}^2 < q_{all} = 170.95\ \text{kN/m}^2$$

So, the allowable bearing capacity of the foundation is $170.95\ \text{kN/m}^2$ ✓.

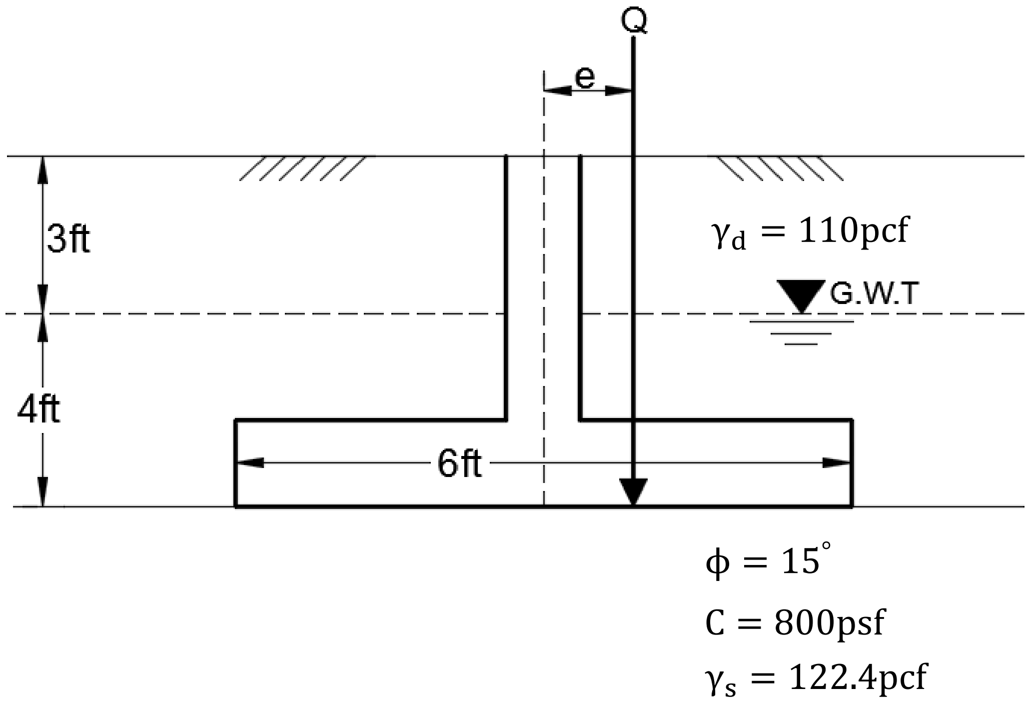
Important Note:

If the previous check is not ok, we say (without calculations): the allowable bearing capacity of 170.95 is not adequate for q_{max}, so the footing dimensions (B or L) must be enlarged to be adequate, the dimension (B or L) is the dimension in the direction of eccentricity (L in this problem).

But, if you are asked to **calculate** the new dimension of the footing:
Put: $q_{max} = q_{all}$ and then substitute in equation of q_{max} to calculate the new dimension

5.

An eccentrically loaded rectangular foundation (6ft x 8ft) shown below. Use factor of safety of 3 and if $e = 0.5$ft, determine the allowable load that the foundation could carry. (The factor of safety is **based on** the maximum stress along the base of the footing).



Solution

Note that the factor of safety is for $q_{max} \rightarrow$ $FS = \frac{q_u}{q_{max}} \geq 3$ (As required)

$$q_u = cN_cF_{cs}F_{cd}F_{ci} + qN_qF_{qs}F_{qd}F_{qi} + 0.5B\gamma N_\gamma F_{\gamma s}F_{\gamma d}F_{\gamma i}$$

$c = 800\ Ib/ft^2$

$q(\text{effective stress}) = 110 \times 3 + (122.4 - 62.4) \times 4 = 570\ Ib/ft^2$

Calculating the new area that maintains q_u uniform:

Note that the eccentricity in the direction of (B=6)

$e = 0.5$ft

$B' = B - 2e = 6 - 2 \times 0.5 = 5\text{ft}\ \ ,\ \ L' = L = 8\text{ft}$

$B'_{used} = \min(B', L') = 5\text{ft}\ \ ,\ \ L'_{used} = 8\text{ft}$

Effective Area $(A') = 5 \times 8 = 40\ ft^2$

Water table is above the bottom of the foundation $\rightarrow \gamma = \gamma' = \gamma_s - \gamma_w$

$\rightarrow \gamma = \gamma' = 122.4 - 62.4 = 60\ \text{Ib/ft}^3$

Bearing Capacity Factors:

For $\phi = 15^{\circ} \rightarrow N_c = 10.98,\ N_q = 3.94,\ N_\gamma = 2.65$ (**Table 3.3**)

Shape Factors:

As we explained previously, use B'_{used} and L'_{used}

$$F_{cs} = 1 + \left(\frac{B'_{used}}{L'_{used}}\right)\left(\frac{N_q}{N_c}\right) = 1 + \left(\frac{5}{8}\right)\left(\frac{3.94}{10.98}\right) = 1.224$$

$$F_{qs} = 1 + \left(\frac{B'_{used}}{L'_{used}}\right)\tan\phi = 1 + \left(\frac{5}{8}\right) \times \tan 15 = 1.167$$

$$F_{\gamma s} = 1 - 0.4\left(\frac{B'_{used}}{L'_{used}}\right) = 1 - 0.4 \times \left(\frac{5}{8}\right) = 0.75$$

Depth Factors:

As we explained previously, use B not B'_{used}

$$\frac{D_f}{B} = \frac{7}{6} = 1.16 > 1 \quad \text{and} \quad \phi = 15 > 0.0 \rightarrow\rightarrow\rightarrow$$

$$F_{qd} = 1 + 2\tan\phi\,(1 - \sin\phi)^2\ \underbrace{\tan^{-1}\left(\frac{D_f}{B}\right)}_{\textbf{radians}}$$

$$\underbrace{\tan^{-1}\left(\frac{D_f}{B}\right)}_{\textbf{radians}} = \tan^{-1}\left(\frac{7}{6}\right) = 0.859$$

$$\rightarrow F_{qd} = 1 + 2\tan(15) \times (1 - \sin 15)^2 \times 0.859 = 1.252$$

$$F_{cd} = F_{qd} - \frac{1 - F_{qd}}{N_c \tan\phi} = 1.252 - \frac{1 - 1.252}{10.98 \times \tan(15)} = 1.337$$

$$F_{\gamma d} = 1$$

Inclination Factors:

The load on the foundation is not inclined, so all inclination factors are (1).

Now substitute from all above factors in Meyerhof equation:

$$q_u = 800 \times 10.98 \times 1.224 \times 1.337 + 570 \times 3.94 \times 1.167 \times 1.252$$
$$+0.5 \times 5 \times 60 \times 2.65 \times 0.75 \times 1$$

$$\rightarrow q_u = 17954.34\ \text{Ib/ft}^2$$

Now, to calculate q_{max} we firstly should check the value of $(e = 0.5ft)$

$$\frac{B}{6} = \frac{6}{6} = 1ft \rightarrow e = 0.5 < \frac{B}{6} = 1 \rightarrow\rightarrow$$

$$q_{max} = \frac{Q}{B \times L}\left(1 + \frac{6e}{B}\right)$$

$$q_{max} = \frac{Q}{6 \times 8}\left(1 + \frac{6 \times 0.5}{6}\right) = 0.03125Q$$

$$FS = \frac{q_u}{q_{max}} = 3 \rightarrow q_u = 3q_{max}$$

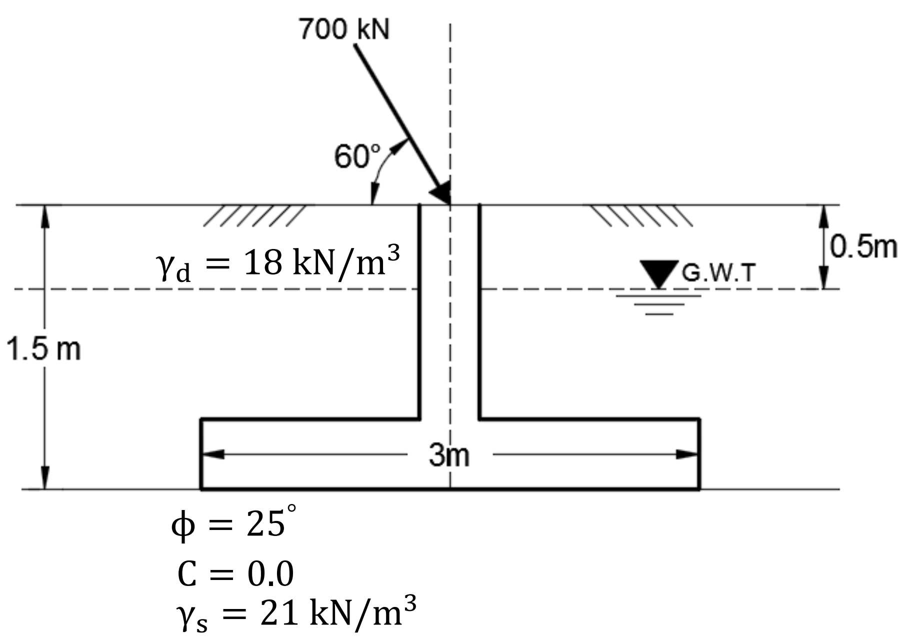
$$\rightarrow 17954.34 = 3 \times 0.03125Q \rightarrow Q = 191512.96\ Ib = 191.5\ Kips\ ✓.$$

6.

For the rectangular foundation (2m x 3m) shown below:

a) Compute the net allowable bearing capacity (FS=3).

b) If the water table is lowered by 2m. What effect on bearing capacity would occur due to the water lowering?

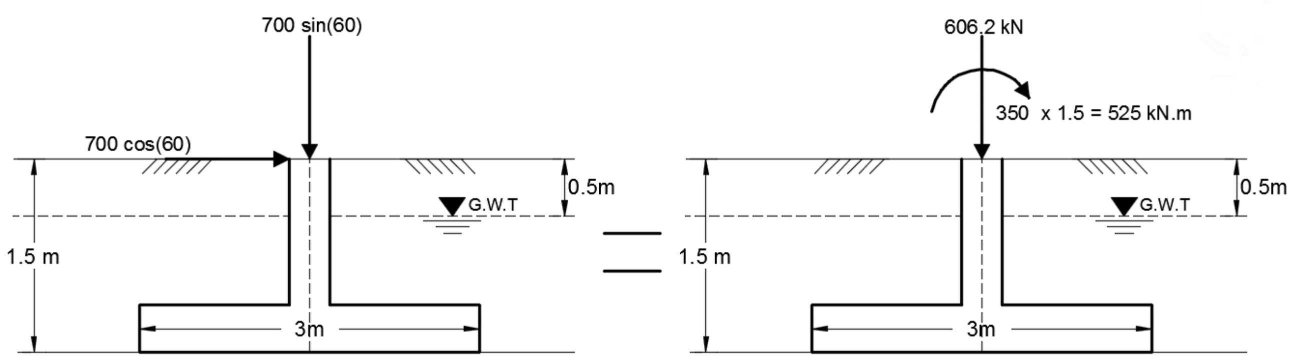


Solution

Important Note:

The load on the foundation is considered inclined when this load is applied directly on the foundation, however if the load does not applied directly on the foundation (like this problem), this load is not considered inclined.

The analysis of the inclined load (700 KN) on the column will be as shown in figure below:



The inclined load on the column will be divided into two components (vertical and horizontal):

Vertical component $= 700 \times \sin 60 = 606.2$ KN

Horizontal component $= 700 \times \cos 60 = 350$ kN

The horizontal component will exerts moment on the foundation in the direction shown in figure above:

$M = 350 \times 1.5 = 525\ \text{kN.m}$

$$e = \frac{\text{Overall moment}}{\text{Vertical Load}} = \frac{525}{606.2} = 0.866\ \text{m}$$

a)

$$q_{all,net} = \frac{q_u - q}{FS}$$

$$q_u = cN_cF_{cs}F_{cd}F_{ci} + qN_qF_{qs}F_{qd}F_{qi} + 0.5B\gamma N_\gamma F_{\gamma s}F_{\gamma d}F_{\gamma i}$$

Note that the value of (c) for the soil under the foundation equal zero, so the first term in the equation will be terminated (because we calculate the bearing capacity for soil below the foundation) and the equation will be:

$$q_u = qN_qF_{qs}F_{qd}F_{qi} + 0.5B\gamma N_\gamma F_{\gamma s}F_{\gamma d}F_{\gamma i}$$

$$q(\text{effective stress}) = 18 \times 0.5 + (21 - 10) \times 1 = 20\ \text{kN/m}^2$$

Calculating the new area that maintains q_u uniform:

Note that the eccentricity in the direction of (L=3)

$e = 0.866\text{m}$

$B' = B = 2\text{ m} \rightarrow \ , \ L' = L - 2e \rightarrow L' = 3 - 2 \times 0.866 = 1.268\text{m}$

$B'_{used} = \min(B', L') = 1.268\text{ m} \ , \ L'_{used} = 2\text{ m}$

$\text{Effective Area } (A') = 1.268 \times 2 = 2.536\text{ m}^2$

Water table is above the bottom of the foundation $\rightarrow \gamma = \gamma' = \gamma_s - \gamma_w$

$\rightarrow \gamma = \gamma' = 21 - 10 = 11\text{ kN/m}^3$

Bearing Capacity Factors:

For $\phi = 25^{\circ} \rightarrow N_c = 20.72, \ N_q = 10.66, \ N_\gamma = 10.88$ (**Table 3.3**)

Shape Factors:

As we explained previously, use B'_{used} and L'_{used}

$F_{cs} = 1 + \left(\frac{B'_{used}}{L'_{used}}\right)\left(\frac{N_q}{N_c}\right)$ does not required (because $c = 0.0$)

$$F_{qs} = 1 + \left(\frac{B'_{used}}{L'_{used}}\right)\tan\phi = 1 + \left(\frac{1.268}{2}\right) \times \tan 25 = 1.296$$

$$F_{\gamma s} = 1 - 0.4\left(\frac{B'_{used}}{L'_{used}}\right) = 1 - 0.4 \times \left(\frac{1.268}{2}\right) = 0.746$$

Depth Factors:

As we explained previously, use B not B'_{used}

$$\frac{D_f}{B} = \frac{1.5}{2} = 0.75 < 1 \quad \text{and} \quad \phi = 25 > 0.0 \rightarrow\rightarrow\rightarrow$$

$F_{cd} = F_{qd} - \frac{1 - F_{qd}}{N_c \tan\phi}$ does not required (because $c = 0.0$)

$$F_{qd} = 1 + 2\tan\phi\,(1 - \sin\phi)^2\left(\frac{D_f}{B}\right)$$

$$= 1 + 2\tan 25 \times (1 - \sin 25)^2 \times 0.75 = 1.233$$

$F_{\gamma d} = 1$

Inclination Factors:

The load on the foundation is not inclined, so all inclination factors are (1).

Now substitute from all above factors in Meyerhof equation:

$$q_u = 20 \times 10.66 \times 1.296 \times 1.233 + 0.5 \times 1.268 \times 11 \times 10.88 \times 0.746$$

$$\rightarrow q_u = 397.29 \text{ kN/m}^2$$

$$q_{all,net} = \frac{q_u - q}{FS} = \frac{397.29 - 20}{3} = 125.76 \text{ kN/m}^2 \checkmark.$$

Now, we check for $q_{max} \rightarrow q_{max} \leq q_{all}$

$$q_{all} = \frac{q_u}{FS} = \frac{397.3}{3} = 132.4 \text{ kN/m}^2$$

To calculate q_{max} we firstly should check the value of $(e = 0.866\text{m})$

$$\frac{L}{6} = \frac{3}{6} = 0.5\text{m} \rightarrow e = 0.866 > \frac{B}{6} = 0.5 \rightarrow\rightarrow$$

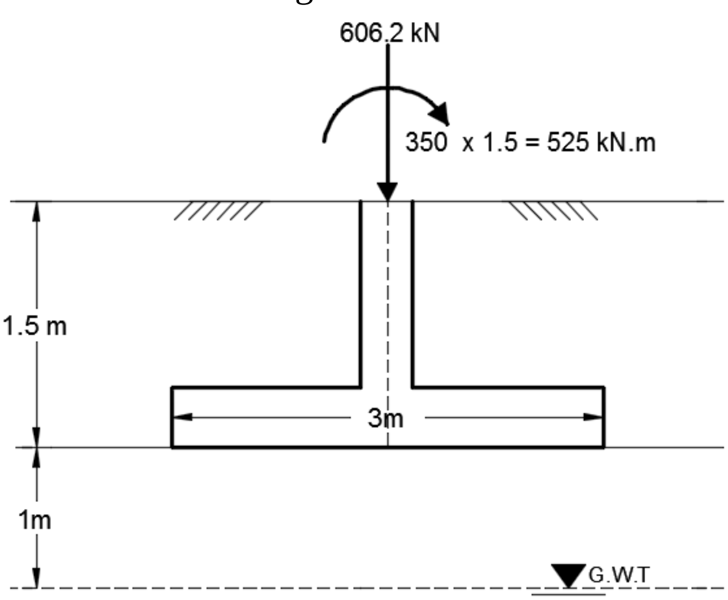
$$q_{max} = q_{max,new} = \frac{4Q}{3B(L - 2e)}$$

$$q_{max,new} = \frac{4 \times 606.2}{3 \times 2 \times (3 - 2 \times 0.866)} = 318.7\text{kN/m}^2 > q_{all} = 132.4$$

So, the allowable bearing capacity of the foundation is 132.4 kN/m^2 is not adequate for q_{max} and the dimensions of the footing must be enlarged.

b)

This case is shown in the below figure:



All factors remain unchanged except q and γ:

$q(\text{effective stress}) = \gamma \times D_f = 18 \times 1.5 = 27 kN/m^2$

$d = 1m \leq B = 2m \rightarrow$ water table will effect on $q_u \rightarrow\rightarrow$

$\gamma = \bar{\gamma} = \gamma' + \frac{d \times (\gamma - \gamma')}{B}$ (Use B not B'_{used} as we explained previously)

$\gamma' = \gamma_{sat} - \gamma_w = 21 - 10 = 11\ kN/m^3\ ,\ d = 1m\ ,\ \gamma = 18\ kN/m^3 \rightarrow$

$\bar{\gamma} = 11 + \frac{1 \times (18 - 11)}{2} = 14.5\ kN/m^3$

Substitute in Meyerhof equation:

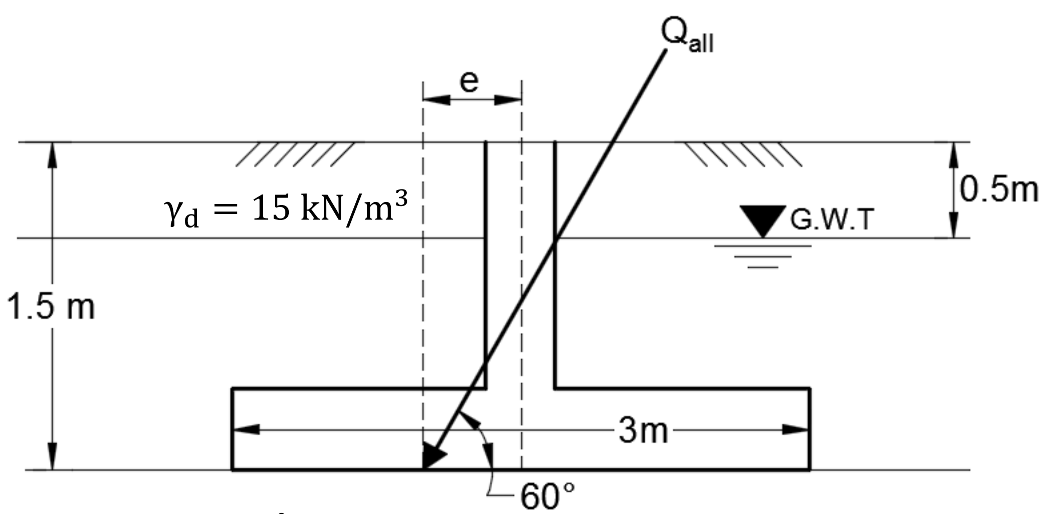
$q_u = 27 \times 10.66 \times 1.296 \times 1.233 + 0.5 \times 1.268 \times 14.5 \times 10.88 \times 0.746$

$\rightarrow q_u = 534.54\ kN/m^2$

The effect of water lowering is increase q_u by $534.5 - 397.3 = 137.2 kN/m^2$ ✓.

7.

For the rectangular footing (2.5m x 3m) shown below, if $e = 0.35m$ and $q_{max} = 410\ kN/m^2$. Calculate the factor of safety against bearing capacity, and determine whether the design is good or not.



$\phi = 30^\circ$

$C = 30\ kN/m^2$

$\gamma_s = 21\ kN/m^3$

Solution

Note that the inclined load is applied directly on the foundation, so it is an inclined load with angle ($\beta = 90 - 60 = 30^{\circ}$ with vertical).

$$\mathrm{FS} = \frac{Q_u}{Q_{all}} \quad , \quad Q_u = q_u \times A' \quad , \quad Q_{all} = ??$$

$$q_u = cN_cF_{cs}F_{cd}F_{ci} + qN_qF_{qs}F_{qd}F_{qi} + 0.5B\gamma N_\gamma F_{\gamma s}F_{\gamma d}F_{\gamma i}$$

Since $\beta = \phi = 30^{\circ}$, the inclination factor $F_{\gamma i}$ will equal zero, so the last term in equation will be terminated and the equation will be:

$$q_u = cN_cF_{cs}F_{cd}F_{ci} + qN_qF_{qs}F_{qd}F_{qi}$$

$$c = 30\ \mathrm{kN/m^2}$$

$$q(\text{effective stress}) = 15 \times 0.5 + (21 - 10) \times 1 = 18.5\ \mathrm{kN/m^2}$$

Calculating the new area that maintains q_u uniform:

Note that the eccentricity in the direction of (L=3)

$e = 0.35\mathrm{m}$

$B' = B = 2.5\ \mathrm{m} \rightarrow \quad , \quad L' = L - 2e \rightarrow L' = 3 - 2 \times 0.35 = 2.3\mathrm{m}$

$B'_{used} = \min(B', L') = 2.3\ \mathrm{m} \quad , \quad L'_{used} = 2.5\ \mathrm{m}$

$$\text{Effective Area } (A') = 2.3 \times 2.5 = 5.75\ \mathrm{m^2}$$

Bearing Capacity Factors:

For $\phi = 30^{\circ} \rightarrow N_c = 30.14,\ N_q = 18.4,\ N_\gamma = 22.4$ (**Table 3.3**)

Shape Factors:

As we explained previously, use B'_{used} and L'_{used}

$$F_{cs} = 1 + \left(\frac{B'_{used}}{L'_{used}}\right)\left(\frac{N_q}{N_c}\right) = 1 + \left(\frac{2.3}{2.5}\right)\left(\frac{18.4}{30.14}\right) = 1.56$$

$$F_{qs} = 1 + \left(\frac{B'_{used}}{L'_{used}}\right)\tan\phi = 1 + \left(\frac{2.3}{2.5}\right) \times \tan 30 = 1.53$$

$$F_{\gamma s} = 1 - 0.4\left(\frac{B'_{used}}{L'_{used}}\right) \quad \text{does not required (because } \beta = \phi = 30^{\circ}\text{)}$$

Depth Factors:

As we explained previously, use B not B'_{used}

$$\frac{D_f}{B} = \frac{1.5}{2.5} = 0.6 < 1 \quad \text{and} \quad \phi = 30 > 0.0 \rightarrow\rightarrow\rightarrow$$

$$F_{qd} = 1 + 2\tan\phi\,(1 - \sin\phi)^2 \left(\frac{D_f}{B}\right)$$

$$F_{qd} = 1 + 2\,\tan(30) \times (1 - \sin 30)^2 \times 0.6 = 1.173$$

$$F_{cd} = F_{qd} - \frac{1 - F_{qd}}{N_c \tan\phi} = 1.173 - \frac{1 - 1.173}{30.14 \times \tan(30)} = 1.183$$

$$F_{\gamma d} = 1$$

Inclination Factors:

$$F_{ci} = F_{qi} = \left(1 - \frac{\beta^{\circ}}{90}\right)^2 = \left(1 - \frac{30}{90}\right)^2 = 0.444$$

$$F_{\gamma i} = 0.0$$

Now substitute from all above factors in Meyerhof equation:

$$q_u = 30 \times 30.14 \times 1.56 \times 1.183 \times 0.444$$
$$+18.5 \times 18.4 \times 1.53 \times 1.173 \times 0.444$$
$$\rightarrow q_u = 1012.14\ \text{kN/m}^2$$

$$Q_u = q_u \times A' = 1012.14 \times 5.75 = 5819.8\ \text{KN}$$

$$e = 0.35\ \text{m} \quad , \quad \frac{L}{6} = \frac{3}{6} = 0.5\text{m} \rightarrow e = 0.35 < \frac{L}{6} = 0.5 \rightarrow\rightarrow\rightarrow$$

We used term (L) because eccentricity in L direction

$$q_{max} = 410 = \frac{Q_{all}}{B \times L}\left(1 + \frac{6e}{L}\right) \rightarrow 410 = \frac{Q_{all}}{2.5 \times 3} \times \left(1 + \frac{6 \times 0.35}{3}\right) \rightarrow\rightarrow$$
$$Q_{all} = 1808.3\text{KN}$$

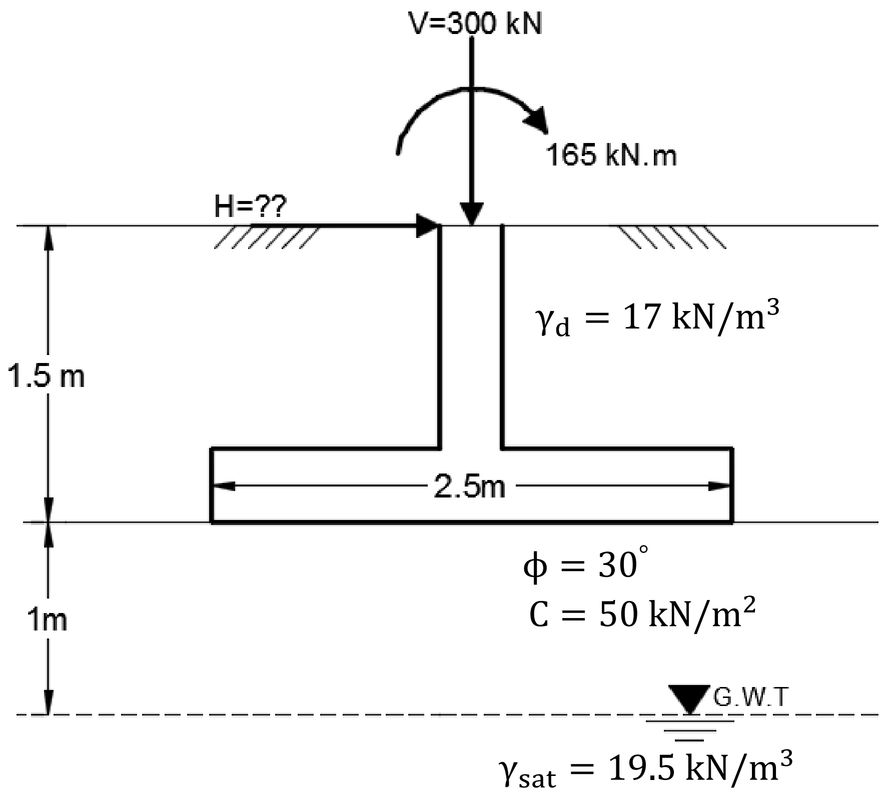
$$FS = \frac{Q_u}{Q_{all}} = \frac{5819.8}{1808.3} = 3.22\checkmark.$$

Since the factor of safety is larger than 3, the design is good✓.

8.

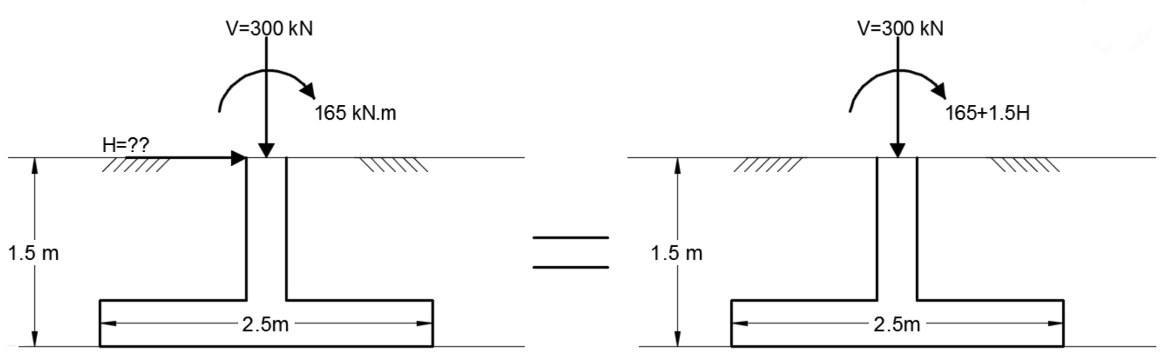
A square footing 2.5m x 2.5m is shown in the figure below. If the maximum pressure on the foundation should not exceed the allowable bearing capacity. Using factor of safety (FS=3), find the maximum horizontal force that the foundation can carry if the water table is 1m below the foundation.

(Use Terzaghi equation)



Solution

The following figure explains the analysis of the given loads:



$$e = \frac{\text{Overall moment}}{\text{Vertical Load}} = \frac{165 + 1.5H}{300} = 0.55 + 0.005H \rightarrow\rightarrow (1)$$

$q_{max} \le q_{all}(\text{given}) \to\to q_{all} = q_{max}$ (To get maximum value of H)

$$\to FS = \frac{q_u}{q_{all}} \to 3 = \frac{q_u}{q_{all}} \to q_u = 3q_{all} \;\; \text{so}, \; q_u = 3q_{max}$$

$$q_u = 1.3cN_c + qN_q + 0.4B\gamma N_\gamma$$

$c = 50 \text{ kN/m}^2$

$q(\text{effective stress}) = 17 \times 1.5 = 25.5 \text{ kN/m}^2$

Calculating the new area that maintains q_u uniform:

$B' = B - 2e = 2.5 - 2e \;\to\;, \quad L' = B = 2.5$

$B'_{used} = \min(B', L') = 2.5 - 2e \;\;, \;\; L'_{used} = 2.5\text{m}$

$\text{Effective Area } (A') = (2.5 - 2e) \times 2.5 = 6.25 - 5e$

$d = 1\text{m} \le B = 2.5\text{m} \to$ water table will effect on $q_u \to\to$

$$\gamma = \bar{\gamma} = \gamma' + \frac{d \times (\gamma - \gamma')}{B} \text{ (Use B not } B'_{used} \text{ as we explained previously)}$$

$\gamma' = \gamma_{sat} - \gamma_w = 19.5 - 10 = 9.5 \text{ kN/m}^3 \;,\;\; d = 1\text{m} \;,\;\; \gamma = 17 \text{ kN/m}^3 \to$

$$\bar{\gamma} = 9.5 + \frac{1 \times (17 - 9.5)}{2.5} = 12.5 \text{ kN/m}^3$$

Bearing Capacity Factors:

For $\phi = 30^\circ \to N_c = 37.16,\; N_q = 22.46,\; N_\gamma = 19.13$ (**Table 3.1**)

Substitute from all above factors in Terzaghi equation:

$$q_u = 1.3 \times 50 \times 37.16 + 25.5 \times 22.46 + 0.4 \times (2.5 - 2e) \times 12.5 \times 19.13$$

$$q_u = 3227.25 - 191.3e$$

Calculating of q_{max}:

$$\frac{B}{6} = \frac{2.5}{6} = 0.416 \;\;, \;\; e = 0.55 + 0.005H$$

(Note that the first term of $e = 0.55 > \frac{B}{6} = 0.416 \;\; \to e > \frac{B}{6} \to\to$

Use the modified equation for q_{max}:

$$q_{max,modified} = \frac{4Q}{3L(B - 2e)} = \frac{4 \times 300}{3 \times 2.5 \times (2.5 - 2e)} = \frac{160}{2.5 - 2e}$$

$$q_u = 3q_{max} \to 3227.25 - 191.3e = 3 \times \frac{160}{2.5 - 2e}$$

Multiply both side by $(2.5 - 2e)$:

$382.6e^2 - 6932.75e + 7588.125 = 0.0 \rightarrow e = 1.17\text{ m}$

Substitute in Eq.(1):

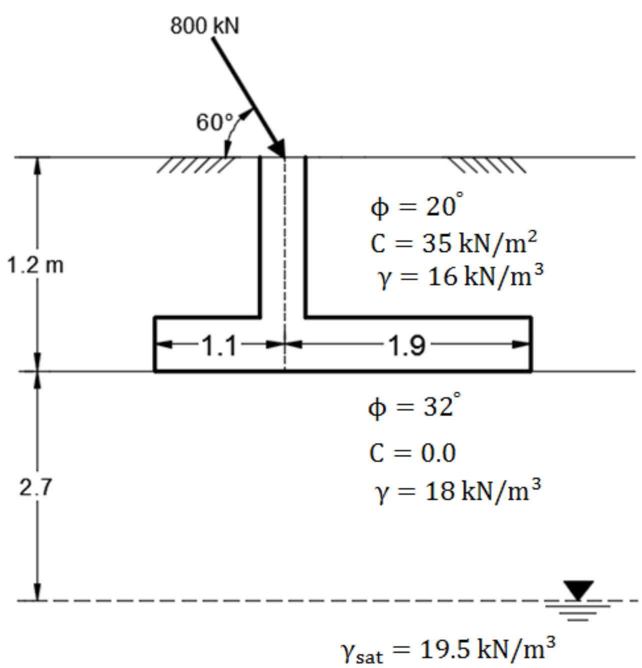
$1.17 = 0.55 + 0.005H \rightarrow H = 124\text{ kN}$✓.

9.

For the soil profile given below, determine the net allowable bearing capacity of the isolated rectangular footing (2.5m x 3m) that subjected to a given load as shown. Use FS=3.

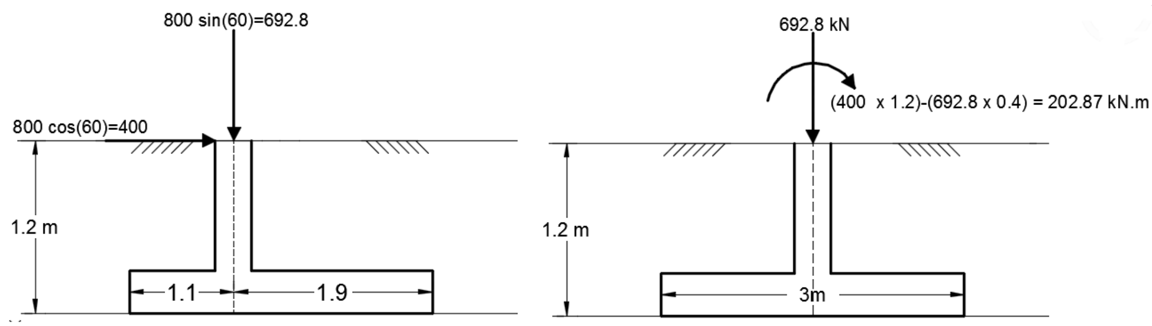
For $\phi = 20^{\circ} \rightarrow N_c = 14.83,\ N_q = 6.4,\ N_\gamma = 5.39$

For $\phi = 32^{\circ} \rightarrow N_c = 35.49,\ N_q = 23.18,\ N_\gamma = 30.22$



Solution

The analysis of the inclined load (800 KN) on the column will be as shown in figure below:



$$e = \frac{\text{Overall moment}}{\text{Vertical Load}} = \frac{202.87}{692.8} = 0.29\text{m}$$

$$q_{all,net} = \frac{q_u - q}{FS}$$

$$q_u = cN_cF_{cs}F_{cd}F_{ci} + qN_qF_{qs}F_{qd}F_{qi} + 0.5B\gamma N_\gamma F_{\gamma s}F_{\gamma d}F_{\gamma i}$$

But $c = 0.0$ for the soil under the foundation $\rightarrow$

$$\rightarrow q_u = qN_qF_{qs}F_{qd}F_{qi} + 0.5B\gamma N_\gamma F_{\gamma s}F_{\gamma d}F_{\gamma i}$$

$q(\text{effective stress}) = 16 \times 1.2 = 19.2\ \text{kN/m}^2$

Calculating the new area that maintains q_u uniform:

Eccentricity in the direction of (L=3)

$e = 0.29\text{m}$

$B' = B = 2.5\ \text{m} \rightarrow\ ,\ \ L' = L - 2e \rightarrow L' = 3 - 2 \times 0.29 = 2.42\text{m}$

$B'_{used} = \min(B', L') = 2.42\ \text{m}\ \ ,\ \ L'_{used} = 2.5\ \text{m}$

Water table is at distance (2.7m) below the foundation base

$\rightarrow B = 2.5\text{m} < 2.7\ \rightarrow$ No effect of water table $\rightarrow$ use $\gamma = 18\text{kN/m}^3$

Bearing Capacity Factors:

For $\phi = 32^\circ \rightarrow N_c = 35.49,\ N_q = 23.18,\ N_\gamma = 30.22$ (**Givens**)

Shape Factors:

$$F_{cs} = 1 + \left(\frac{B'_{used}}{L'_{used}}\right)\left(\frac{N_q}{N_c}\right) \text{ does not required (because } c = 0.0)$$

$$F_{qs} = 1 + \left(\frac{B'_{used}}{L'_{used}}\right)\tan\phi = 1 + \left(\frac{2.42}{2.5}\right) \times \tan 32 = 1.6$$

$$F_{\gamma s} = 1 - 0.4\left(\frac{B'_{used}}{L'_{used}}\right) = 1 - 0.4 \times \left(\frac{2.42}{2.5}\right) = 0.61$$

Depth Factors:

$$\frac{D_f}{B} = \frac{1.2}{2.5} = 0.48 < 1 \quad \text{and} \quad \phi = 32 > 0.0 \rightarrow\rightarrow\rightarrow$$

$$F_{cd} = F_{qd} - \frac{1 - F_{qd}}{N_c \tan\phi} \quad \text{does not required (because } c = 0.0)$$

$$F_{qd} = 1 + 2\tan\phi\,(1 - \sin\phi)^2\left(\frac{D_f}{B}\right)$$

$$= 1 + 2\tan 32 \times (1 - \sin 32)^2 \times 0.48 = 1.13$$

$$F_{\gamma d} = 1$$

Inclination Factors:

The load on the foundation is not inclined, so all inclination factors are (1).

Now substitute from all above factors in Meyerhof equation:

$$q_u = 19.2 \times 23.18 \times 1.6 \times 1.13 + 0.5 \times 2.42 \times 18 \times 30.22 \times 0.61 \times 1$$

$$= 1206.16 \text{ kN/m}^2$$

$$q_{all,net} = \frac{q_u - q}{FS} = \frac{1206.16 - 19.2}{3} = 395.65 \text{ kN/m}^2 \ \checkmark.$$

Now, we check for $q_{max} \rightarrow q_{max} \leq q_{all}$

$$q_{all} = \frac{q_u}{FS} = \frac{1206.16}{3} = 402\ kN/m^2$$

Now, to calculate q_{max} we firstly should check the value of $(e = 0.29m)$

$$\frac{L}{6} = \frac{3}{6} = 0.5m \rightarrow e = 0.29 < \frac{B}{6} = 0.5 \rightarrow\rightarrow$$

$$q_{max} = \frac{Q}{B \times L}\left(1 + \frac{6e}{L}\right)$$

$$q_{max} = \frac{692.8}{2.5 \times 3}\left(1 + \frac{6 \times 0.29}{3}\right) = 145.95\ kN/m^2 < q_{all} = 402\ kN/m^2$$

So, the allowable bearing capacity of the foundation is $402\ kN/m^2$ ✓.

Chapter (4)
Ultimate Bearing Capacity of Shallow Foundations
(Special Cases)

Introduction

The ultimate bearing capacity theories discussed in Chapter 3 assumed that the soil supporting the foundation is homogeneous (i.e. one layer) and extends to a great depth below the bottom of the foundation. They also assume that the ground surface is horizontal. However, that is not true in all cases: It is possible to encounter a soil may be layered and have different shear strength parameters, and in some cases it may be necessary to construct foundations on or near a slope.
All of above cases are special cases from Chapter 3, and will be discussed in this Chapter.

Bearing Capacity of Layered Soils: Stronger soil Underlain by Weaker Soil

The bearing capacity equations presented in Chapter 3 involved cases in which the soil supporting the foundation is homogeneous and extend to a great depth (i.e. the cohesion, angle of friction, and unit weight of soil were assumed to remain constant for the bearing capacity analysis). However, in practice, layered soil profiles are often encountered (more than one layer). In such instances, the failure surface at ultimate load may extend in **two** or more soil layers. This section features the procedures for estimating the bearing capacity for layered soils (stronger soil layer, underlain by a weaker soil layer that extends to a great depth).

Notes:

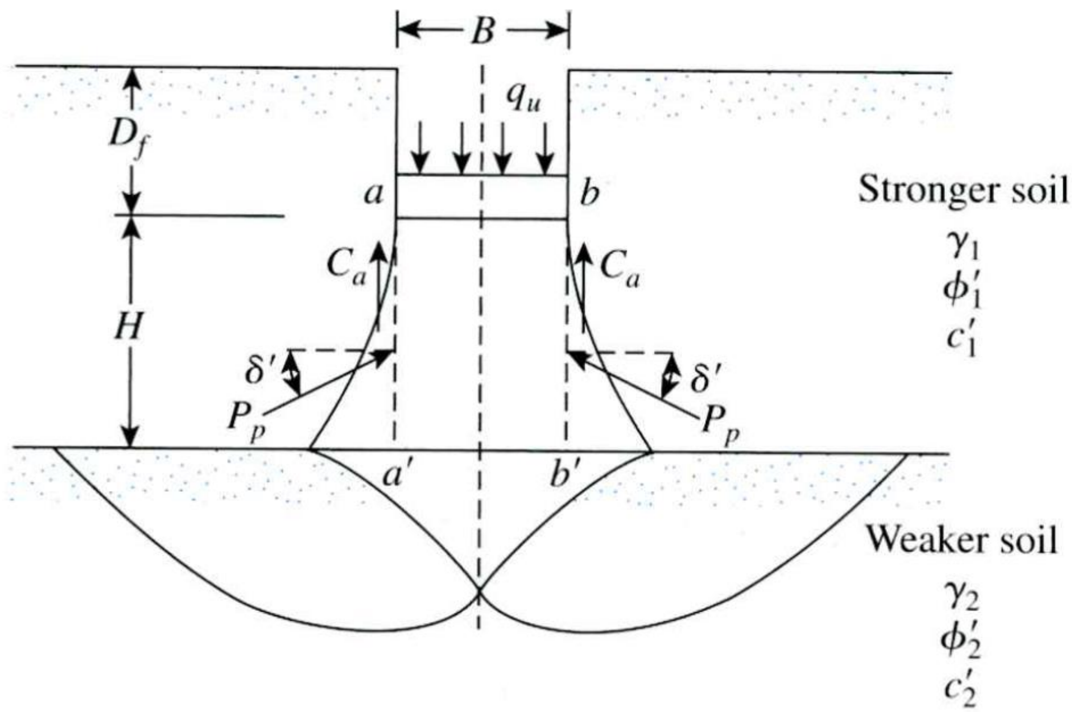
1. Always the factors of top soil are termed by (1) and factors of bottom soil are termed by (2) as shown in the following table:

Soil Properties			
Layer	Unit weight	Friction angle	Cohesion
Top	γ_1	ϕ_1	c_1
Bottom	γ_2	ϕ_2	c_2

2. The equation will be derived for continuous or strip footing and then will be modified to be valid for rectangular, square, and circular footings.

Let the depth ,H, is the distance from the bottom of the foundation to the top of weaker soil (bottom soil layer) and ,B, is the width of continuous or strip footing (i.e. equation will be derived for continuous footing), the failure surface in layered soil below the foundation may have two cases:

Case I: If the depth H is relatively small compared with the foundation width B (upper layer can't resist overall failure due to its small thickness), a punching shear failure will occur in the top soil layer, followed by a general shear failure in the bottom soil layer (due to its large extend downward), so the ultimate bearing capacity in this case will equal the ultimate bearing capacity of bottom layer (because general shear failure occur on it) in addition to punching shear resistance from top layer.



$q_u = q_b +$ Punching shear resistance from top layer ($q_{punching}$)

($q_{punching}$)can be calculated as following (see the above figure):

$$q_{punching} = \underbrace{\frac{(2C_a + 2P_P \sin\delta)}{B \times 1}}_{\text{Upward}} - \underbrace{\gamma_1 \times H}_{\text{Downward}}$$

$C_a =$ adhesive force (between concrete and soil) $\rightarrow C_a = c_a \times H$

$c_a =$ adhesion between concrete and soil along the thickness H

$\delta =$ inclination of the passive, P_P, force with the horizontal

$P_P =$ passive force per unit length along the thickness H applied from soil to the foundation and can be calculated as following:

$$P_P = \frac{1}{2} H \times \text{vertical effective stress} \times K$$

$$\frac{1}{2} H \times \text{vertical effective stress} = \text{area of the vertical pressure diagram}$$

$$\text{vertical effective stress} = \gamma_1 \times H$$

$$K_{PH} = K\cos\delta \rightarrow K = \frac{K_{PH}}{\cos\delta}$$

$K =$ Coefficient used to transform vertical pressure to the direction of passive force

$K_{PH} =$ Horizontal component of passive earth pressure coefficient(K)

Now the equation of P_P will be:

$$P_P = \frac{1}{2} H \times (\gamma_1 \times H) \times K = \frac{1}{2} \times \gamma_1 \times H^2 \times \frac{K_{PH}}{\cos\delta}$$

Now substitute in equation of $q_{punching}$:

$$q_{punching} = \frac{2c_a \times H}{B} + \frac{2 \times \left(\frac{1}{2} \times \gamma_1 \times H^2 \times \frac{K_{PH}}{\cos\delta}\right) \times \sin\delta}{B} - \gamma_1 \times H$$

$$q_{punching} = \frac{2c_a \times H}{B} + \gamma_1 H^2 \times \frac{K_{PH}\tan\delta}{B} - \gamma_1 \times H$$

Now correction for depth factors (according Terzaghi assumption) should be established. This modification will be in punching shear term as following:

$$q_{punching} = \frac{2c_a \times H}{B} + \gamma_1 H^2 \left(1 + \frac{2D_f}{H}\right) \times \frac{K_{PH}\tan\delta}{B} - \gamma_1 \times H$$

From several experiments, investigators found that $K_{PH}\tan\delta = K_s \tan\phi_1$

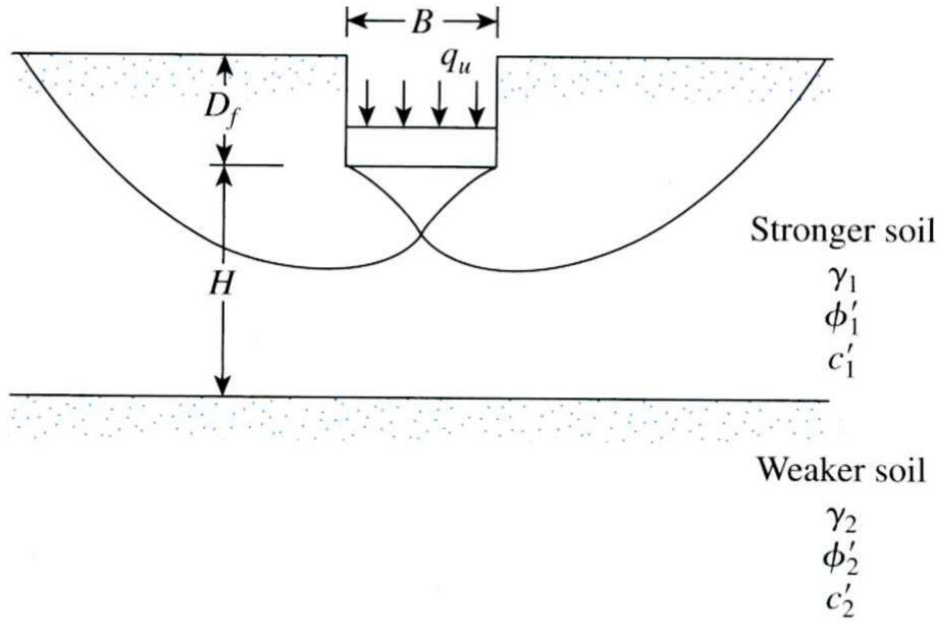
$K_s =$ Punching shear coefficient

$$\rightarrow q_{punching} = \frac{2c_a \times H}{B} + \gamma_1 H^2 \left(1 + \frac{2D_f}{H}\right) \times \frac{K_s \tan\phi_1}{B} - \gamma_1 \times H$$

Now substitute in equation of (q_u):

$$q_u = q_b + \frac{2c_a \times H}{B} + \gamma_1 H^2 \left(1 + \frac{2D_f}{H}\right) \times \frac{K_s \tan\phi_1}{B} - \gamma_1 \times H$$

Case II: If the depth ,H, is relatively large (thickness off top layer is large), then the failure surface will be **completely** located in the top soil layer and the ultimate bearing capacity for this case will be the ultimate bearing capacity for top layer alone (q_t).



$$q_u = q_t = c_1 N_{c(1)} + q\, N_{q(1)} + 0.5B\gamma_1 N_{\gamma(1)}$$

$N_{c(1)}, N_{q(1)}, N_{\gamma(1)}$ = Meyerhof bearing capacity factors (for ϕ_1)(**Table3.3**)

All depth factors will equal (1) because their considered in punching term.
All shape factors will equal (1) because strip or continuous footing.
Assume no inclination so, all inclination factors equal (1).

Combination of two cases:

As mentioned above, the value of q_t is the maximum value of q_u can be reached, so it should be an upper limit for equation of q_u:

$$q_u = q_b + \frac{2c_a \times H}{B} + \gamma_1 H^2 \left(1 + \frac{2D_f}{H}\right) \times \frac{K_s \tan\phi_1}{B} - \gamma_1 \times H \leq q_t$$

The above equation is the derived equation for strip or continuous footing, but if the foundation is square, circular and rectangular the equation will be modified to be **general equation for all shapes of footings:**

$$q_u = q_b + \left(1 + \frac{B}{L}\right) \times \frac{2c_a \times H}{B}$$
$$+\gamma_1 H^2 \times \left(1 + \frac{B}{L}\right)\left(1 + \frac{2D_f}{H}\right) \times \frac{K_s \tan\phi_1}{B} - \gamma_1 \times H \leq q_t$$

$q_t = c_1 N_{c(1)} F_{cs(1)} + q\, N_{q(1)} F_{qs(1)} + 0.5B\gamma_1 N_{\gamma(1)} F_{\gamma s(1)}$
$q = \text{effective stress at the top of layer}(1) = \gamma_1 \times D_f$

$q_b = c_2 N_{c(2)} F_{cs(2)} + q\, N_{q(2)} F_{qs(2)} + 0.5B\gamma_2 N_{\gamma(2)} F_{\gamma s(2)}$
$q = \text{effective stress at the top of layer}(2) = \gamma_1 \times (D_f + H)$

All depth factors will equal (1) because their considered in punching term.
Assume no inclination so, all inclination factors equal (1).

Note:
All factors and equations mentioned above are based on Meyerhof theory discussed in Chapter 3.

All of above factors are known except K_s and c_a
$K_s = f\left(\frac{q_2}{q_1}, \phi_1\right)$ and $\frac{c_a}{c_1} = f\left(\frac{q_2}{q_1}\right)$ → to find K_s and c_a: $\left(\frac{q_2}{q_1}\right)$ must be known.

Calculating of q_1 and q_2 is based on the following three main assumptions:
1. The foundation is always strip foundation even if it's not strip
2. The foundation exists on the ground surface ($D_f = 0.0$) and the second term on equation will be terminated.
3. In calculating q_1 we assume the top layer only exists below the foundation to a great depth, and the same in calculating of q_2.
$q_1 = c_1 N_{c(1)} + 0.5B\gamma_1 N_{\gamma(1)}$
$q_2 = c_2 N_{c(2)} + 0.5B\gamma_2 N_{\gamma(2)}$
$\rightarrow \left(\frac{q_2}{q_1}\right) = \checkmark$

Calculating of K_s:

$$K_s = f\left(\frac{q_2}{q_1}, \phi_1\right)$$

K_s can be calculated easily from **(Figure 4.9)** according the values of $\left(\frac{q_2}{q_1} \text{ and} \phi_1\right)$

Calculating of c_a:

$$\frac{c_a}{c_1} = f\left(\frac{q_2}{q_1}\right)$$

$\frac{c_a}{c_1}$ can be calculated easily from **(Figure 4.10)** according the value of $\left(\frac{q_2}{q_1}\right)$

$$\frac{c_a}{c_1} = \checkmark \text{ and } c_1 = \checkmark \rightarrow\rightarrow c_a = \checkmark$$

Important Notes:

1. If there is a water table near the foundation (above or below foundation), the three cases discussed in Chapter 3 should be considered (i.e. the factor q for top and bottom layers may be modified and $\gamma_1$and γ_2 for top and bottom layers may also be modified according to the existing case of water table.

2. If the strong layer and the weak layer are not clear (cohesion and friction angle for each layer are convergent), to know the strong and the weak layer do the following:

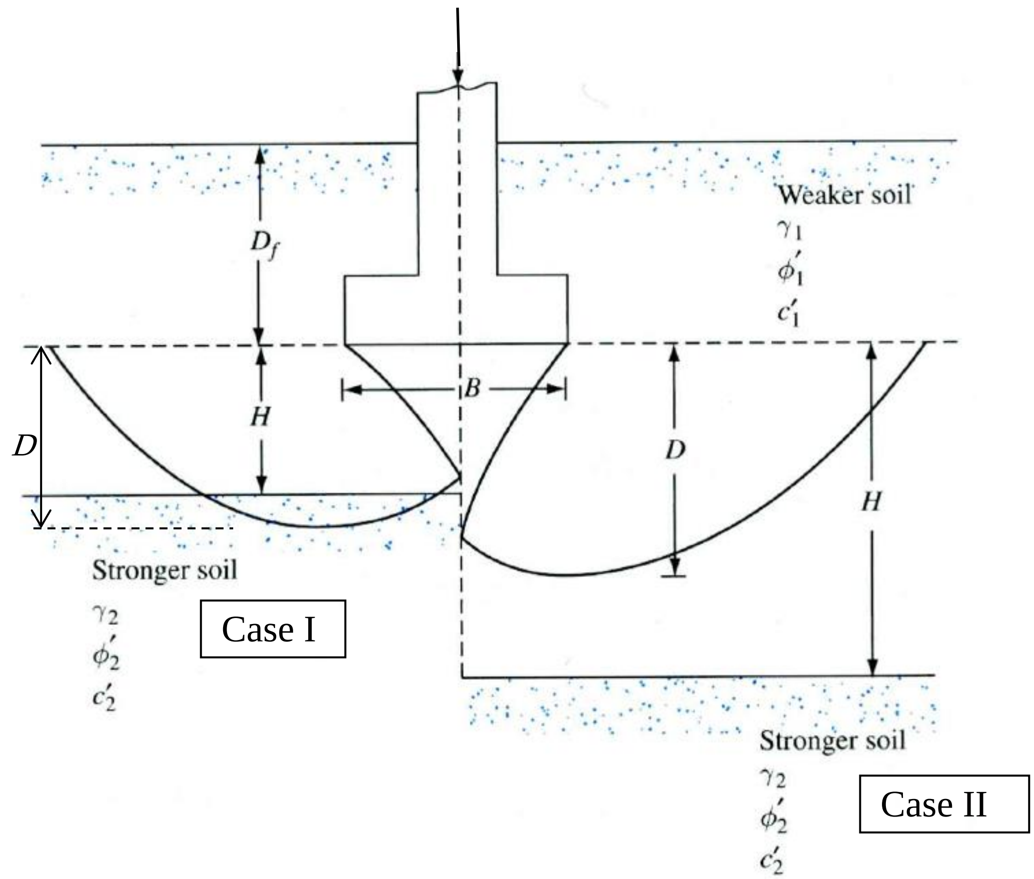
✓ Calculate q_1 and q_2 and then calculate $\left(\frac{q_2}{q_1}\right)$

✓ If $\left(\frac{q_2}{q_1}\right) < 1 \rightarrow$ The top layer is the stronger layer and the bottom is the weaker layer

✓ If $\left(\frac{q_2}{q_1}\right) > 1 \rightarrow$ The top layer is the weaker layer and the bottom is the stronger layer.

3. Any special cases can be derived from the general equation above.

Bearing Capacity of Layered Soils: Weaker soil Underlain by Stronger Soil



Let the depth ,H, is the distance from the bottom of the foundation to the top of stronger soil (bottom soil layer) and ,B, is the width of the foundation and ,D, is the depth of failure beneath the foundation.

As shown on the above figure, there are two cases:

Case I: For $(\mathrm{H} < \mathrm{D} \rightarrow \frac{\mathrm{H}}{\mathrm{D}} < 1)$ →The failure surface in soil at ultimate load will pass through both soil layers (i.e. the ultimate bearing capacity of soil will be greater than the ultimate bearing capacity for bottom layer alone).

Case II: $(\mathrm{H} > \mathrm{D} \rightarrow \frac{\mathrm{H}}{\mathrm{D}} > 1)$ →The failure surface on soil will be fully located on top ,weaker soil layer, (i.e. the ultimate bearing capacity in this case is equal the ultimate bearing capacity for top layer alone).

For these two cases, the ultimate bearing capacity can be given as following:

For $\left(H \leq D \rightarrow \frac{H}{D} \leq 1\right)$

$$q_u = q_t + (q_b - q_t)\left(1 - \frac{H}{D}\right)^2$$

Note that if $\frac{H}{D} = 1$ →the value of q_u will equal q_t, and this is logical, because in this special case the failure surface will be exist on whole depth of top (weaker layer).

For $\left(H > D \rightarrow \frac{H}{D} > 1\right)$

$q_u = q_t$

Because failure surface is fully located on top (weaker soil).

$q_{t,weak} = c_1 N_{c(1)} F_{cs(1)} + q\, N_{q(1)} F_{qs(1)} + 0.5B\gamma_1 N_{\gamma(1)} F_{\gamma s(1)}$

$q =$ effective stress at the top of layer$(1) = \gamma_1 \times D_f$

$q_{b,strong} = c_2 N_{c(2)} F_{cs(2)} + q\, N_{q(2)} F_{qs(2)} + 0.5B\gamma_1 N_{\gamma(2)} F_{\gamma s(2)}$

$q =$ effective stress at the top of layer(1) by assuming the foundation is located directly above stronger soil layer at depth of D_f

$\rightarrow q = \gamma_2 \times D_f$

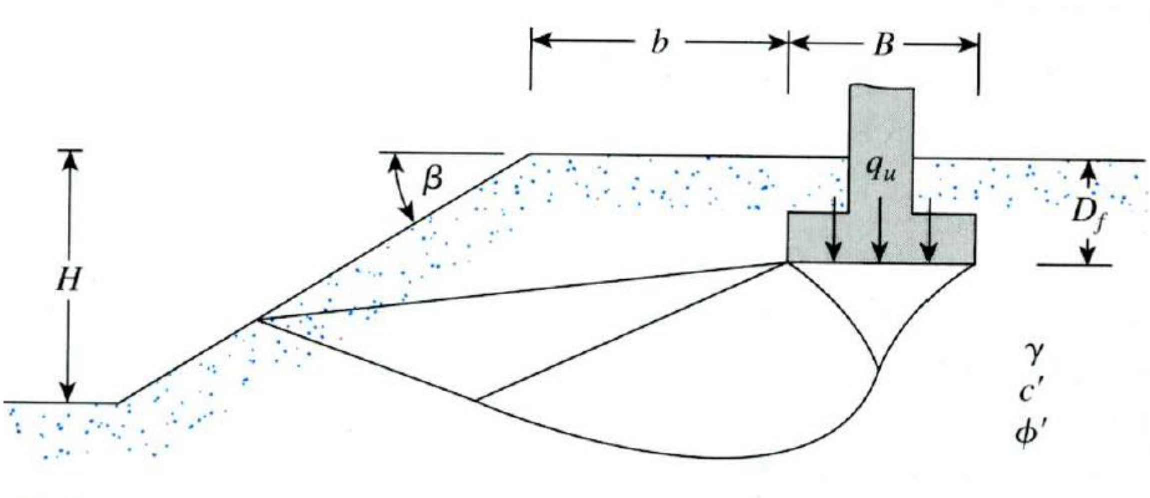
Important Note:

$D = B$ (for **loose sand** and **clay**)

$D = 2B$ (for **dense sand**)

Bearing Capacity of Foundations on Top of a Slope

In some instances, foundations need to be constructed on top of a slope, thus calculating of bearing capacity of soil under such conditions will differ from Chapter 3. This section explains how we can calculate the bearing capacity of soil under these conditions.



$H =$ height of slope , $\beta =$ angle between the slope and horizontal
$b =$ distance from the edge of the foundation to the top of the slope

The ultimate bearing capacity for **continuous or strip** footing can be calculated by the following theoretical relation:

$$q_u = cN_{cq} + 0.5B\gamma N_{\gamma q}$$

For purely granular soil ($c = 0.0$):
$$q_u = 0.5B\gamma N_{\gamma q}$$

For purely cohesive soil ($\phi = 0.0$):
$$q_u = cN_{cq}$$

Calculating of $N_{\gamma q}$:

The value of $N_{\gamma q}$ can be calculated from **(Figure 4.15 P.204)** according the following steps:

1. Calculate the value of $\left(\frac{D_f}{B}\right)$.
2. If $\left(\frac{D_f}{B}\right) = 0.0 \rightarrow$ use **solid lines** on the figure.

3. If $\left(\frac{D_f}{B}\right) = 1 \rightarrow$ use **dashed lines** on the figure.

4. Calculate the value of $\left(\frac{b}{B}\right)$ which the horizontal axis aof the figure.

5. According the values of (ϕ, β and factors mentioned above) we can calculate the value of $N_{\gamma q}$ on vertical axis of the figure.

Note:

If the value of $\frac{D_f}{B}$ is in the following range: $\left(0 < \frac{D_f}{B} < 1\right)$ do the following:

- ✓ Calculate $N_{\gamma q}$ at $\left(\frac{D_f}{B}\right) = 1$.
- ✓ Calculate $N_{\gamma q}$ at $\left(\frac{D_f}{B}\right) = 0$.
- ✓ Do interpolation between the above two values of $N_{\gamma q}$ to get the required value of $N_{\gamma q}$.

Calculating of N_{cq}:

The value of N_{cq} can be calculated from **(Figure 4.16 P.205)** according the following steps:

1. Calculate the value of $\left(\frac{D_f}{B}\right)$.

2. If $\left(\frac{D_f}{B}\right) = 0.0 \rightarrow$ use **solid lines** on the figure.

3. If $\left(\frac{D_f}{B}\right) = 1 \rightarrow$ use **dashed lines** on the figure.

4. Determining the horizontal axis of the figure:
   - ✓ If $B < H \rightarrow$ the horizontal axis of the figure is $\left(\frac{b}{B}\right)$
   - ✓ If $B \geq H \rightarrow$ the horizontal axis of the figure is $\left(\frac{b}{H}\right)$

5. Calculating the value of stability number for clay (N_s):
   - ✓ If $B < H \rightarrow$ use $N_s = 0.0$ in the figure
   - ✓ If $B \geq H \rightarrow$ calculate N_s from this relation $N_s = \frac{\gamma H}{c}$ to be used in the figure.

6. According the values of (ϕ, β and factors mentioned above) we can calculate the value of N_{cq} on vertical axis of the figure.

Note:

If the value of $\frac{D_f}{B}$ is in the following range: $\left(0 < \frac{D_f}{B} < 1\right) \rightarrow$
Do interpolation as mentiond above.

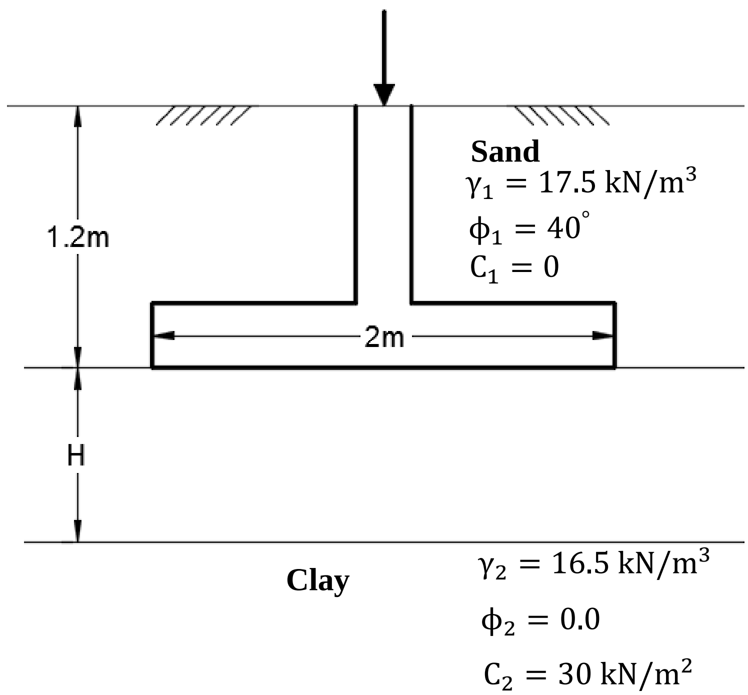
Problems

1.

The figure below shows a continuous foundation.

a) If H=1.5 m, determine the ultimate bearing capacity, q_u

b) At what minimum depth ,H, will the clay layer not have any effect on the ultimate bearing capacity of the foundation?



Solution

The first step in all problems like this one is determining whether the two soils are stronger soil and weaker soil as following:

$q_1 = c_1 N_{c(1)} + 0.5B\gamma_1 N_{\gamma(1)} \quad (c_1 = 0.0) \rightarrow q_1 = 0.5B\gamma_1 N_{\gamma(1)}$

$B = 2m \ , \ \gamma_1 = 17.5\ kN/m^3$

For $\phi_1 = 40^{\circ} \rightarrow N_{\gamma(1)} = 109.41\ (\mathbf{Table3.3})$

$\rightarrow q_1 = 0.5 \times 2 \times 17.5 \times 109.41 = 1914.675\ kN/m^2$

$q_2 = c_2 N_{c(2)} + 0.5B\gamma_2 N_{\gamma(2)} \quad (\phi_2 = 0.0) \rightarrow q_2 = c_2 N_{c(2)}$

$c_2 = 30\ kN/m^2 \ , \ \text{For } \phi_2 = 0^{\circ} \rightarrow N_{c(2)} = 5.14\ (\mathbf{Table3.3})$

$q_2 = 30 \times 5.14 = 154.2\ kN/m^2$

$\frac{q_2}{q_1} = \frac{154.2}{1914.675} = 0.08 < 1 \rightarrow$ The top layer is stronger soil and bottom layer is weaker soil.

1. For strip footing:

$$q_u = q_b + \frac{2c_a \times H}{B} + \gamma_1 H^2 \left(1 + \frac{2D_f}{H}\right) \times \frac{K_s \tan\phi_1}{B} - \gamma_1 \times H \leq q_t$$

$q_t = c_1 N_{c(1)} + q\, N_{q(1)} + 0.5B\gamma_1 N_{\gamma(1)}$

$c_1 = 0.0$, $q = \gamma_1 \times D_f = 17.5 \times 1.2 = 21 kN/m^2$, $B = 2m$

For $\phi_1 = 40^\circ \rightarrow N_{c(1)} = 75.31, N_{q(1)} = 64.2 , N_{\gamma(1)} = 109.41$ (**Table3.3**)

$q_t = 0 + 21 \times 64.2 + 0.5 \times 2 \times 17.5 \times 109.41 = 3262.875\ kN/m^2$

$q_b = c_2 N_{c(2)} + q\, N_{q(2)} + 0.5B\gamma_2 N_{\gamma(2)}$

$c_2 = 30$, $q = \gamma_2 \times (D_f + H) = 17.5 \times (1.2 + 1.5) = 47.25\ kN/m^2$

For $\phi_2 = 0^\circ \rightarrow N_{c(2)} = 5.14, N_{q(2)} = 1 , N_{\gamma(2)} = 0$ (**Table3.3**)

$q_b = 30 \times 5.14 + 47.25 \times 1 + 0 = 201.45\ kN/m^2$

Calculating of c_a:

$\frac{q_2}{q_1} = 0.08$

From figure **(4.10)** $\rightarrow \frac{c_a}{c_1} = 0.7 \rightarrow c_a = 0.7 \times 0 = 0$

Calculating of K_s:

$\frac{q_2}{q_1} = 0.08$

From figure **(4.9)** $\rightarrow K_s = 2.4$

$$q_u = 201.45 + 0 + 17.5 \times 1.5^2 \left(1 + \frac{2 \times 1.2}{1.5}\right) \times \frac{2.4 \tan 40}{2} - 17.5 \times 1.5$$

$q_u = 278\ kN/m^2$ ✓.

2. The **minimum** depth that make the clay layer have no effect on q_u is occur when $q_u = q_t$ and $q_b = 0.0$

$$q_u = q_t = 0 + \frac{2c_a \times H}{B} + \gamma_1 H^2 \left(1 + \frac{2D_f}{H}\right) \times \frac{K_s \tan\phi_1}{B} - \gamma_1 \times H$$

$$3262.875 = 0 + 0 + 17.5 \times H^2 \left(1 + \frac{2 \times 1.2}{H}\right) \times \frac{2.4 \tan 40}{2} - 17.5 \times H$$

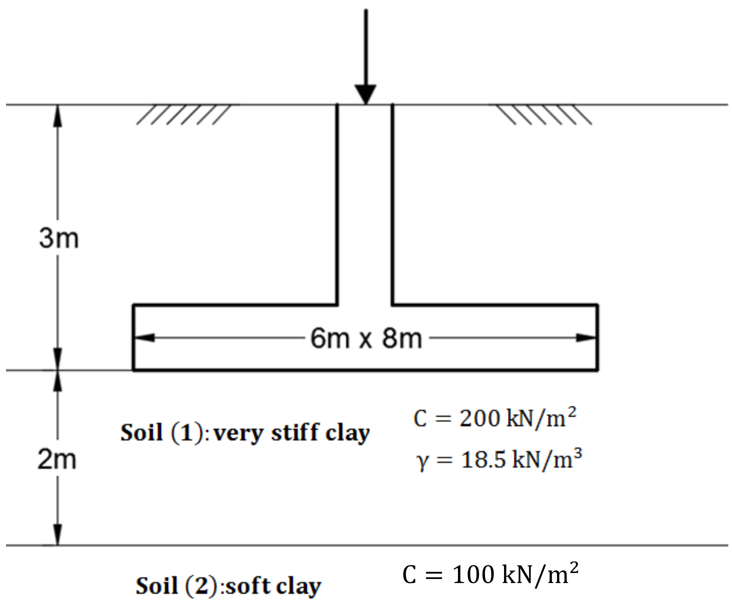
$\rightarrow H = 12.92\ m$ ✓.

2.

A rectangular footing of size 6m x 8m is founded at a depth of 3 m in a clay stratum of very stiff consistency overlying a softer clay stratum at a depth of 5 m from the ground surface. The soil parameters of the two layers of soil are as shown in the Figure below. If the top layer has been removed and replaced by dense sand. The soil parameters of the dense sand are □ = 19 kN/m³ and □ = 35 degrees. All other data remain the same. Estimate the ultimate bearing capacity of the footing.

For $\phi = 35^{\circ} \rightarrow N_c = 46.12,\ N_q = 33.3,\ N_\gamma = 48.03$

For $\phi = 0 \rightarrow N_c = 5.14,\ N_q = 1,\ N_\gamma = 0$



Solution

The top layer is dense sand with $\phi_1 = 35^{\circ}$ and $\gamma_1 = 19\ \text{kN/m}^3$

Determine which layer is strong:

$q_1 = c_1 N_{c(1)} + 0.5B\gamma_1 N_{\gamma(1)}$ but $c_1 = 0.0 \rightarrow q_1 = 0.5B\gamma_1 N_{\gamma(1)}$

at $\phi_1 = 35^{\circ} \rightarrow N_{\gamma(1)} = 48.03$

$\rightarrow q_1 = 0.5 \times 6 \times 19 \times 48.03 = 2737.71\text{kN/m}^2$

$q_2 = c_2 N_{c(2)} + 0.5B\gamma_2 N_{\gamma(2)}$ but $\phi_2 = 0.0 \rightarrow q_2 = c_2 N_{c(2)}$

at $\phi_2 = 0 \rightarrow N_{c(2)} = 5.14 \rightarrow q_2 = 100 \times 5.14 = 514\ \text{kN/m}^2$

$$\frac{q_2}{q_1} = \frac{514}{2737.7} = 0.187 < 1$$

$\rightarrow$ The top layer is strong and the bottom is weak

General equation fir rectangular foundation:

$$q_u = q_b + \left(1 + \frac{B}{L}\right) \times \frac{2c_a \times H}{B} + \gamma_1 H^2 \times \left(1 + \frac{B}{L}\right)\left(1 + \frac{2D_f}{H}\right) \times \frac{K_s \tan\phi_1}{B}$$

$$-\gamma_1 \times H \le q_t$$

But $c_1 = 0.0 \rightarrow c_a = 0.0 \rightarrow$ so the equation will be:

$$q_u = q_b + \gamma_1 H^2 \times \left(1 + \frac{B}{L}\right)\left(1 + \frac{2D_f}{H}\right) \times \frac{K_s \tan\phi_1}{B} - \gamma_1 \times H \le q_t$$

Calculation of q_t:

$$q_t = c_1 N_{c(1)} F_{cs(1)} + q\, N_{q(1)} F_{qs(1)} + 0.5B\gamma_1 N_{\gamma(1)} F_{\gamma s(1)} \;\; \text{but} c_1 = 0.0$$

$$\rightarrow q_t = q\, N_{q(1)} F_{qs(1)} + 0.5B\gamma_1 N_{\gamma(1)} F_{\gamma s(1)}$$

$$q = 3 \times 19 = 57 \text{ kN/m}^2$$

$$\text{at } \phi_1 = 35^\circ \rightarrow N_{q(1)} = 33.3 \;\;,\;\; N_{\gamma(1)} = 48.03$$

$$F_{qs(1)} = 1 + \left(\frac{B}{L}\right) \tan\phi_1 = 1 + \left(\frac{6}{8}\right) \tan 35 = 1.525$$

$$F_{\gamma s(1)} = 1 - 0.4\left(\frac{B}{L}\right) = 1 - 0.4\left(\frac{6}{8}\right) = 0.7$$

$$\rightarrow q_t = 57 \times 33.3 \times 1.525 + 0.5 \times 6 \times 19 \times 48.03 \times 0.7$$

$$= 4811 \text{ kN/m}^2$$

Calculation of q_b:

$$q_b = c_2 N_{c(2)} F_{cs(2)} + q\, N_{q(2)} F_{qs(2)} + 0.5B\gamma_2 N_{\gamma(2)} F_{\gamma s(2)} \;\; \text{but } \phi_2 = 0.0$$

$$\rightarrow q_b = c_2 N_{c(2)} F_{cs(2)} + q\, N_{q(2)} F_{qs(2)}$$

$$q = (3 + 2) \times 19 = 95 \text{ kN/m}^2$$

at $\phi_2 = 0 \rightarrow N_{c(2)} = 5.14$, $N_{q(2)} = 1$

$$F_{cs(2)} = 1 + \left(\frac{B}{L}\right)\left(\frac{N_{q(2)}}{N_{c(2)}}\right) = 1 + \frac{6}{8} \times \frac{1}{5.14} = 1.146$$

$$F_{qs(2)} = 1 + \left(\frac{B}{L}\right)\tan\phi_2 = 1 + \left(\frac{6}{8}\right)\tan 0 = 1$$

$$\rightarrow q_b = 100 \times 5.14 \times 1.146 + 95 \times 1 \times 1 = 684 \text{ kN/m}^2$$

Determination of K_s

$$\frac{q_2}{q_1} = 0.187 \text{ and } \phi_1 = 35^{\circ} \rightarrow K_s = 2.5 \text{ (from the given chart).}$$

Now apply in the equation an calculate q_u

$$q_u = 684 + 19 \times 2^2 \times \left(1 + \frac{6}{8}\right)\left(1 + \frac{2 \times 3}{2}\right) \times \frac{2.5 \times \tan 35}{6} - 19 \times 2$$

$$= 801.21 \text{ kN/m}^2 < q_t = 4811 \text{ kN/m}^2 \rightarrow q_u = 801.21 \text{ kN/m}^2 \checkmark.$$

3.

Solve examples **4.4** and **4.5** in your text book.

4.

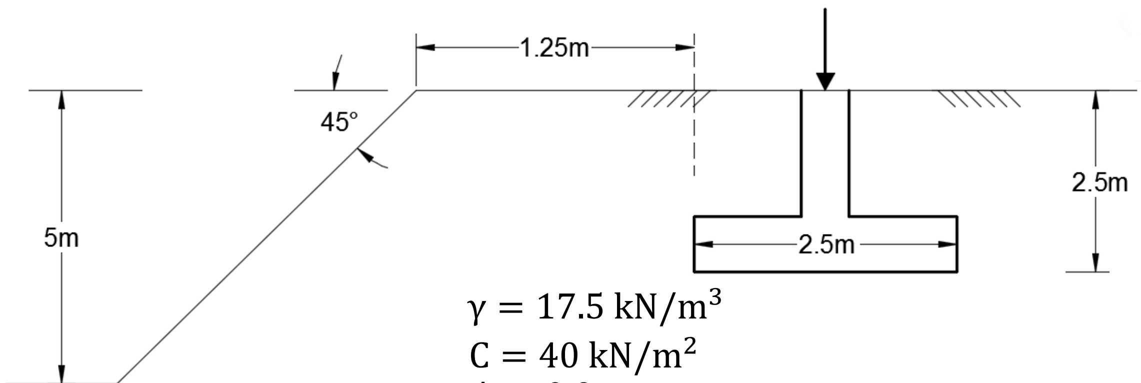
Solve example **4.6** in your text book, **but** use this equation for calculating(q_u):

$$q_u = q_t + (q_b - q_t)\left(1 - \frac{H}{D}\right)^2$$

Because the equation in text book for this case doesn't true.

5.

For the soil profile shown below, determine the ultimate bearing capacity of the continuous footing.



$\gamma = 17.5\ \text{kN/m}^3$
$C = 40\ \text{kN/m}^2$
$\phi = 0.0$

Solution

From the figure: $B = 2.5\text{m}$, $b = 1.25$ m, $H = 5\text{m}$, $D_f = 2.5\text{m}$, $\beta = 45^\circ$

$q_u = cN_{cq} + 0.5B\gamma N_{\gamma q}$ but $\phi = 0.0 \rightarrow q_u = cN_{cq}$

$c = 40\ \text{kN/m}^2$

Calculating of N_{cq} (Figure 4.16):

$\frac{D_f}{B} = \frac{2.5}{2.5} = 1 \rightarrow$ use dashed lines on figure

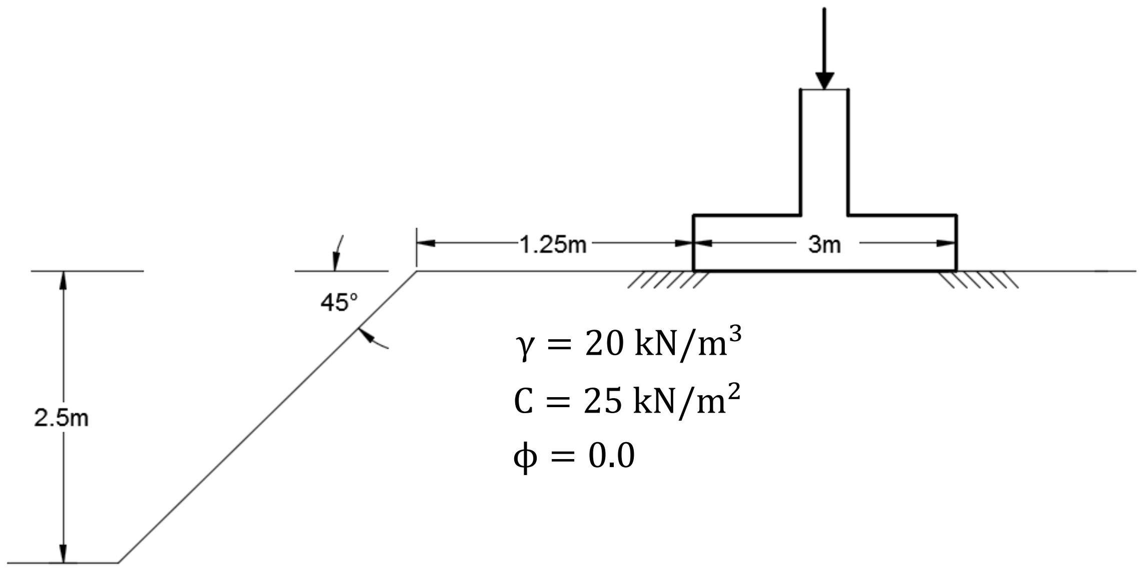
$B = 2.5 < H = 5 \rightarrow$ the horizontal axis of the figure is $\left(\frac{b}{B}\right) = \frac{1.25}{2.5} = 0.5$

$B = 2.5 < H = 5 \rightarrow$ use $N_s = 0.0$ in the figure

From the figure, the value of $N_{cq} \cong 5.7 \rightarrow q_u = 40 \times 5.7 = 228\ \text{kN/m}^2$ ✓.

6.

For the soil profile shown below, determine the ultimate bearing capacity of the continuous footing.



Solution

From the figure: $B = 3\mathrm{m}$, $b = 1.25\ \mathrm{m}$, $H = 2.5\mathrm{m}$, $D_f = 0.0\mathrm{m}$, $\beta = 45^{\circ}$

$q_u = cN_{cq} + 0.5B\gamma N_{\gamma q}$ but $\phi = 0.0 \rightarrow q_u = cN_{cq}$

$c = 25\ \mathrm{kN/m^2}$

Calculating of N_{cq} (Figure 4.16):

$\frac{D_f}{B} = \frac{0}{2.5} = 0 \rightarrow$ use solid lines on figure

$B = 3 > H = 2.5 \rightarrow$ the horizontal axis of the figure is $\left(\frac{b}{H}\right) = \frac{1.25}{2.5} = 0.5$

$B = 3 > H = 2.5 \rightarrow$ use $N_s = \frac{\gamma \times H}{c} = \frac{20 \times 2.5}{25} = 2$ in the figure

From the figure, the value of $N_{cq} \cong 3 \rightarrow q_u = 25 \times 3 = 75\ \mathrm{kN/m^2}$ ✓.

Chapter (5)
Allowable Bearing Capacity and Settlement

Introduction

As we discussed previously in Chapter 3, foundations should be designed for both shear failure and allowable settlement. So the allowable settlement of shallow foundations may control the allowable bearing capacity. The allowable settlement itself may be controlled by local building codes.
For example; the maximum allowable settlement for mat foundation is 50 mm, and 25 mm for isolated footing. These foundations should be designed for these limiting values of settlement (by calculating the allowable bearing capacity from the allowable settlement). Thus, the allowable bearing capacity is the smaller of the following two conditions:

$$q_{all} = \text{smallest of} \begin{cases} \dfrac{q_u}{FS} \text{(to control shear failure Ch.3)} \\ \\ q_{all,settlement} \text{(to control settlement)} \end{cases}$$

In this Chapter, we will learn how to calculate the allowable bearing capacity for settlement ($q_{all,settlement}$), but firstly we want to calculate the total settlement of the foundation.
The settlement of a foundation can be divided into two major categories:

a) Immediate or elastic settlement (S_e):
Elastic or immediate settlement occurs during or immediately after the application of the load (construction of structure) without change in the moisture content of the soil.

b) Consolidation Settlement (S_c):
Consolidation settlement occur over time, such that pore water is extruded from the void spaces of saturated clayey soil submerged in water.
Consolidation settlement comprises two phases: **Primary** and **secondary**.

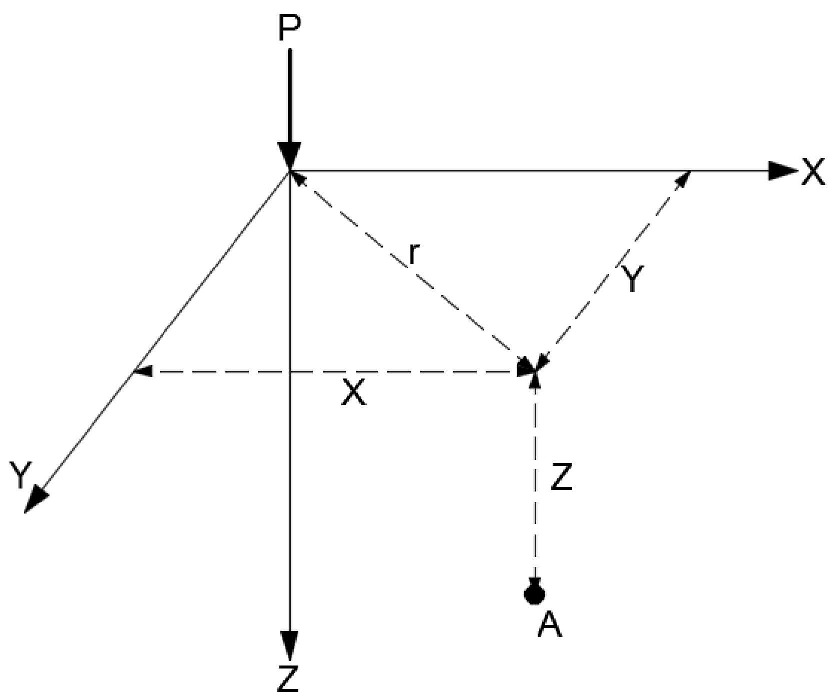
To calculate foundation settlement (both elastic and consolidation), it is required to estimate the **vertical stress increase** in the soil mass due to the net load applied on the foundation (exactly as discussed previously in soil mechanics course “Ch.10”). Hence, this chapter is divided into the following three parts:
1. Calculation of vertical stress increase (Ch.10 in soil mechanics course).

2. Elastic Settlement Calculation (Main topic of this chapter).
3. Consolidation settlement calculation (Ch.11 in soil mechanics course).

Vertical Stress Increase in a Soil Mass Caused by Foundation Load

Stress Due to a Concentrated (Point) Load:

We calculate the vertical stress increase at any point at any depth due to the applied point load as following:
Consider we want to calculate the vertical stress increase at point **A** in figure below:



$$\Delta\sigma_{z,A} = \frac{3.P.Z^3}{2\pi(r^2+Z^2)^{\frac{5}{2}}}$$, and the same at any point.

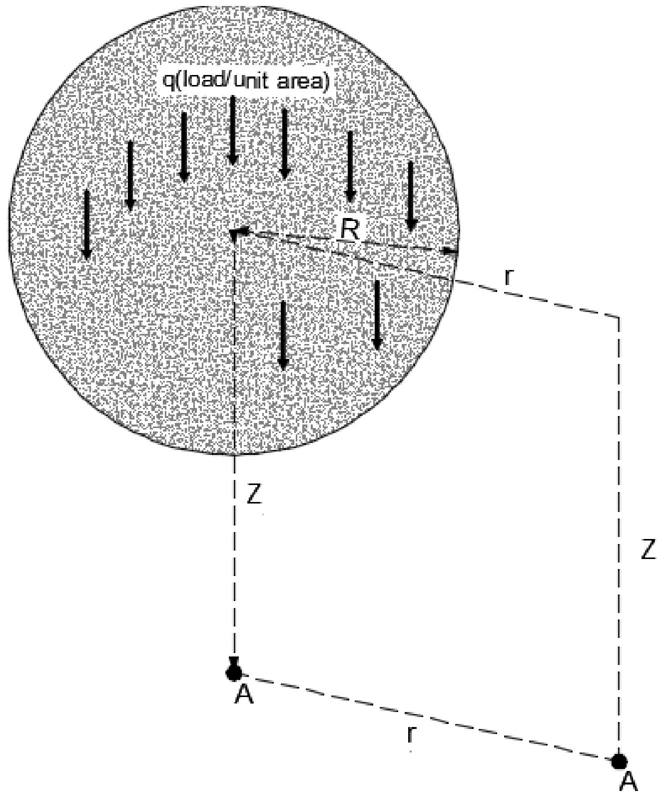
$r = \sqrt{X^2+Y^2}$

X , Y and Z are measured from the point of applied load as shown in figure above.

Note: If there are more than one point load applied on the soil profile at different positions , you should calculate $\Delta\sigma_z$ for each load and then :

$$\Delta\sigma_{z,t} = \Delta\sigma_{z,1} + \Delta\sigma_{z,2} + \Delta\sigma_{z,3} + \cdots + \Delta\sigma_{z,n}$$

Stress Due to a Circularly Loaded Area:



If we want to calculate the vertical stress below the **center** of the circular foundation or at **any point** at distance (r) from the center of the foundation, do the following:

Let $R = \frac{B}{2} = \text{Radius of circular area}$

$r = \text{distance from the center of the foundation to the required point}$

$z = \text{depth of point at which the vertical stress increas is required}$

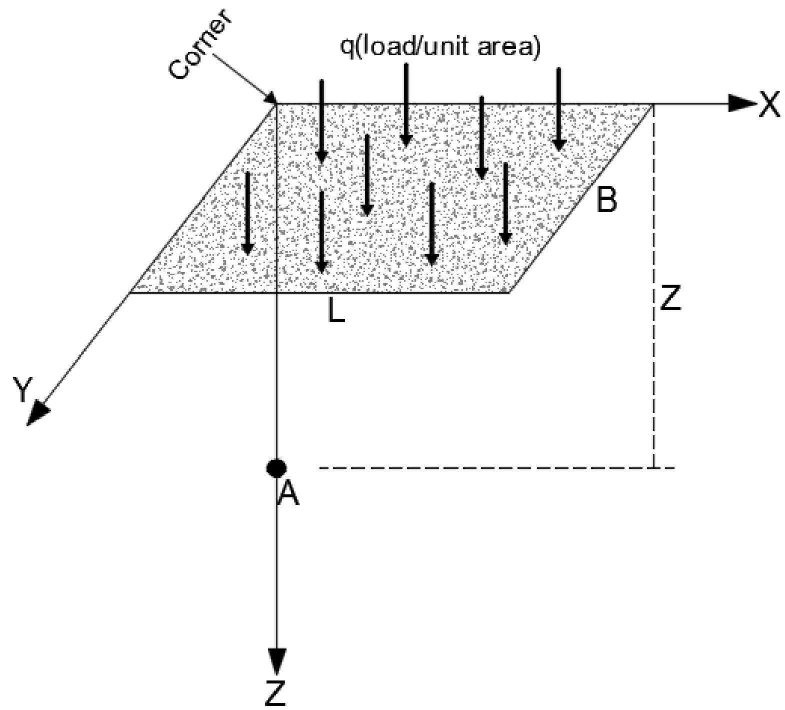
We can calculate the value of $\left(\frac{\Delta\sigma_z}{q_o}\right)$ from **(Table 5.1 P. 226)** which gives the variation of $\left(\frac{\Delta\sigma_z}{q_o}\right)$ with $\left(\frac{r}{B/2}\right)$ and $\left(\frac{z}{B/2}\right)$ $\left\{\text{For } 0 \le \left(\frac{r}{B/2}\right) \le 1\right\}$

$q_o = \text{the stress at the base of the foundation } \left(\frac{\text{column load}}{\text{foundation area}}\right)$

Note that, if the value of $\left(\frac{r}{B/2}\right) = 0.0$ →The point is below the center of the foundation

Vertical Stress Caused by a Rectangularly Loaded Area

Consider we want to calculate the vertical stress increase at point **A** in figure below:



We calculate the vertical stress increase at point below the **corner** of rectangular loaded area as following:

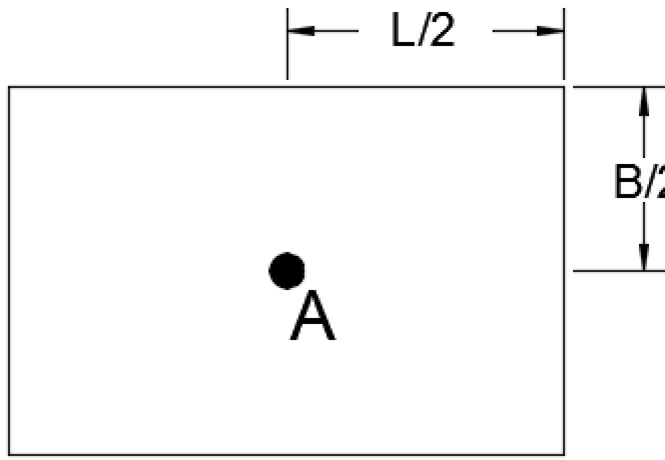
$\Delta\sigma_z = qI$

$I = \text{Influence factor} = f(m, n)$ (From **Table 5.2 P. 228 and 229)**

$m = \frac{B}{Z}$, $n = \frac{L}{Z}$

B: Smaller dimension , L: Larger dimension

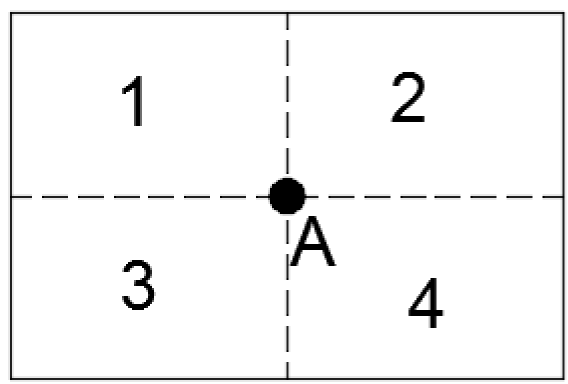
If we want to calculate $\Delta\sigma_z$ below the center of rectangular area there are two methods:



1. Divide this area into 4 areas to make point "**A**" under the corner of each area:

We note that, point "**A**" is under the corner of each rectangular area, so:

$$\Delta\sigma_{z,total} = q(I_1 + I_2 + I_3 + I_4)$$



Because the total area is rectangular and divided into 4 areas it is clear that the four areas are equal so:

$$I_1 = I_2 = I_3 = I_4 = I$$

$$\Delta\sigma_{z,total} = q(4I)$$

2.

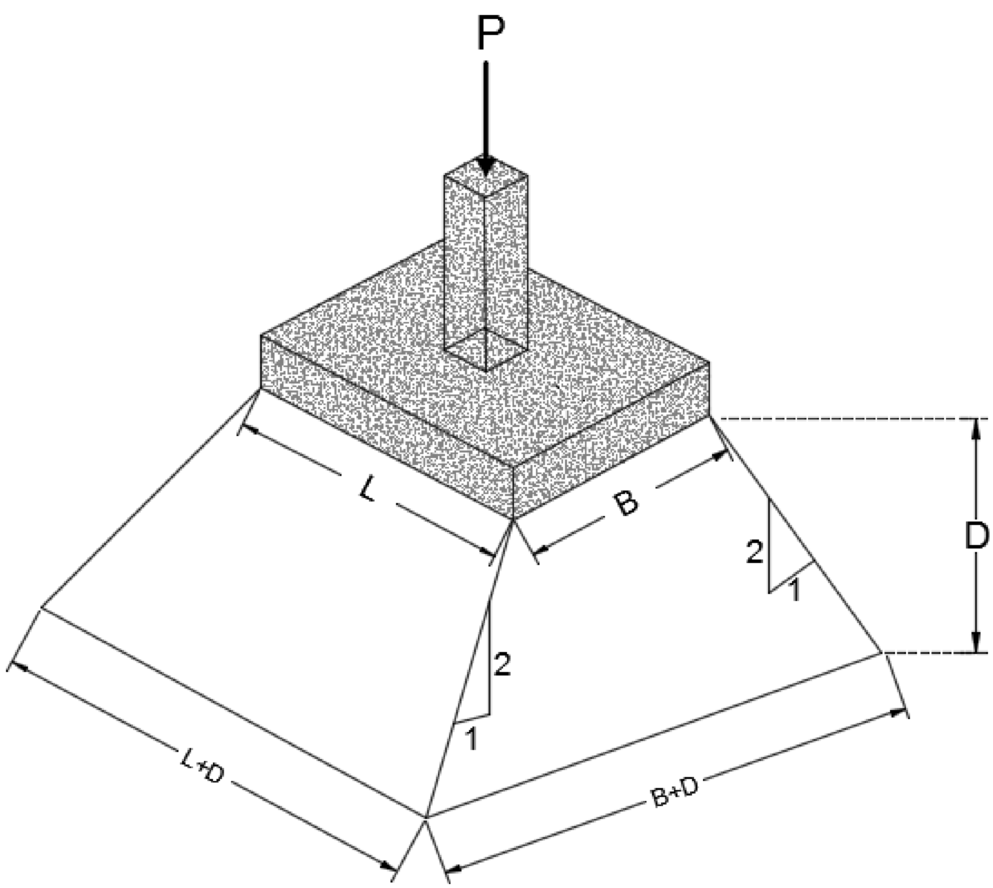
$$\Delta\sigma_{z,total} = qI_c$$

$I_c = f(m_1, n_1)$ (From **Table 5.3 P. 230)**

$$m_1 = \frac{L}{B} \quad , \quad n_1 = \frac{Z}{b} = \frac{2Z}{B}$$

Approximate Method (2:1 Method)

An alternative approximate method can be used rather than (Ch.10) in soil mechanics course, this method is easier and faster than methods in (Ch.10). This method called **(2:1 Method)**. The value of $(\Delta\sigma')$at depth D can be determined using (2:1 method) as following:

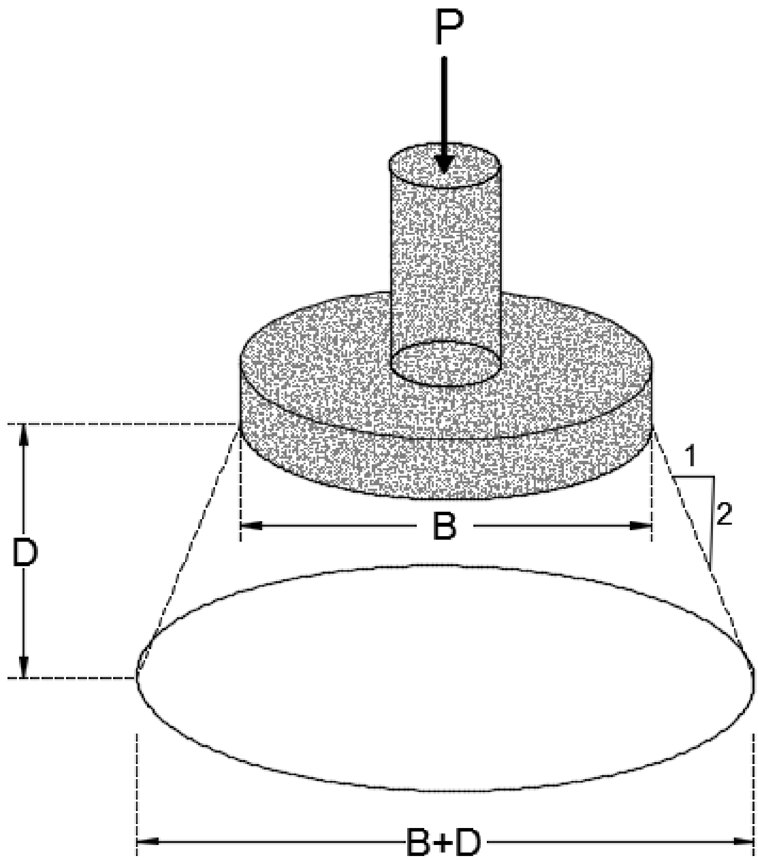


According to this method, the value of $(\Delta\sigma')$ at depth (D) under the **center** of the foundation is:

$$\Delta\sigma'_D = \frac{P}{A} = \frac{P}{(B+D)\times(L+D)}$$

$P =$ the load applied on the foundation (KN).
$A =$ the area of the stress distribution at **depth** (**D**).
Note that the above equation is based on the assumption that the stress from the foundation **spreads out** with a **vertical**-to-**horizontal** slope of **2:1.**

Note: if the foundation is **circular** the value of ($\Delta\sigma'$) at depth (D) under the center of the foundation can be determined as following:

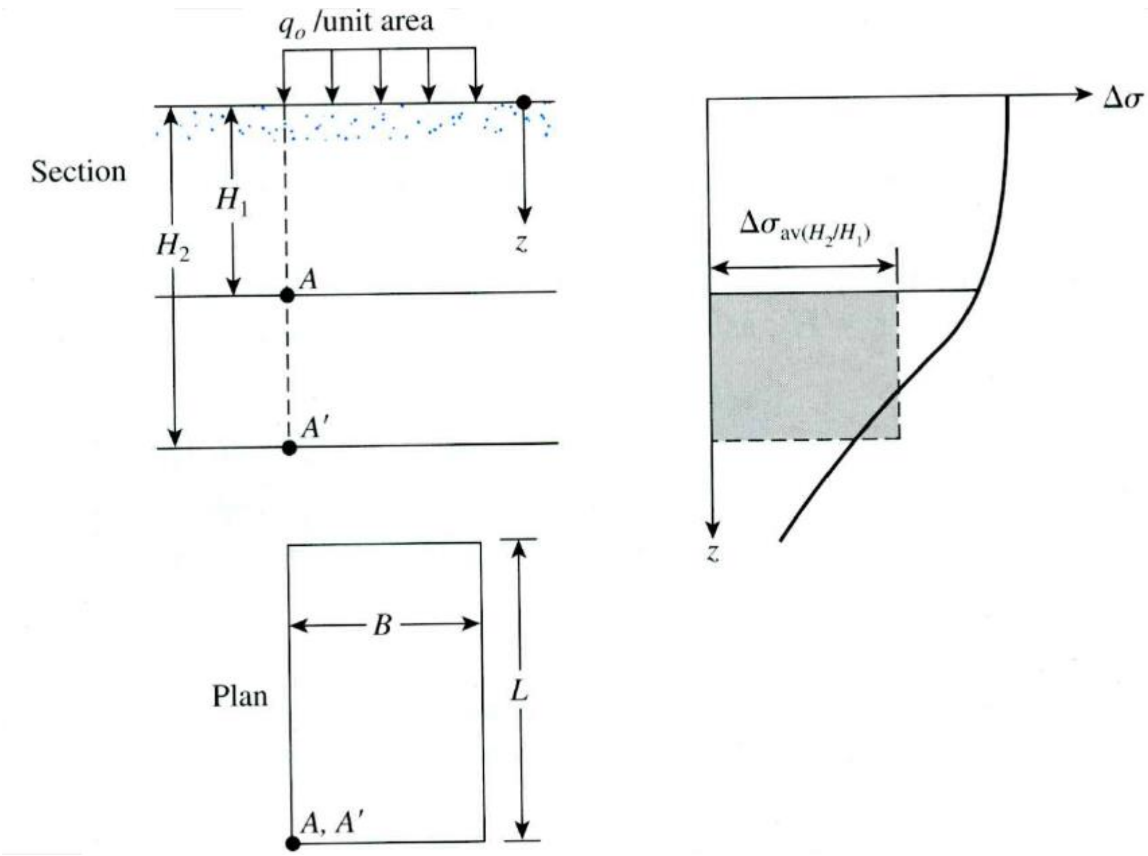


$$\Delta\sigma'_D = \frac{P}{\text{Area at depth (D)}} = \frac{P}{\frac{\pi}{4} \times (B + D)^2}$$

$P =$ the load applied on the foundation (KN).
$B =$ diameter of the foundation(m).

Average Vertical Stress Increase Due to a Rectangularly Loaded Area

In many cases, the average stress increase ($\Delta\sigma'_{av}$) below the **corner** of the rectangular foundation is required (to calculate the consolidation settlement below the corner of rectangular foundation), this can be calculated by the following method:



The average stress increase for layer between $z = H_1$ and $z = H_2$ can be calculated using the following equation:

$$\Delta\sigma'_{av(H_2/H_1)} = q_o \left[\frac{H_2 I_{a(H_2)} - H_1 I_{a(H_1)}}{H_2 - H_1}\right]$$

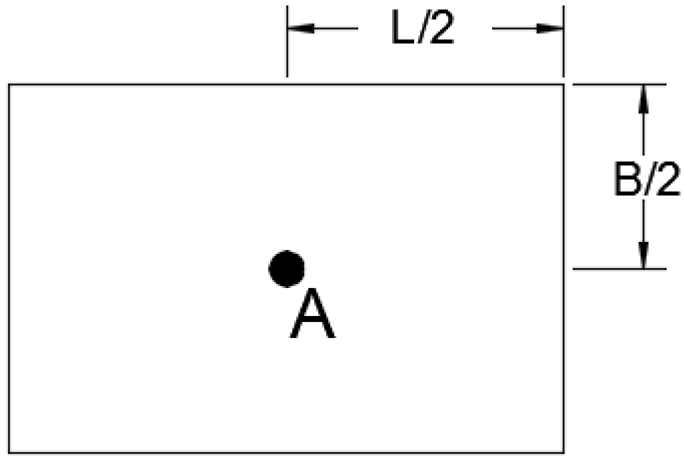
q_o = Stress at the base of the foundation

$$I_{a(H_2)} = I_a \text{ for } z = 0 \text{ to } z = H_2 = f\left(m_2 = \frac{B}{H_2}, n_2 = \frac{L}{H_2}\right)$$

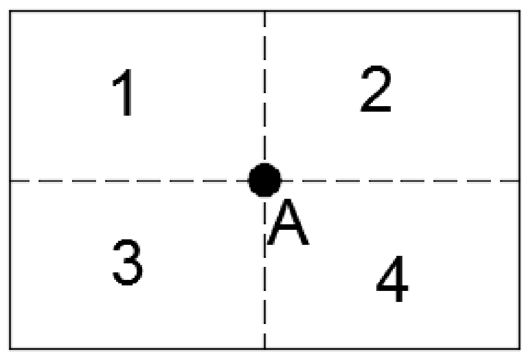
$$I_{a(H_1)} = I_a \text{ for } z = 0 \text{ to } z = H_1 = f\left(m_2 = \frac{B}{H_1}, n_2 = \frac{L}{H_1}\right)$$

Values of $I_{a(H_2)}$ and $I_{a(H_1)}$ can be calculated from (**Figure 5.7 P.234**)

If we want to calculate $\Delta\sigma'_{av(H_2/H_1)}$ below the center of rectangular area:



Divide this area into 4 areas to make point "**A**" under the corner of each area:



We note that, point "**A**" is under the corner of each rectangular area, so:

Because the total area is rectangular and divided into 4 areas it is clear that the four areas are equal so:

$$\Delta\sigma'_{av(H_2/H_1),center} = 4 \times \Delta\sigma'_{av(H_2/H_1),corner}$$

Solve Example (5.2)

There is another method used to calculate $\Delta\sigma'_{av}$:

$$\Delta\sigma'_{av} = \frac{\Delta\sigma'_t + 4\Delta\sigma'_m + \Delta\sigma'_b}{6} \quad \text{(under the center or under the cornere)}$$

$\Delta\sigma'_t =$ Increase in effective stress at the top of soil layer.

$\Delta\sigma'_m =$ Increase in effective stress at the middle of soil layer.

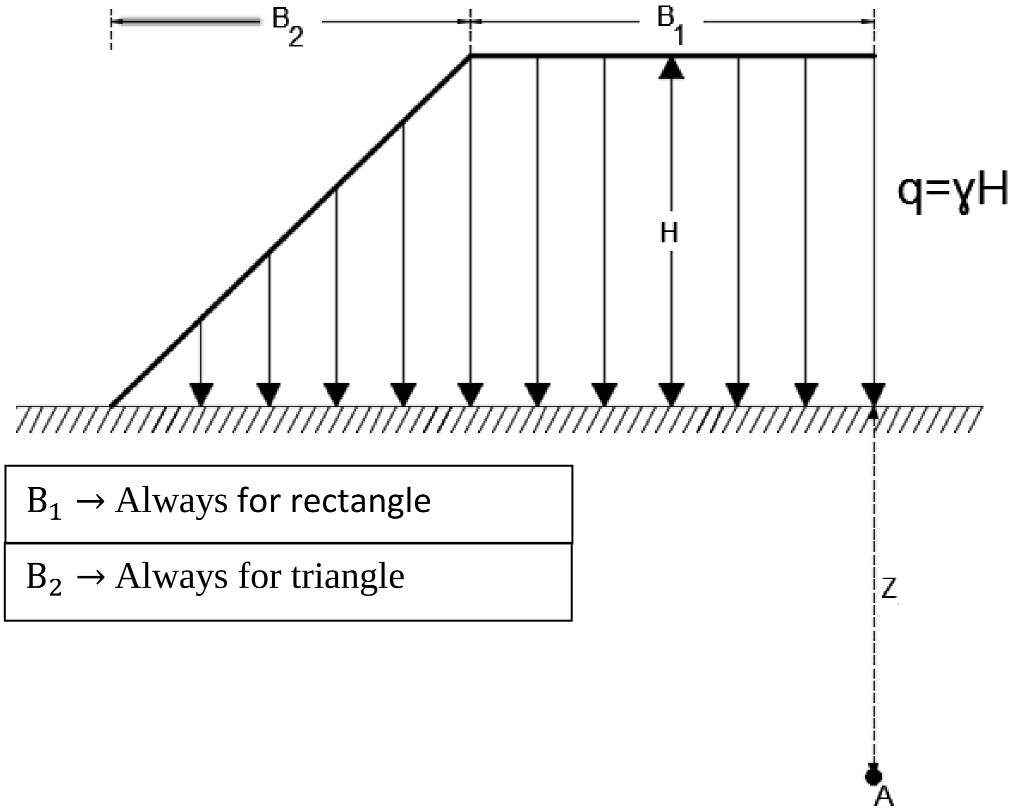
$\Delta\sigma'_b =$ Increase in effective stress at the bottom of soil layer.

$\Delta\sigma'_t, \Delta\sigma'_m$ and $\Delta\sigma'_b$ can be caculated under the **center** or **cornere** of the foundation (as required).

If $\Delta\sigma'_{av}$ is required under the center of the foundation, we can use (2:1) method to calculate $\Delta\sigma'_t, \Delta\sigma'_m$ and $\Delta\sigma'_b$.

Stress Increase under Embankment Loading

Consider we want to calculate the vertical stress increase at point **A** in figure below:



$B_1 \rightarrow$ Always for rectangle	
$B_2 \rightarrow$ Always for triangle	

We can calculate the vertical stress increase at any point due to the embankment load as following:

$\Delta\sigma_z = qI'$

$q = \text{Embankment Load} = \gamma H$

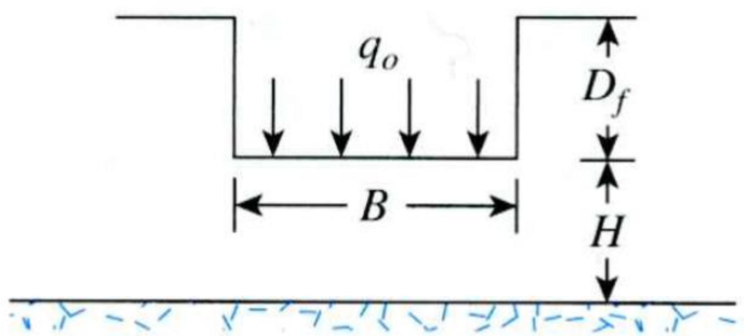
The value of I' can be taken from (**Figure 5.11 P.237**) according to the values of $\frac{B_1}{z}$ and $\frac{B_2}{z}$

Elastic Settlement

Elastic Settlement of Foundations on Saturated Clay

This method used for calculating elastic settlement only for saturated clay from the following equation:

$$S_e = A_1 A_2 \frac{q_o B}{E_s}$$



$A_1 = f\left(\frac{H}{B} \& \frac{L}{B}\right)$ = factor depends on the shape of the loaded area and can be estimated from (**Figure 5.14 P.244**)

$A_2 = f\left(\frac{D_f}{B}\right)$ = factor depends on the depth of the footing and can be estimated from (**Figure 5.14 P.244**)

H = Thickness of clay layer under the bottom of the foundation

B = Foundation width , L = Foundation length

E_s = Modulus of elsticity of clay layer $= \beta c_u$

c_u = undrained shear strength for clay

$\beta = f(PI, OCR)$

PI = Plasticity Index $= LL - PL$

$$OCR = \text{Overconsolidated Ratio} = \frac{\text{Preconsolidation Pressure}}{\text{Present Effective Pressure}} = \frac{\sigma'_c}{\sigma'_o}$$

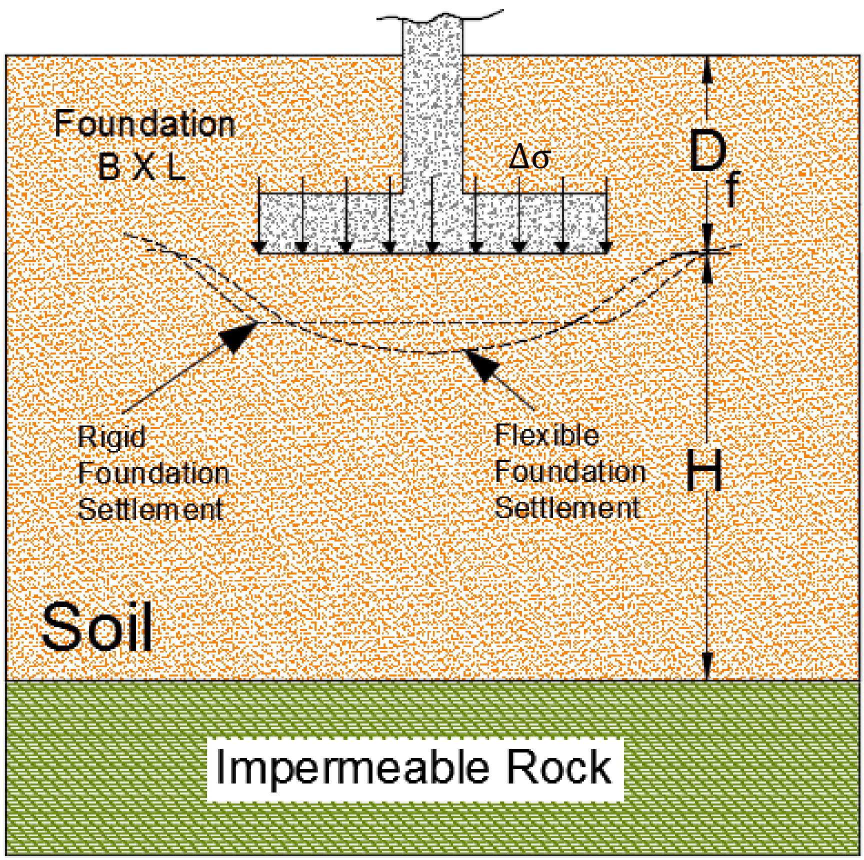
Now, the value of β can be taken from **(Table 5.7 P.244)** and then calculate the value of E_s.

Settlement Based on the Theory of Elasticity

Elastic or immediate settlement (S_e) occurs directly after the application of the load without change in the moisture content of the soil.
The magnitude of the elastic settlement will depend on the **flexibility** of the foundation (flexible or rigid), and on the type of material (soil) that the foundation will rest on it (i.e. this method is valid for both **sand** and **clay**).

Elastic Settlement under a **flexible** foundation can be calculated from the following equation:

$$S_e = \Delta\sigma(\alpha B')\frac{1-\mu_s^2}{E_s}I_s I_f$$



$L =$ Larger dimension of the foundation

$B =$ Smaller dimension of the foundation

$\Delta\sigma =$ Applied Net Pressure $=$ Load/Area

$E_s =$ Modulus of Elsticity of the soil through the depth H.

If there are more than one soil layer through the depth H we can find the modulus of elasticity for all layers by weighted average method:

$$E_s = \frac{\sum E_{s(i)} \times \Delta Z}{Z}$$

$E_{s(i)}$ = modulus of elasticity of each layer.

$\Delta Z =$ depth of each layer.

$Z = H$ or $5B$ whichever is **smaller**

$H =$ Distance from the face of the footing to the nearest rock layer (as shown in figure above)

Now, as shown in figure above, the elastic settlement under a flexible foundation at the center is larger than at the corner, thus there are some differences in calculating (S_e) under the **center** and under the **corner** of the footing.

These differences can be considered in the values of $(B'$ and $\alpha)$.

For calculating S_e under the center of the foundation:

$$B' = \frac{B}{2} \quad \text{and} \quad \alpha = 4$$

For calculating S_e under the corner of the foundation:

$$B' = B \quad \text{and} \quad \alpha = 1$$

Where, α = factor depends on the location on the foundation where settlement is being calculated.

I_s = shape factor.

$$I_s = F_1 + \frac{1-2\mu_s}{1-\mu_s} F_2$$

F_1(can be calculated from **Table 5.8 P.248**)

F_2(can be calculated from **Table 5.9 P.250**)

To get the values of $F_1$and F_2 from tables we need the values of $m'$and n'

$$m' = \frac{L}{B} \quad , \quad n' = \frac{H}{B'}$$

I_f = factor depens on depth of excavation, footing dimensions and soil type

The value of (I_f) can be calculated from (**Table 5.10 P.252)**.

Note: If $D_f = 0.0 \rightarrow\rightarrow I_f = 1$

Elastic Settlement under a **rigid** foundation:
From the figure above (page 93) it is noted that the elastic settlement under a rigid foundation is **constant** and **less than** S_e for flexible foundation (at center).
So, the value of S_e under a rigid foundation can be estimated as following:
$S_{e(rigid)} = 0.93S_{e(flexible,center)}$

See **(example 5.5 P.252)** in your textbook.

Settlement of Sandy Soil: Using Strain Influence Factor

This method is one of the most famous methods used to calculate elastic settlement for **sandy soil.** According to this method, the settlement is:

$$S_e = C_1 C_2 (\bar{q} - q) \sum_0^{z_2} \frac{I_z}{E_s} \Delta z$$

$C_1 =$ correction factor for depth of foundation embedment

$$C_1 = 1 - 0.5 \times \frac{q}{\bar{q} - q}$$

$C_2 =$ correction factor to account for creep in soil
(i. e. at what time after construction, you want to calculate S_e?)

$$C_2 = 1 + 0.2 \log\left(\frac{\text{Time in years}}{0.1}\right)$$

$\bar{q} =$ gross stress at the level of the foundation $= \frac{\text{Applied gross Load}}{\text{Foundation Area}} = \frac{P}{A}$

$q = \textbf{effective stress}$ at the base of the foundation
$E_s =$ modulus of elasticity of soil
$\Delta z =$ thickness of each soil layer
$I_z =$ influence line factor

Note:
$\bar{q} - q = \textbf{net stress}$ applied at the base of the foundation, so if you are given the **net load** $\rightarrow \bar{q} - q = \frac{\text{Net Load}}{\text{Foundation Area}}$

Procedures for calculating elastic settlement using strain influence factor:

1. Plot the variation of I_z with depth (under the base of the foundation):

To do this**, three values** of I_z at three depths should be calculated:

A- Calculate the value of $I_{z,initial}$ at depth $z = 0.0$ (i.e.at the base of the foundation) by the following equation:

$$I_{z,initial} = 0.1 + 0.0111\left(\frac{L}{B} - 1\right) \leq 0.2$$

Special cases:

✓ For square or circular footings $\rightarrow \frac{L}{B} = 1 \rightarrow I_{z,initial} = 0.1$

✓ For strip or continuous footings→

$$\rightarrow \frac{L}{B} \geq 10 \rightarrow I_{z,initial} = 0.2 \text{ (max value at } \frac{L}{B} = 10)$$

✓ For rectangular footing→ apply on the equation by $\frac{L}{B}$ to get $I_{z,initial}$

B- Calculate the value of $I_{z,max}$ at depth $z = z_1$ from the base of the foundation using the following equation:

$$I_{z,max} = 0.5 + 0.1\sqrt{\frac{\bar{q} - q}{q'_{z(1)}}}$$

$q'_{z(1)}$ = effective stress from the ground surface to depth z_1 befor construction the foundation

So to calculate $q'_{z(1)}$ and then calculate $I_{z,max}$, we fiestly must calculate (z_1) by the following equation:

$$\frac{z_1}{B} = 0.5 + 0.0555\left(\frac{L}{B} - 1\right) \leq 1$$

Special cases:

✓ For square or circular footings $\rightarrow \frac{L}{B} = 1 \rightarrow z_1 = 0.5\ B$

✓ For strip or continuous footings→

$$\rightarrow \frac{L}{B} \geq 10 \rightarrow z_1 = B \text{ (max value at } \frac{L}{B} = 10)$$

✓ For rectangular footing→ apply on the equation by $\frac{L}{B}$ to get z_1

Now, you can easily calculate $q'_{z(1)}$ an then calculate $I_{z,max}$

C- The value of $I_{z,final} = 0.0$ at depth $z = z_2$ from the base of the foundation, so we need to calculate the value of (z_2) by the following equation:

$$\frac{z_2}{B} = 2 + 0.222\left(\frac{L}{B} - 1\right) \leq 4$$

Special cases:

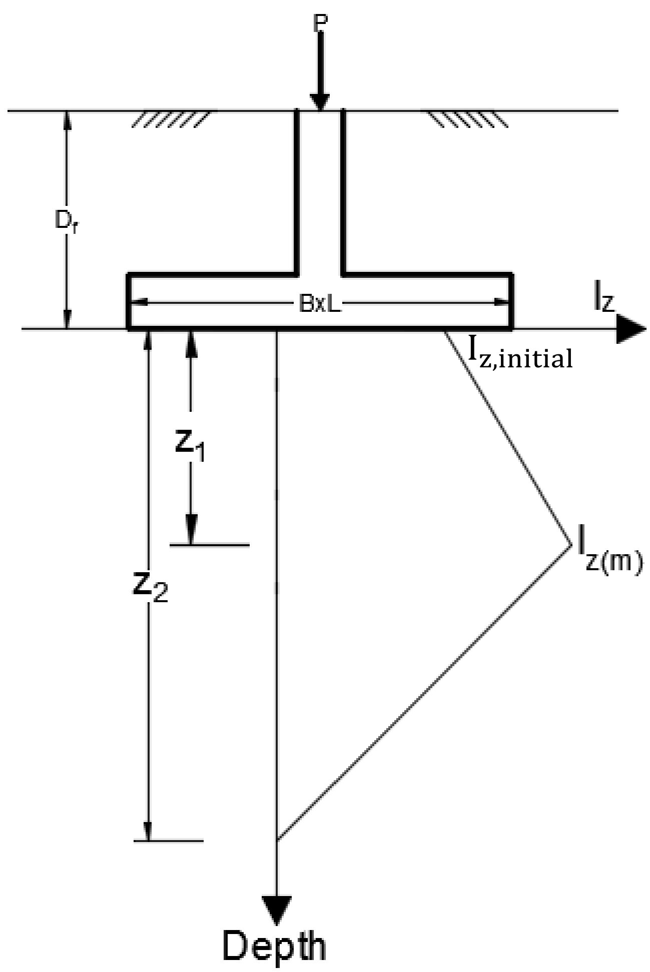
✓ For square or circular footings $\rightarrow \frac{L}{B} = 1 \rightarrow z_2 = 2\,B$

✓ For strip or continuous footings→

$$\rightarrow \frac{L}{B} \geq 10 \rightarrow z_2 = 4B \ \ (\text{max value at } \frac{L}{B} = 10)$$

✓ For rectangular footing→ apply on the equation by $\frac{L}{B}$ to get z_2

Now, you can draw the variation of I_z with depth as following:



2. Plot the variation of E_s with depth (under the base of the foundation):

Firstly we want to calculate E_s for each layer under the base of the foundations as following:

➢ Using correlation from **cone penetration test**:

$E_s = 2.5\, q_c$ (For square or circular foundation)

$E_s = 3.5\, q_c$ (For strip or continuous foundation)

$q_c =$ cone penetration resistance

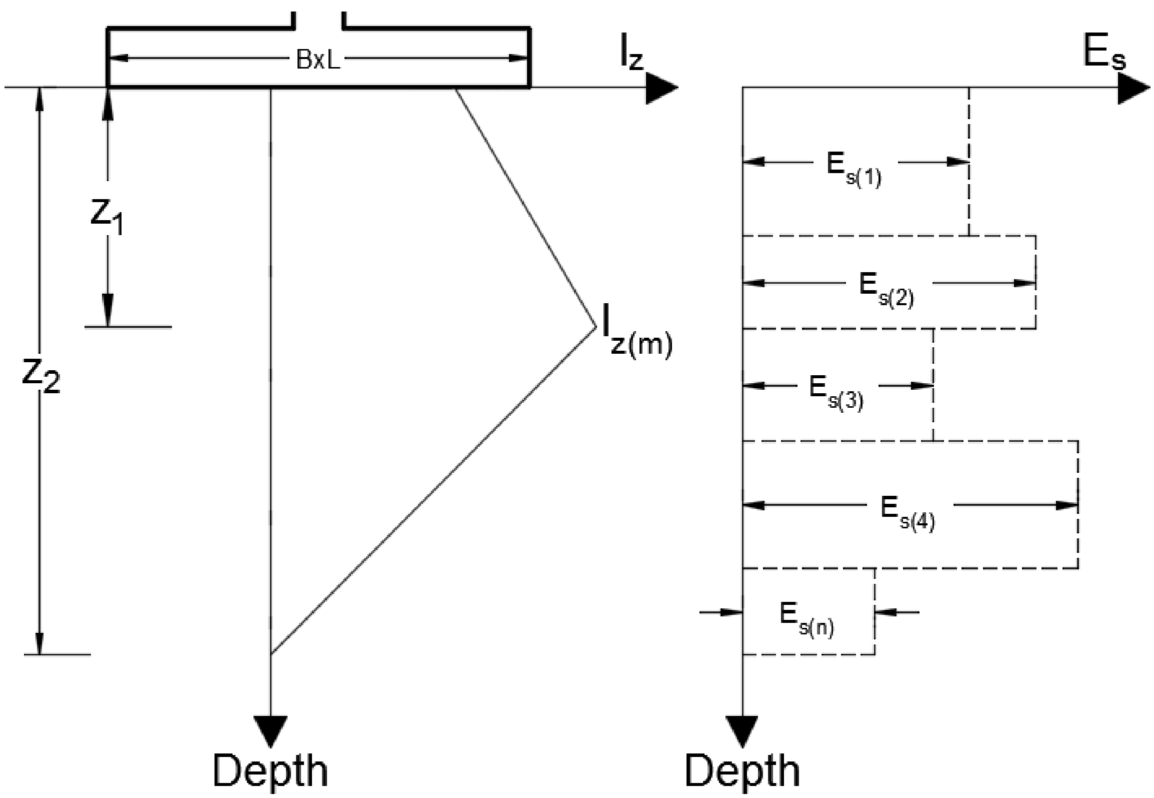
For rectangular foundation:

$$E_{s(rectangle)} = \left(1 + 0.4\log\frac{L}{B}\right) E_{s(square)}$$

➢ Using correlation from standard penetration test:

$E_s = 500\, N_{60}$ (KN/m^2) for all types of foundations, or you may given different rule according the type of sand.

Now, calculate the value of E_s for each soil layer and then draw the variation of E_s with depth on the same graph of I_z with depth as following:



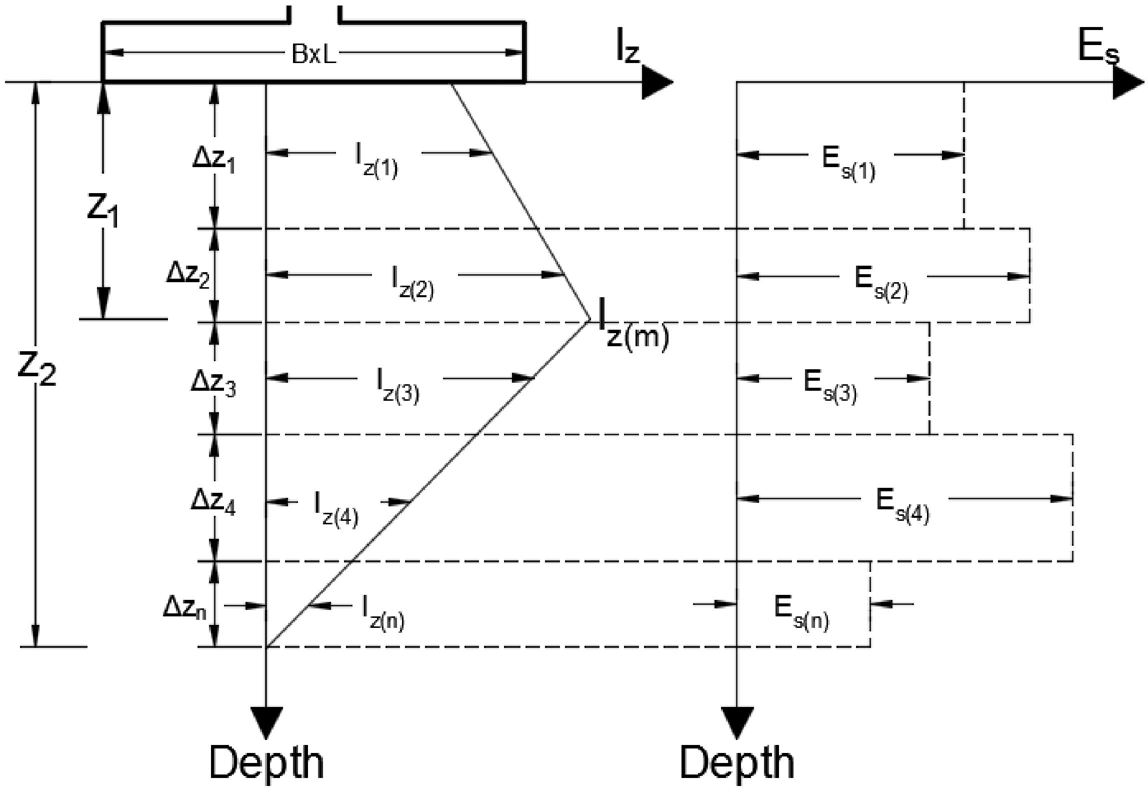
3. Divide the soil layer from $z = 0$ to $z = z_2$ into a number of layers:

Each layer must satisfy the following two conditions:

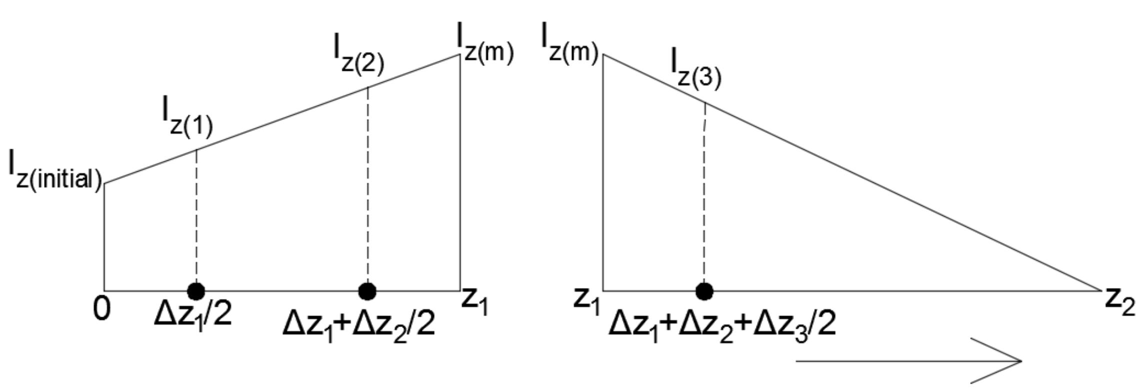
✓ The value of E_s must be constant along the layer thickness.

✓ The slope of I_z line must be constant along the layer thickness, if not you have to divide this layer into two layers although the value of E_s is constant.

After divided the layers, match between E_s and I_z charts by horizontal line as shown below:



4. Calculate the value of I_z at the middle of each layer by interpolation:



5. Prepare table (such the following table) to obtain $\sum_0^{z_2} \frac{I_z}{E_s} \Delta z$:

Layer No.	Δz	E_s	I_z at the middle of each layer	$\frac{I_z}{E_s} \Delta z$
1	Δz_1	$E_{s(1)}$	$I_{z(1)}$	$\frac{I_{I_{z(1)}}}{E_{s(1)}} \Delta z_1$
2	Δz_2	$E_{s(2)}$	$I_{z(2)}$	$\frac{I_{I_{z(2)}}}{E_{s(2)}} \Delta z_2$
3	Δz_3	$E_{s(3)}$	$I_{z(3)}$	$\frac{I_{I_{z(3)}}}{E_{s(3)}} \Delta z_3$
4	Δz_4	$E_{s(4)}$	$I_{z(4)}$	$\frac{I_{I_{z(4)}}}{E_{s(4)}} \Delta z_4$
n	Δz_n	$E_{s(n)}$	$I_{z(n)}$	$\frac{I_{I_{z(n)}}}{E_{s(n)}} \Delta z_n$
				$\sum_0^{z_2} \frac{I_z}{E_s} \Delta z$

6. Finally calculate the value of S_e:

$$S_e = C_1 C_2 (\bar{q} - q) \sum_0^{z_2} \frac{I_z}{E_s} \Delta z$$

Settlement of Foundation on Sand Based on Standard Penetration Resistance

Bowles (1977) proposed a correlation for a net allowable bearing capacity for foundations:

$$q_{all,net}(kN/m^2) = \frac{N_{60}}{0.05} \times F_d \left(\frac{S_e}{25}\right) \quad \text{For } B \leq 1.22 \text{ m}$$

$$q_{all,net}(kN/m^2) = \frac{N_{60}}{0.08}\left(\frac{B+0.3}{B}\right)^2 F_d \left(\frac{S_e}{25}\right) \quad \text{For } B > 1.22 \text{ m}$$

$$F_d = 1 + 0.33\frac{Df}{B} \leq 1.33$$

$S_e =$ elastic settlement in (mm)

$N_{60} =$ corrected standard penetration resistance (number).

$$S_e(mm) = \frac{1.25 \times q_{all,net}}{N_{60} \times F_d}\left(\frac{B}{B+0.3}\right)^2 \quad \text{For } B \leq 1.22\text{m}$$

$$S_e(mm) = \frac{2 \times q_{all,net}}{N_{60} \times F_d}\left(\frac{B}{B+0.3}\right)^2 \quad \text{For } B > 1.22\text{m}$$

Consolidation Settlement

Primary Consolidation Settlement

As we discussed previously in soil mechanics course (Ch.11), the consolidation settlement is the process of water expulsion from the voids of saturated clay with time. But here, we want to calculate total primary consolidation settlement (i.e. @ $t = \infty$) using the following three equations:

1. For normally consolidated clay:

$$\text{For Normally consolidated clay} \rightarrow OCR = \frac{\sigma'_c}{\sigma'_o} = 1$$

$$S_c = \frac{C_c \times H}{1 + e_o} \times \log\left(\frac{\sigma'_o + \Delta\sigma'_{av}}{\sigma'_o}\right)$$

$S_c =$ primary consolidation settlement

$C_s =$ swell index , $H =$ thickness of clay layer , $e_o =$ initial void ratio

$\sigma'_o =$ present effective overburden pressure (at the middle of clay layer)

$\Delta\sigma'_{av} =$ average effective stress increase for clay layer

2. For Overconsolidated clay

For Overconsolidated clay $\rightarrow \mathrm{OCR} = \frac{\sigma'_c}{\sigma'_o} > 1$

There are two cases:

Case One ($\sigma'_c \geq \sigma'_o + \Delta\sigma'_{av}$):

$$S_c = \frac{C_s \times H}{1+e_o} \times \log\left(\frac{\sigma'_o + \Delta\sigma'_{av}}{\sigma'_o}\right)$$

C_s = swell index

Case Two ($\sigma'_c < \sigma'_o + \Delta\sigma'_{av}$):

$$S_c = \frac{C_s \times H}{1+e_o} \times \log\left(\frac{\sigma'_c}{\sigma'_o}\right) + \frac{C_c \times H}{1+e_o} \times \log\left(\frac{\sigma'_o + \Delta\sigma'_{av}}{\sigma'_c}\right)$$

If the stress at the base of the foundation is decreased with depth, we can calculate the value of $\Delta\sigma'_{av}$ by Simpson's rule as following:

$$\Delta\sigma'_{av} = \frac{\Delta\sigma'_t + 4\Delta\sigma'_m + \Delta\sigma'_b}{6}$$

$\Delta\sigma'_t$ = Increase in effective stress at the top of clay layer.

$\Delta\sigma'_m$ = Increase in effective stress at the middle of clay layer.

$\Delta\sigma'_b$ = Increase in effective stress at the bottom of clay layer.

Secondary Consolidation Settlement

At the end of primary consolidation (after complete dissipation of pore water pressure), some settlement is observed that is due to the plastic adjustment of soil fabrics. This stage of consolidation is called secondary consolidation.

$$S_s = \frac{H \times C_\alpha}{1+e_p} \times \log\left(\frac{t_2}{t_1}\right)$$

e_p = void ratio at the end of primary consolidation stage.

t_1 = time at the end of primary consolidation settlement (start secondary consolidation settlement).

t_2 =any time after beginning secondary consolidation settlement.

the value of C_α is depend on type of clay (N.C.Clay or O.C.Clay or Organic clay) and there is a typical values for each type.

Plate Load Test

The ultimate bearing capacity of foundations, as well as the allowable bearing capacity based on tolerable settlement considerations, can be **effectively** determined from the field load test, generally referred as **plate load test.**

Plate Properties:

The plate used in this test is made of steel and have the following dimensions:

✓ If the plate is circular, the diameter will be (150mm to 762mm) with 25mm thickness.

✓ If the plate is square, the dimensions are (305mm x 305mm) with 25mm thickness.

Test Mechanism:

✓ To conduct a test, a hole is excavated with a minimum diameter of 4B (B is the diameter of the test plate) to depth D_f (depth of proposed foundation).

✓ The plate is at the center of the hole, and the load is applied on the plate and increased gradually.

✓ As the load increase, the settlement of the plate is observed on dial gauge.

✓ The test should be conducted until failure, or the settlement of the plate became 25mm.

✓ The value of load at which the test is finished is the ultimate load can be resisted by the plate.

✓ Divide the ultimate load on the plate area to get ultimate bearing capacity of the plate $q_{u(P)}$.

For test in clay:

$$q_{u(F)} = q_{u(P)}$$

$q_{u(F)} =$ ultimate bearing capacity of the proposed foundation

For test in sandy soil:

$$q_{u(F)} = q_{u(P)} \frac{B_F}{B_P}$$

$B_F =$ width of the foundation , $B_P =$ width of the test plate

Problems

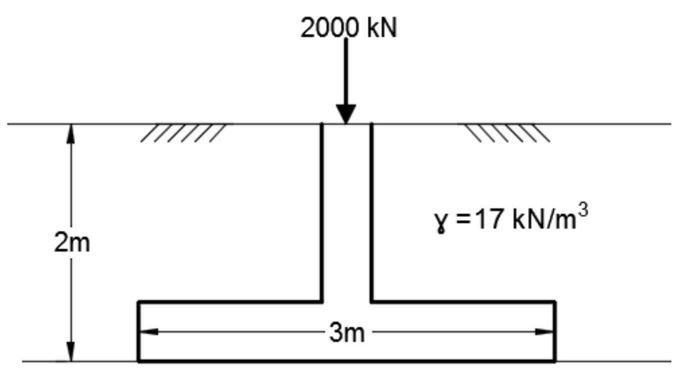
1.

A 3m×3m square footing is shown in the figure below, if the net load on the foundation is 2000 kN.

A. Use strain influence factor method to calculate elastic settlement of the footing after 6 years of construction. Given to you values of tip resistance from cone penetration test:

Depth (m) from the ground surface	$q_c(kN/m^2)$
0-2	8000
2-4	10000
4-6	9000
6-8	8500

B. If the corrected standard penetration number $(N_{60}) = 10$ and the allowble settlement is 25.4mm. Calculate the net allowable bearing capacity using Bowles equations.



Solution

A. $S_e = ??$ (using influence line factor method).

$$S_e = C_1 C_2(\bar{q} - q)\sum_{0}^{z_2}\frac{I_z}{E_s}\Delta z$$

$$C_1 = 1 - 0.5 \times \frac{q}{\bar{q} - q}$$

$q = \gamma D_f = 17 \times 2 = 34\ \text{kN/m}^2$

$$\bar{q} - q = \frac{\text{Net Load}}{\text{Foundation Area}} = \frac{2000}{3 \times 3} = 222.22\ \text{kN/m}^2$$

$$\rightarrow C_1 = 1 - 0.5 \times \frac{34}{222.22} = 0.923$$

$$C_2 = 1 + 0.2 \log\left(\frac{\text{Time in years}}{0.1}\right) \quad \text{time} = 6\text{years}$$

$$\rightarrow C_2 = 1 + 0.2 \log\left(\frac{6}{0.1}\right) = 1.356$$

Calculation of $\sum_0^{z_2} \frac{I_z}{E_s} \Delta z$:

1. Calculation of I_z with depth:

✓ At depth $z = 0$ (at the base of the foundation) $\rightarrow I_z = I_{z,\text{initial}}$

$$I_{z,\text{initial}} = 0.1 + 0.0111\left(\frac{L}{B} - 1\right) \text{ but for square footing } \rightarrow I_{z,\text{initial}} = 0.1$$

✓ At depth $z = z_1 \rightarrow I_z = I_{z(\max)}$

$$I_{z(\max)} = 0.5 + 0.1\sqrt{\frac{\bar{q} - q}{q'_{z(1)}}}$$

Before calculating $I_{z(\max)}$ we need to calculate $q'_{z(1)}$ at depth z_1

$$\frac{z_1}{B} = 0.5 + 0.0555\left(\frac{L}{B} - 1\right) \text{ but for square footing} \rightarrow z_1 = 0.5\ B$$

$\rightarrow z_1 = 0.5\ B = 0.5 \times 3 = 1.5\text{m}$

$q'_{z(1)} = 17 \times (2 + 1.5) = 59.5\ \text{kN/m}^2$

$$\rightarrow I_{z(\max)} = 0.5 + 0.1\sqrt{\frac{222.22}{59.5}} = 0.69$$

✓ At depth $z = z_2 \rightarrow I_z = 0.0$

$$\frac{z_2}{B} = 2 + 0.222\left(\frac{L}{B} - 1\right) \text{ but for square footing} \rightarrow z_2 = 2\ B$$

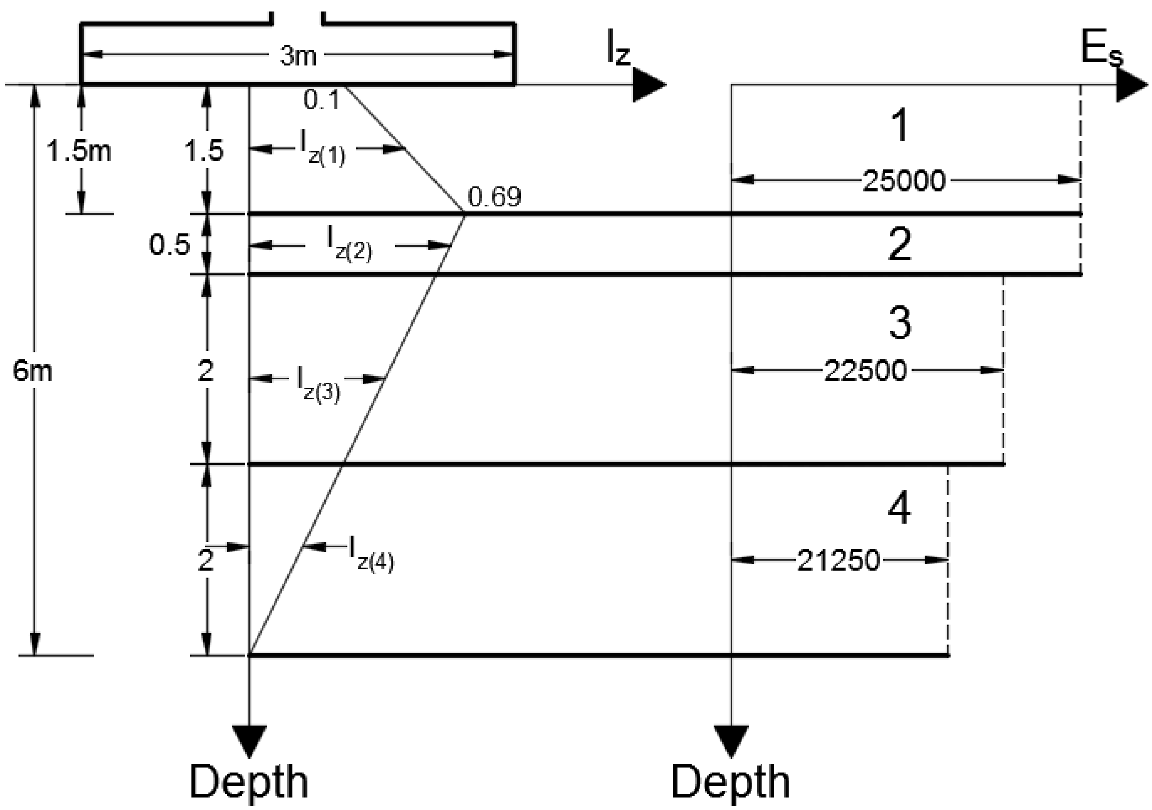
$\rightarrow z_2 = 2B = 2 \times 3 = 6\text{m}$

I.e. we concerned about soil to depth of 6m below the foundation or (6+2) m from the ground surface.

2. Calculation of E_s with depth:
The value of E_s will vary with depth according the values of q_c and for square footing → $E_s = 2.5\, q_c$ →→

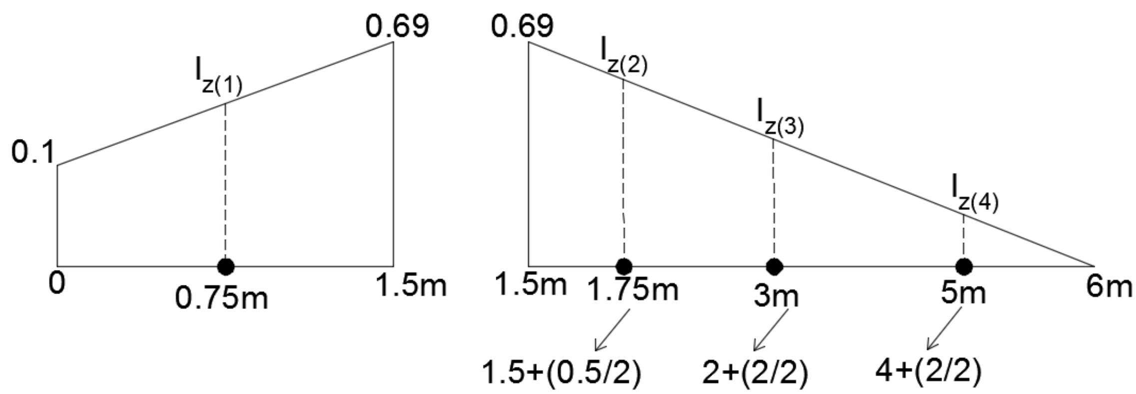
Depth (m) from the ground surface	$q_c (kN/m^2)$	$E_s = 2.5\, q_c$ (kN/m^2)
0-2	8000	20000
2-4	10000	25000
4-6	9000	22500
6-8	8500	21250

3. Now we can draw the variation of I_z and E_s with depth and then divide the soil for layers according the values of E_s and the slope of I_z:



Note that the layer (2) has the same value of E_s for layer (1), but because the slope is differing, we divided them into two layers.

Now we want to calculate the value of I_z at the middle of each layer by interpolation:



By interpolation (using calculator):

$I_{z(1)} = 0.395$, $I_{z(2)} = 0.651$, $I_{z(3)} = 0.46$, $I_{z(4)} = 0.153$

4. Now we can calculate the value of $\sum_0^{z_2} \frac{I_z}{E_s} \Delta z$:

Layer No.	$\Delta z(m)$	$E_s(kN/m^2)$	I_z at the middle of each layer	$\frac{I_z}{E_s}\Delta z$
1	1.5	25000	0.395	2.37×10^{-5}
2	0.5	25000	0.651	1.30×10^{-5}
3	2	22500	0.46	4.08×10^{-5}
4	2	21250	0.153	1.44×10^{-5}
			$\sum_0^{z_2} \frac{I_z}{E_s}\Delta z = 9.192 \times 10^{-5}$	

$$\rightarrow S_e = 0.923 \times 1.356 \times 222.222 \times 9.192 \times 10^{-5}$$
$$= 0.02557m = 25.57\ mm\checkmark.$$

B. $q_{all,net} = ??\ (\text{if } S_e = 25.4mm \text{ and } N_{60} = 10)$

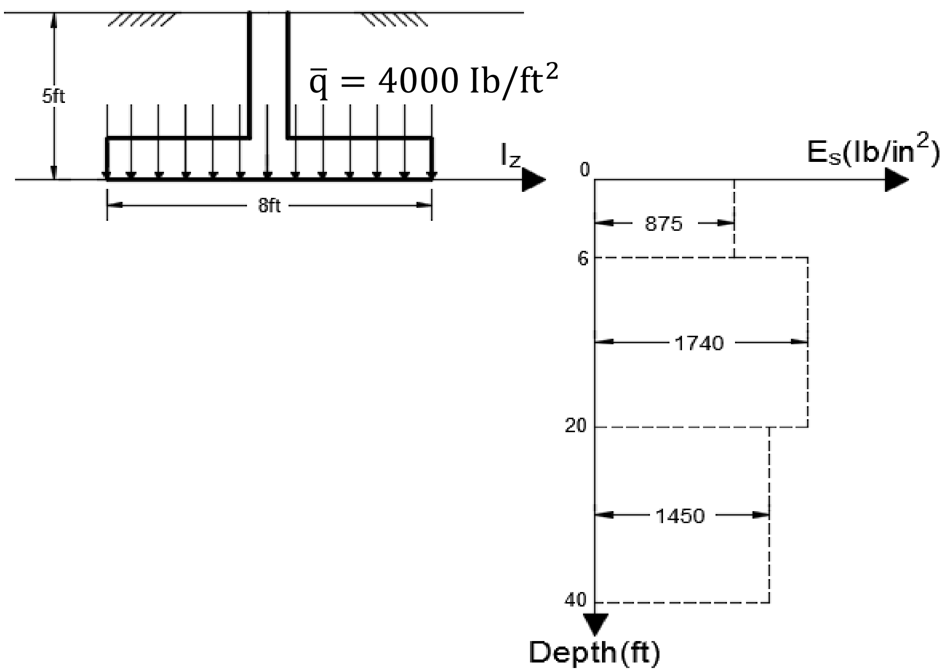
$$q_{all,net}(kN/m^2) = \frac{N_{60}}{0.08}\left(\frac{B+0.3}{B}\right)^2 F_d \left(\frac{S_e}{25}\right) \quad \text{For } B > 1.22\ m$$

$$F_d = 1 + 0.33\frac{Df}{B} \leq 1.33 \rightarrow F_d = 1 + 0.33 \times \frac{2}{3} = 1.22$$

$$q_{all,net}(kN/m^2) = \frac{10}{0.08}\left(\frac{3+0.3}{3}\right)^2 \times 1.22\left(\frac{25.4}{25}\right) = 184.52\ kN/m^2\checkmark.$$

2.

A continuous foundation resting on a deposit of sand layer is shown in the figure below, along with the variation of the modulus of elasticity of soil (E_s). Assuming $\gamma = 115\ \mathrm{Ib/ft^3}$ and the time = 10 years. Calculate the elastic settlement using strain influence factor.



Solution

$$S_e = C_1 C_2 (\bar{q} - q) \sum_{0}^{z_2} \frac{I_z}{E_s} \Delta z$$

$$C_1 = 1 - 0.5 \times \frac{q}{\bar{q} - q}$$

$$q = \gamma D_f = 115 \times 5 = 575 \mathrm{b/ft^2}\ ,\ \ \bar{q} = 4000\ \mathrm{Ib/ft^2}$$

$$\bar{q} - q = 4000 - 575 = 3425\ \mathrm{b/ft^2}$$

$$\rightarrow C_1 = 1 - 0.5 \times \frac{575}{3425} = 0.916$$

$$C_2 = 1 + 0.2 \log\left(\frac{\text{Time in years}}{0.1}\right) \quad \text{time} = 10\text{years}$$

$$\rightarrow C_2 = 1 + 0.2\log\left(\frac{10}{0.1}\right) = 1.4$$

Calculation of $\sum_0^{z_2}\frac{I_z}{E_s}\Delta z$:

1. Calculation of I_z with depth:

✓ At depth $z = 0$ (at the base of the foundation) $\rightarrow I_z = I_{z,initial}$

$I_{z,i} = 0.1 + 0.0111\left(\frac{L}{B} - 1\right)$ but for continuous footing $\rightarrow I_{z,i} = 0.2$

✓ At depth $z = z_1 \rightarrow I_z = I_{z(max)}$

$$I_{z(max)} = 0.5 + 0.1\sqrt{\frac{\bar{q} - q}{q'_{z(1)}}}$$

Before calculating $I_{z(max)}$ we need to calculate $q'_{z(1)}$ at depth z_1

$\frac{z_1}{B} = 0.5 + 0.0555\left(\frac{L}{B} - 1\right)$ but for square footing $\rightarrow z_1 = B$

$\rightarrow z_1 = B = 8$ ft.

$q'_{z(1)} = 115 \times (5 + 8) = 1495\ \text{Ib/ft}^2$

$$\rightarrow I_{z(max)} = 0.5 + 0.1\sqrt{\frac{3425}{1495}} = 0.651$$

✓ At depth $z = z_2 \rightarrow I_z = 0.0$

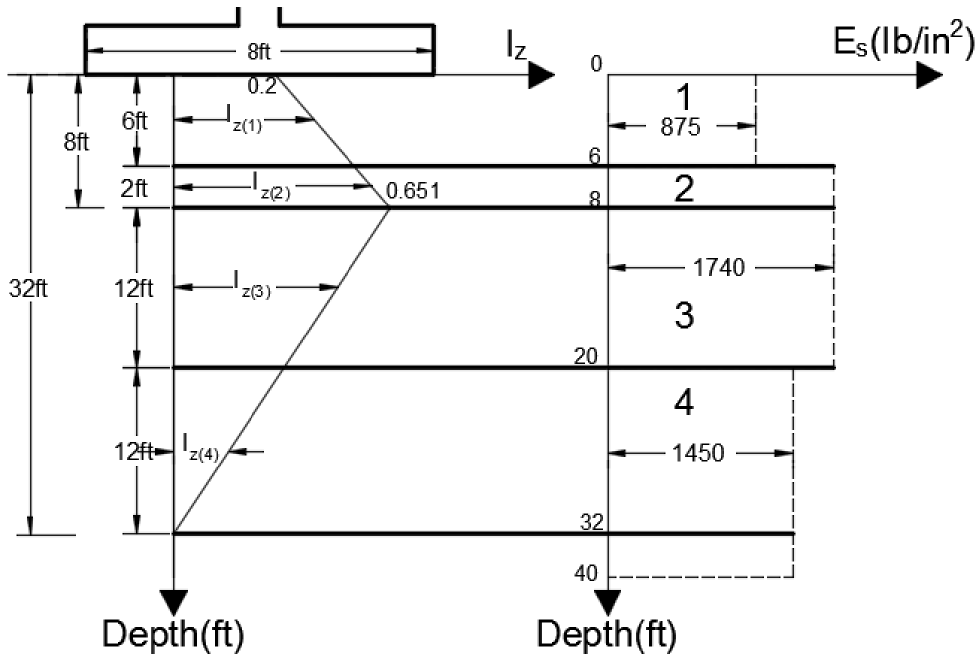
$\frac{z_2}{B} = 2 + 0.222\left(\frac{L}{B} - 1\right)$ but for continuous footing $\rightarrow z_2 = 4\ B$

$\rightarrow z_2 = 4B = 4 \times 8 = 32$ ft

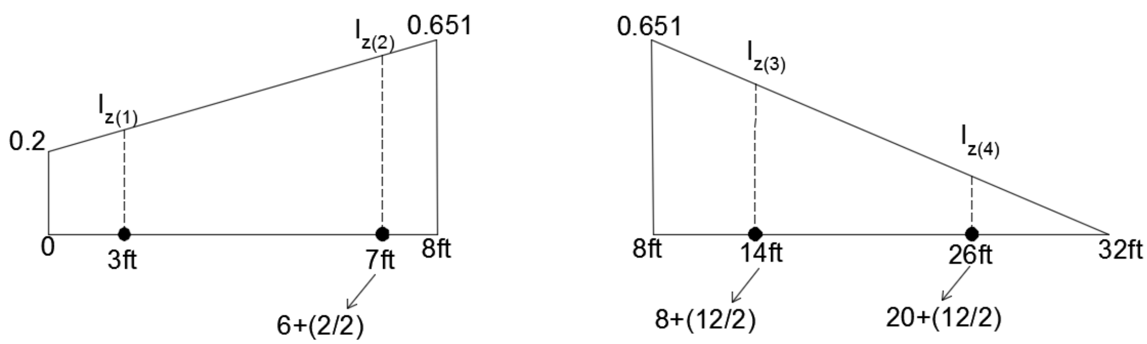
2. Calculation of E_s with depth:

The variations of E_s with depth is given.

Now we can draw the variation of I_z and E_s with depth:



Now we want to calculate the value of I_z at the middle of each layer by interpolation:



By interpolation (using calculator):

$I_{z(1)} = 0.369$, $I_{z(2)} = 0.595$, $I_{z(3)} = 0.488$, $I_{z(4)} = 0.163$

Now, don't forget to transform the unit of E_s from $(lb/in^2$ to $lb/ft^2) \rightarrow$

$\rightarrow 875 lb/in^2 = 875 \times 144 = 126000\ lb/ft^2$ and so on,,,

3. Now we can calculate the value of $\sum_0^{z_2} \frac{I_z}{E_s} \Delta z$:

Layer No.	$\Delta z(\mathrm{ft})$	$E_s(\mathrm{Ib/ft^2})$	I_z at the middle of each layer	$\frac{I_z}{E_s}\Delta z$
1	6	126000	0.369	1.757×10^{-5}
2	2	250560	0.595	0.449×10^{-5}
3	12	250560	0.488	2.337×10^{-5}
4	12	208800	0.163	0.937×10^{-5}
				$\sum_0^{z_2} \frac{I_z}{E_s}\Delta z = 5.53 \times 10^{-5}$

$$\rightarrow S_e = 0.916 \times 1.4 \times 3425 \times 5.53 \times 10^{-5}$$
$$= 0.242\mathrm{ft} = 0.074\mathrm{m} = 74\mathrm{mm}\checkmark.$$

3.

Solve example 5.7 in your textbook

Chapter (6)

Geometric Design of Shallow Foundations

Introduction

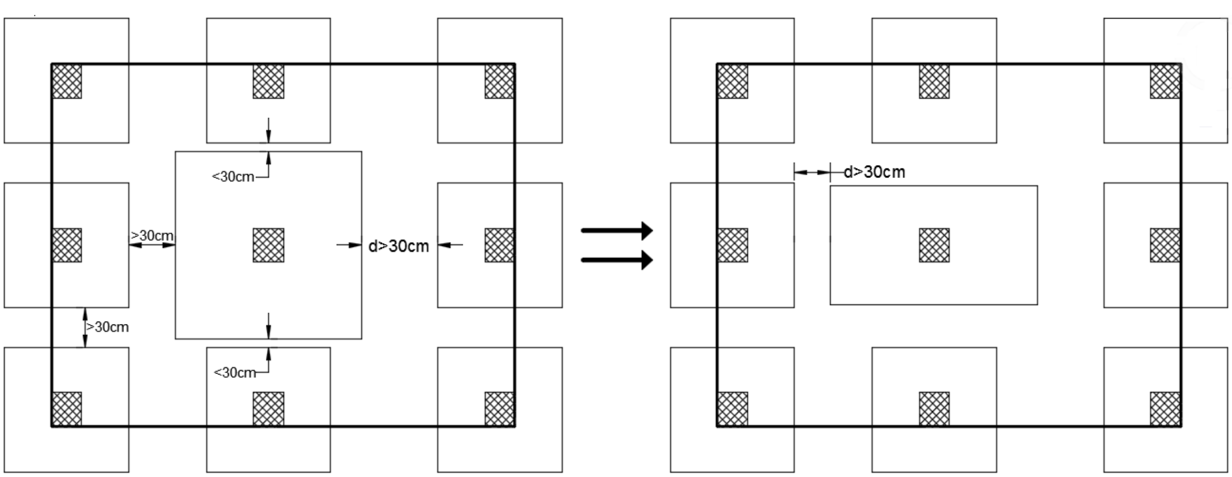
As we stated in Chapter 3, foundations are considered to be shallow if if $[D_f \le (3 \rightarrow 4)B]$. Shallow foundations have several advantages: minimum cost of materials and construction, easy in construction "labor don't need high experience to construct shallow foundations". On the other hand, the main disadvantage of shallow foundations that if the bearing capacity of the soil supporting the foundation is small, the amount of settlement will be large.

Types of Sallow Foundations

1. Isolated Footings (spread footings).
2. Combined Footings.
3. Strap Footings.
4. Mat "Raft" Foundations.

Geometric Design of Isolated Footings

The most economical type of foundations, and usually used when the loads on the columns are relatively small and the bearing capacity of the soil supporting the foundations is large.
In practice, we usually use isolated square footing because is the most economical type if the following condition is satisfied:
The distance between each footing should be more than 30 cm from all direction, if not; we use isolated rectangular footing (if possible) to make the distance more than 30 cm. The following figure explains this condition:



Note that for the middle foundation the distance from two sides is less than 30, so we use rectangular isolated footing as shown in figure (right side). If using of rectangular footing is not possible, we use combined footing as we will explain later.

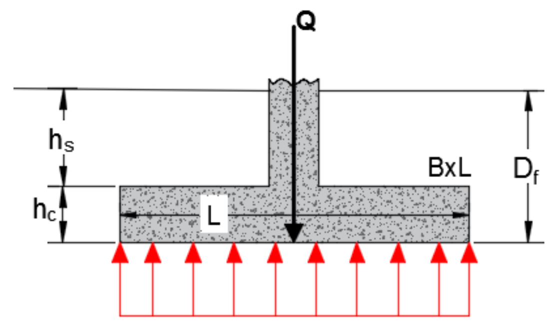
Design Procedures:

1. Calculate the net allowable bearing capacity:

The first step for geometric design of foundations is to calculate the allowable bearing capacity of the foundations as we discussed in previous chapters.

$$q_{all,net} = \frac{q_{u,net}}{FS}$$

$$q_{u,net} = q_{u,gross} - \gamma_c h_c - \gamma_s h_s$$



2. Calculate the required area of the footing:

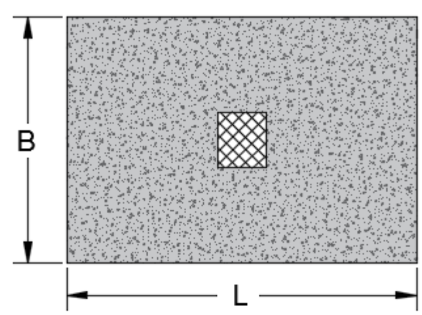
$$A_{req} = \frac{Q_{service}}{q_{all,net}} = B \times L$$

Assume B or L then find the other dimension.

If the footing is square:

$$A_{req} = B^2 \rightarrow B = \sqrt{A_{req}}$$

$$Q_{service} = P_D + P_L$$



Why we use $Q_{service}$:

$$A_{req} = \frac{Q_{service}}{q_{all,net}} = A_{req} = \frac{Q_{service}}{\frac{q_{u,net}}{FS}}$$

$$= \frac{FS \times Q_{service}}{q_{u,net}}$$

In the above equation the service load "$Q_{service}$" is multiplied by FS and when we multiply it by load factor, this is not economical and doesn't make sense.

Note:

The equation of calculating the required area $\left(A_{req} = \frac{Q_{service}}{q_{all,net}}\right)$ is valid only if the pressure under the base of the foundation is uniform.

Geometric Design of Combined Footings

Types:

1. Rectangular Combined Footing (two columns).
2. Trapezoidal Combined Footing (two columns).
3. Strip Footing (more than two columns and may be rectangular or trapezoidal).

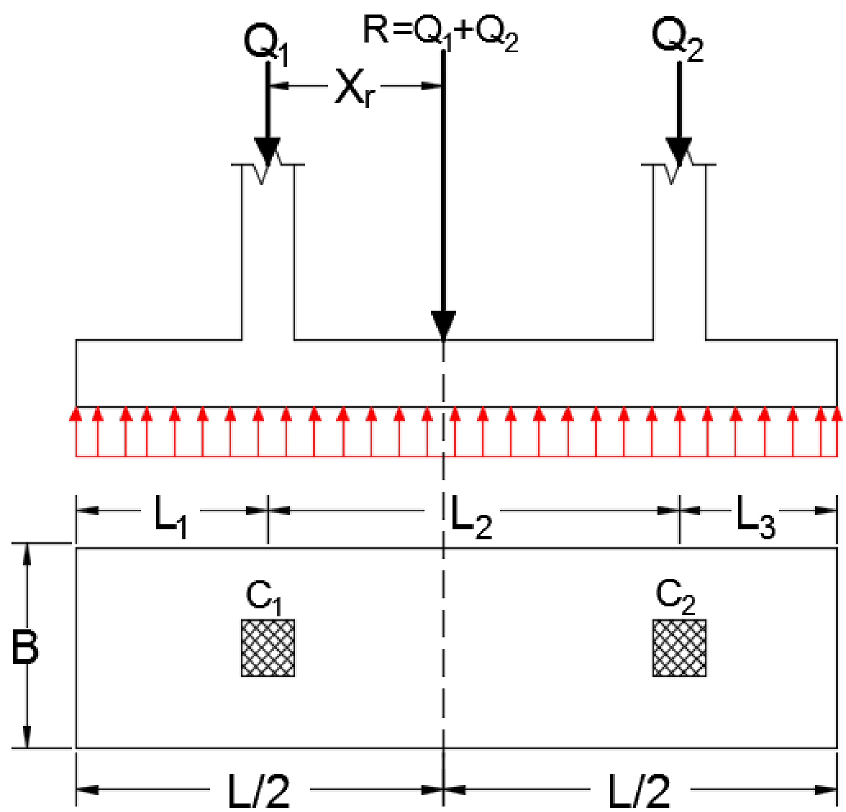
Usage:

1. Used when the loads on the columns are heavy and the distance between these columns is relatively small (i.e. when the distance between isolated footings is less than 30 cm).
2. Used as an alternative to neighbor footing which is an eccentrically loaded footing and it's danger if used when the load on the column is heavy.

Design of Rectangular Combined Footings:

There are three cases:

1. Extension is permitted from both side of the footing:



The resultant force R is more closed to the column which have largest load.

To keep the pressure under the foundation uniform, the resultant force of all columns loads (R) must be at the center of the footing, and since the footing is rectangular, R must be at the middle of the footing (at distance L/2) from each edge to keep uniform pressure.

$$A_{req} = \frac{\sum Q_{service}}{q_{all,net}} = \frac{Q_1+Q_2}{q_{all,net}} = B \times L$$

How we can find L:

$L_2 =$ distance between centers of the two columns and it's always known

$X_r =$ distance between the resultant force and column (1) OR column (2) as u like ☺.

L_1 and $L_3 =$ extensions from left and right " usually un knowns"

Now take summation moments at C_1 equals zero to find X_r:

$$\sum M_{C_1} = 0.0 \rightarrow Q_2 L_2 + (W_{footing} + W_{soil}) \times X_r = R \times X_r \rightarrow X_r = \checkmark.$$

$(W_{footing} + W_{soil})$ are located at the center of the footing

If we are not given any information about $(W_{footing} + W_{soil}) \rightarrow$

$Q_2 L_2 = R \times X_r \rightarrow X_r = \checkmark.$

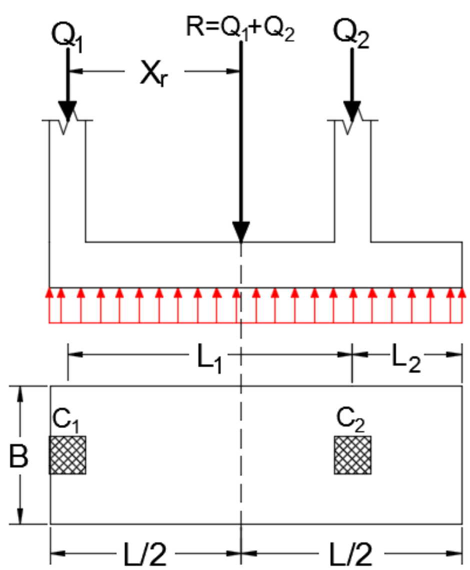
Now, to keep uniform pressure under the foundation:

$$X_r + L_1 = \frac{L}{2} \quad \text{(Two unknowns "}L_1\text{" and "L")}$$

The value of L_1 can be assumed according the permitted extension in site.

$$\rightarrow L = \checkmark \quad \rightarrow\rightarrow B = \frac{A_{req}}{L} = \checkmark.$$

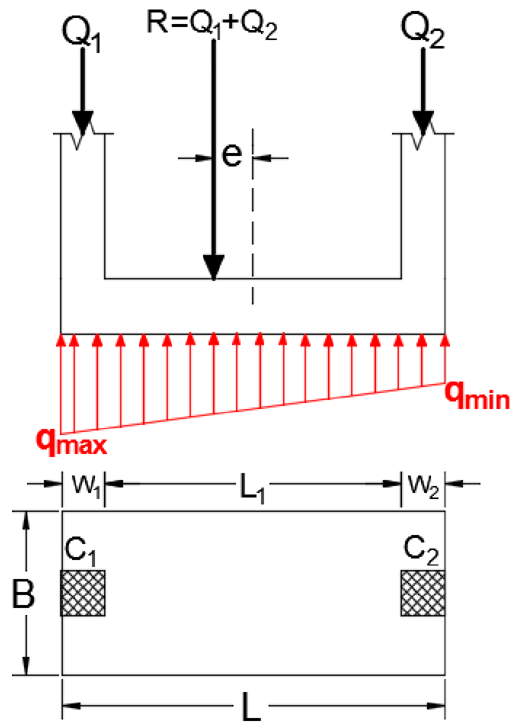
2. Extension is permitted from one side and prevented from other side:



The only difference between this case and the previous case that the extension exists from one side and when we find X_r we can easily find L:

To keep the pressure uniform $\rightarrow X_r + \frac{\text{column width}}{2} = \frac{L}{2} \rightarrow L = \checkmark.$

3. Extension is not permitted from both sides of the footing:



In this case the resultant force R doesn't in the center of rectangular footing because Q_1 and Q_2 are not equals and no extensions from both sides. So the pressure under the foundation is not uniform and we design the footing in this case as following:

$L = L_1 + W_1 + W_2 = \checkmark.$

How we can find e:

$$\sum M_{\text{foundation center}} = 0.0$$

$$Q_1 \times \left(\frac{L}{2} - \frac{W_1}{2}\right) - Q_2 \times \left(\frac{L}{2} - \frac{W_2}{2}\right) = R \times e$$

$\rightarrow e = \checkmark.$

Note: the moment of $W_f + W_s$ is zero because they located at the center of footing.

The eccentricity in the direction of L:

Usually $e < \frac{L}{6}$ (because L is large)

$$q_{max} = \frac{R}{B \times L}\left(1 + \frac{6e}{L}\right)$$

$q_{all,gross} \geq q_{max} \rightarrow q_{all,gross} = q_{max}$ (critical case)

$$q_{all,gross} = \frac{R}{B \times L}\left(1 + \frac{6e}{L}\right) \rightarrow B = \checkmark.$$

Check for B:

$$q_{min} = \frac{R}{B \times L}\left(1 - \frac{6e}{L}\right) \text{ must be } \geq 0.0$$

If this condition doesn't satisfied, use the modified equation for q_{max} to find B:

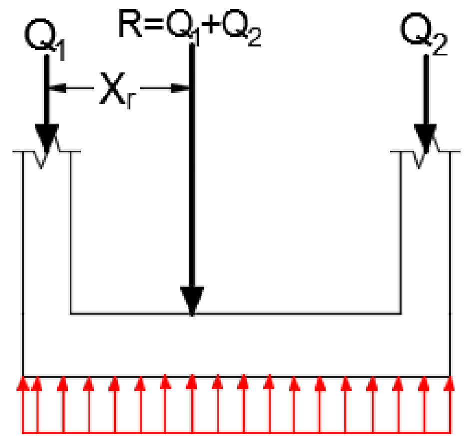
$$q_{max,modified} = \frac{4R}{3B(L - 2e)} \rightarrow B = \checkmark.$$

Design of Trapezoidal Combined Footings:

Advantages:

1. More economical than rectangular combined footing if the extension is not permitted from both sides especially if there is a large difference between columns loads.
2. We can keep uniform contact pressure in case of "extension is not permitted from both sides" if we use trapezoidal footing because the resultant force "R" can be located at the centroid of trapezoidal footing.

Design:



$Q_1 > Q_2 \rightarrow B_1 \text{at } Q_1$ and $B_2 \text{at } Q_2$

$L = L_1 + W_1 + W_2 = \checkmark.$

$$A_{req} = \frac{\sum Q_{service}}{q_{all,net}} = \frac{Q_1 + Q_2}{q_{all,net}}$$

$$\frac{Q_1 + Q_2}{q_{all,net}} = \frac{L}{2}(B_1 + B_2) \rightarrow\rightarrow \text{Eq.}(1)$$

Now take summation moments at C_1 equals zero to find X_r:

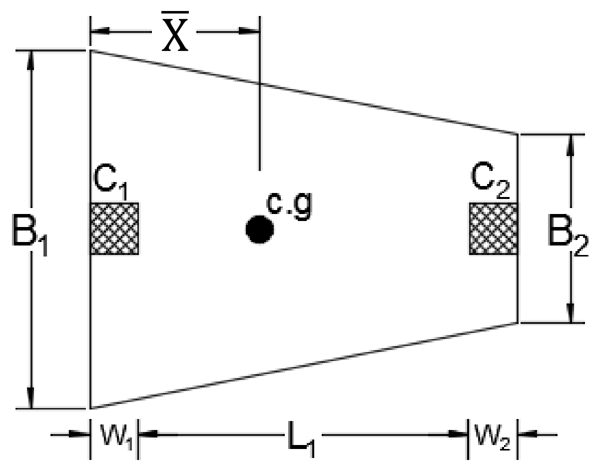
$$\sum M_{C_1} = 0.0 \rightarrow Q_2 L_1 + (W_f + W_s) \times X_r = R \times X_r \rightarrow X_r = \checkmark.$$

$$X_r + \frac{W_1}{2} = \bar{X} = \checkmark.$$

$$\bar{X} = \frac{L}{3}\left(\frac{B_1 + 2B_2}{B_1 + B_2}\right) \rightarrow\rightarrow \text{Eq.}(2)$$

Solve Eq. (1) and Eq. (2) $\rightarrow\rightarrow$

$B_1 = \checkmark$ and $B_2 = \checkmark.$

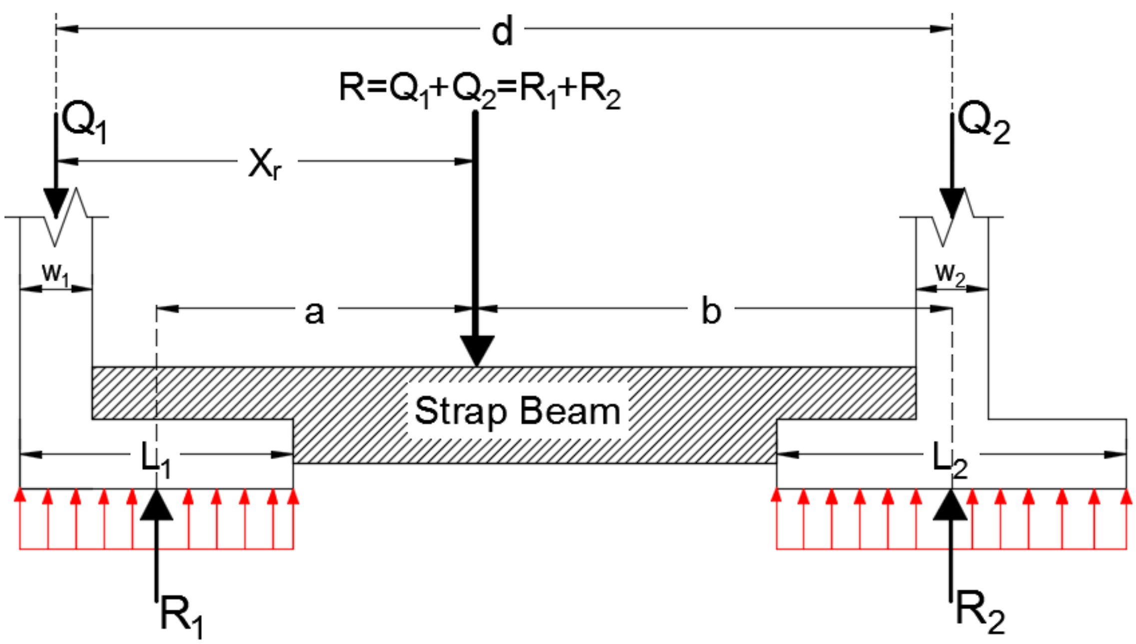


Geometric Design of Strap Footing (Cantilever Footing)

Usage:

1. Used when there is a **property line** which prevents the footing to be extended beyond the face of the edge column. In addition to that the edge column is relatively far from the interior column so that the rectangular and trapezoidal combined footings will be too narrow and long which increases the cost. And may be used to connect between two interior foundations one of them have a large load require a large area but this area not available, and the other foundation have a small load and there is available area to enlarge this footing, so we use strap beam to connect between these two foundations to transfer the load from largest to the smallest foundation.

2. There is a "strap beam" which connects two separated footings. The edge Footing is usually eccentrically loaded and the interior footing is centrically loaded. The **purpose of the beam is to prevent overturning of the eccentrically loaded footing and to keep uniform pressure under this foundation** as shown in figure below**.**



Note that the strap beam doesn't touch the ground (i.e. there is no contact between the strap beam and soil, so no bearing pressure applied on it).
This footing also called "cantilever footing" because the overall moment on the strap beam is negative moment.

Design:

$R = Q_1 + Q_2 = R_1 + R_2 \quad \text{but}, Q_1 \neq R_1 \text{ and } Q_2 \neq R_2$

Q_1 and Q_2 are knowns but R_1 and R_2 are unknowns

Finding X_r:

$$\sum M_{Q_1} = 0.0 \text{ (before use of strap beam)} \rightarrow R \times X_r = Q_2 \times d \rightarrow X_r = \checkmark.$$

$$a = X_r + \frac{w_1}{2} - \frac{L_1}{2} \qquad (L_1 \text{ should be assumed "if not given"})$$

$$b = d - X_r$$

Finding R_1:

$$\sum M_{R_2} = 0.0\text{(after use of strap beam)} \rightarrow R_1 \times (a + b) = R \times b \rightarrow R_1 = \checkmark.$$

Finding R_2:

$$R_2 = R - R_1$$

Design:

$$A_1 = \frac{R_1}{q_{all,net}} \quad , \quad A_2 = \frac{R_2}{q_{all,net}}$$

Mat Foundation

Usage:

We use mate foundation in the following cases:

1. If the area of isolated and combined footing > 50% of the structure area, because this means the loads are very large and the bearing capacity of the soil is relatively small.
2. If the bearing capacity of the soil is small (usually < 15 t/m^2).
3. If the soil supporting the structure classified as (bad soils) such as:
   - **Expansive Soil:** Expansive soils are characterized by clayey material that shrinks and swells as it dries or becomes wet respectively. It is recognized from high values of Plasticity Index, Plastic Limit and Shrinkage Limit.

- **Compressible soil:** It contains a high content of organic material and not exposed to great pressure during its geological history, so it will be exposed to a significant settlement, so mat foundation is used to avoid differential settlement.
- **Collapsible soil:** Collapsible soils are those that appear to be strong and stable in their natural (dry) state, but which rapidly consolidate under wetting, generating large and often unexpected settlements. This can yield disastrous consequences for structures unwittingly built on such deposits.

Types:

- Flat Plate (uniform thickness).
- Flat plate thickened under columns.
- Beams and slabs.
- Slabs with basement walls as a part of the mat.

(To see these types open Page 295 in your textbook)

Compensated Footing

The net allowable pressure **applied** on the **mat foundation** may be expressed as:

$$q = \frac{Q}{A} - \gamma D_f$$

$Q =$ Service Loads of columns on the mat + own weight of mat

$A =$ Area of the mat "raft" foundation.

In all cases q should be less than $q_{all,net}$ of the soil.

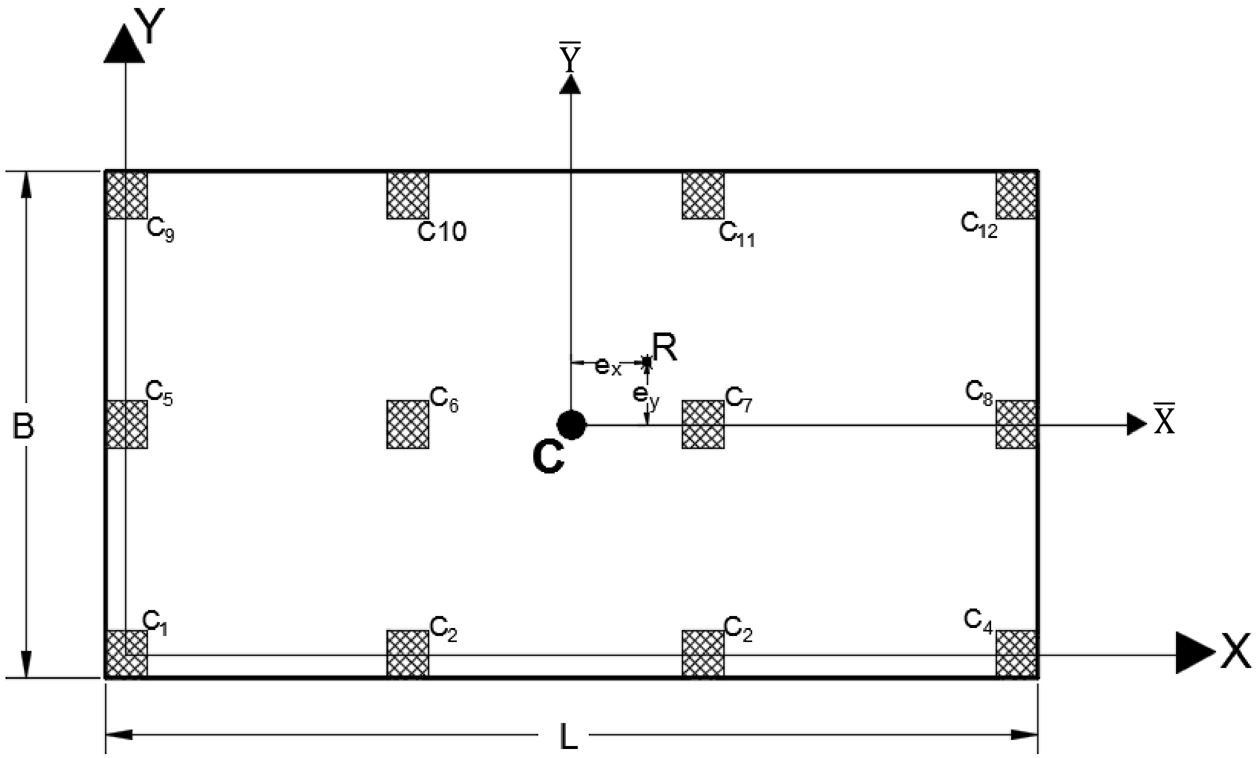
The value of q can be reduced by increasing the depth D_f of the mat, this called **compensated footing** design (i.e. replacing (substituting) the weight of the soil by the weight of the building) and is extremely useful when the when structures are to be built on very soft clay.

At the point which $q = 0.0 \rightarrow$ the overall weight of the soil above this point will replaced (substituted) by the weight of the structure and this case is called **fully compensated footing** (fully safe).

The relationship for **fully compensated depth** D_f can be determined as following:

$$0.0 = \frac{Q}{A} - \gamma D_f \rightarrow D_f = \frac{Q}{A\gamma} \text{ (fully compensated depth)}$$

Geometric Design of Mat Foundation (Working Loads)



Procedures:

1. Determine the horizontal and vertical axes (usually at the center line of the horizontal and vertical edge columns) as shown.

2. Calculate the centroid of the mat [point C $(\overline{X}, \overline{Y})$]with respect to X and Y axes:

$$\overline{X} = \frac{\sum X_i \times A_i}{\sum A_i} \qquad \overline{Y} = \frac{\sum Y_i \times A_i}{\sum A_i}$$

A_i = shapes areas.

X_i = distance between y − axis and the center of the shape.

Y_i = distance between y − axis and the center of the shape.

If the mat is rectangular:

$$\overline{X} = \frac{L}{2} - \frac{w_{\text{vertical edge columns}}}{2}$$

$$\overline{Y} = \frac{B}{2} - \frac{w_{\text{horizontal edge columns}}}{2}$$

3. Calculate the resultant force R:

$$R = \sum Q_i$$

4. Calculate the location of resultant force R (X_R, Y_R) with respect to X and Y axes:

To find X_R take summation moments about Y-axis:

$$X_R = \frac{\sum Q_i \times X_{ri}}{\sum Q_i}$$

To find Y_R take summation moments about X-axis:

$$Y_R = \frac{\sum Q_i \times Y_{ri}}{\sum Q_i}$$

$Q_i = \text{load on column}$

$X_{ri} = \text{distance between columns center and } Y-\text{axis}$

$Y_{ri} = \text{distance between columns center and } X-\text{axis}$

5. Calculate the eccentricities:

$$e_x = |X_R - \overline{X}| \qquad e_y = |Y_R - \overline{Y}|$$

6. Calculate moments in X and Y directions:

$$M_x = e_y \times \sum Q_i \qquad M_y = e_x \times \sum Q_i$$

7. Calculate the stress under each corner of the mat:

$$q = \frac{\sum Q_i}{A} \pm \frac{M_y}{I_y} X \pm \frac{M_x}{I_x} Y$$

How we can know the sign $(+ \text{ or } -)$:

If compression $(+)$ If tension $(-)$

Compression if the arrow of moment is at the required point

Tension if the arrow of moment is far away from the required point

X and Y are distances from the **centroid to the required point**

X and Y (take them always positive)

$I_x = I_{\overline{X}} = \text{moment of inertia about centoid of mat (in } x-\text{direction)}$

$I_y = I_{\overline{Y}} = \text{moment of inertia about centoid of mat (in } y-\text{direction)}$

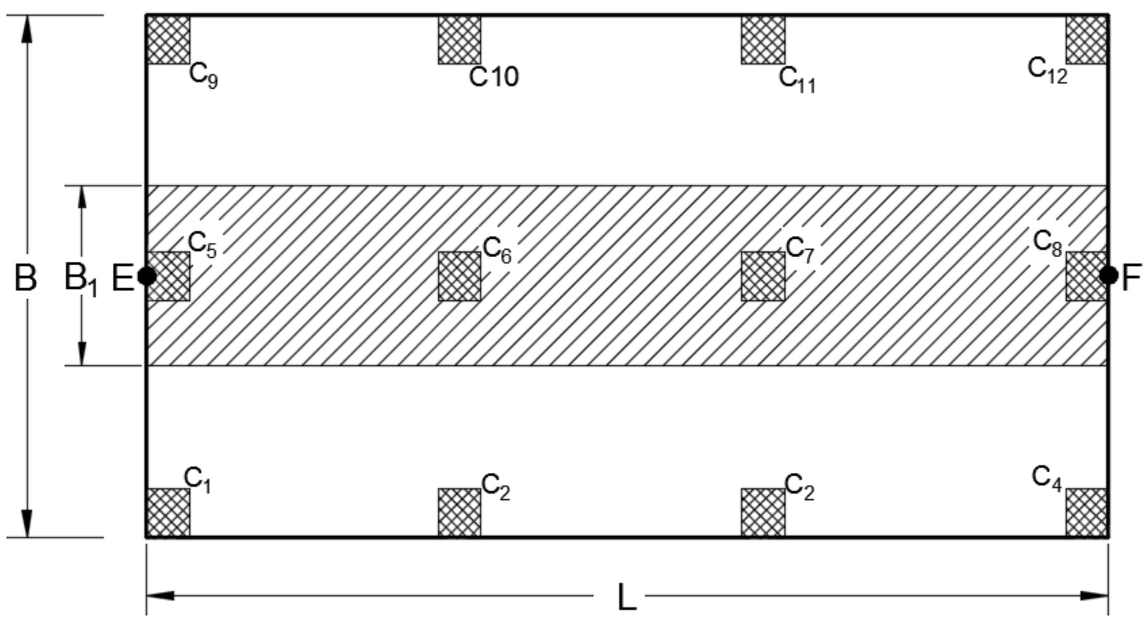
$$I_x = \frac{B^3 L}{12} \qquad I_Y = \frac{L^3 B}{12} \quad \text{(in case of rectangular foundation}$$

8. Check the adequacy of the dimensions of mat foundation:
Calculate q_{max} (maximum stress among all corners of the mat)
Calculate q_{min} (minimum stress among all corners of the mat)
$q_{max} \le q_{all,net}$
$q_{min} \ge 0.0$
If one of the two conditions doesn't satisfied, increase the dimensions of the footing.

Structural Design of Mat Foundation (Ultimate Loads)

In structural design we want to draw shear force and bending moment diagrams, to do this we have to subdivide mat foundations into a strips in both directions, each strip must contains a line of columns, such that the width of strip is related to the loads of the columns including in this stip.

For the previous mat, let we take a strip of width B_1 for the columns 5,6,7 and 8 as shown in figure below:



Procedures for structural design (drawing shear and moments diagrams):

We will take the strip as shown in the figure above, and the following procedures are the same for any other strip:

1. Locate the points E and F at the middle of strip edges.

2. Calculate the factored resultant force (R_u):

$$R_u = \sum Q_{ui}$$

3. The eccentricities in X and Y directions remains unchanged because the location of the resultant force will not change since we factored all columns by the same factor.

4. Calculate the factored moment in X and Y directions:

$$M_{u,x} = e_y \times \sum Q_{ui} \qquad M_{u,y} = e_x \times \sum Q_{ui}$$

5. Calculate the stresses at points E and F (using factored loads and moments):

$$q = \frac{\sum Q_{ui}}{A} \pm \frac{M_{u,y}}{I_y} X \pm \frac{M_{u,x}}{I_x} Y$$

6. Since the stress at points E and F is not equal, the pressure under the strip in not uniform, so we find the average stress under the strip:

$$q_{u,avg} = \frac{q_E + q_F}{2}$$

7. Now, we check the stability of the strip:

$$\left(\sum Q_{ui}\right)_{strip} = q_{u,avg} \times A_{strip}$$

If this check is ok, draw the SFD and BMD and then design the strip.
If not ☹:>>>> Go to step 8.

8. We have to modified the loads to make the strip stable by the following steps:

- Calculate the average load on the strip:

$$\text{Average Load} = \frac{(\sum Q_{ui})_{strip} + q_{u,avg} \times A_{strip}}{2}$$

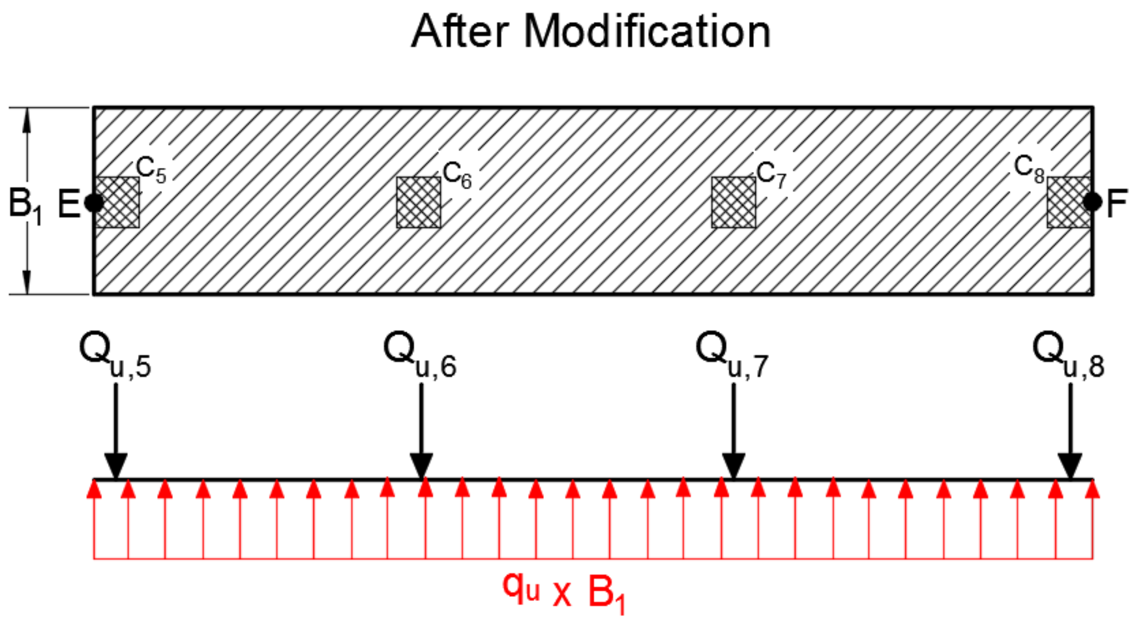
- Calculate the modified columns loads:

$$(Q_{ui})_{mod} = Q_{ui} \times \frac{\text{Average Load}}{(\sum Q_{ui})_{strip}}$$

- Calculate the modified soil pressure:

$$(q_{u,avg})_{mod} = q_{u,avg} \times \frac{\text{Average Load}}{q_{u,avg} \times A_{strip}}$$

$$\text{Now, } \left(\sum Q_{ui}\right)_{strip} = q_{u,avg} \times A_{strip} \rightarrow \text{Draw SFD and BMD}$$



Important Note:

If the SFD and BMD and you are given only service loads, do the above steps for service loads and draw SFD and BMD for service loads.

Problems

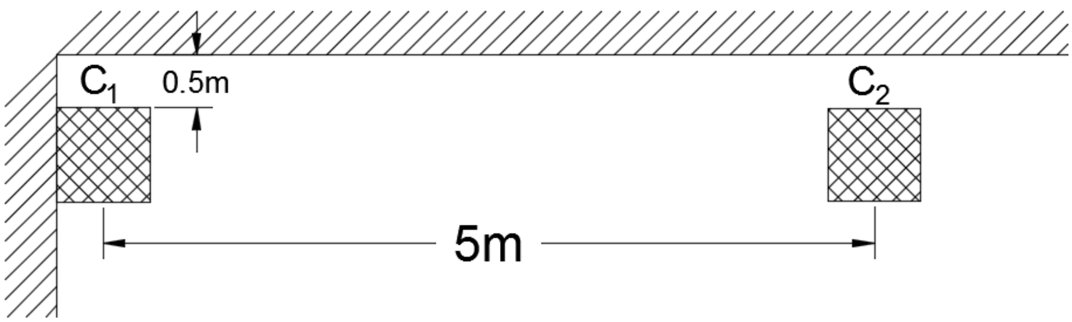
1.

Design the foundation shown below to support the following two columns with uniform contact pressure:

Column (1): P_D=500 kN , P_L=250 kN , cross section (50cm × 50cm).

Column (2): P_D=700 kN , P_L=350 kN , cross section (50cm × 50cm).

Assume the net allowable soil pressure is 200 kN/m^2



Solution

To keep uniform contact pressure under the base, the resultant force R must be at the center of the foundation.

Since the extension is permitted from right side, we can use rectangular combined footing.

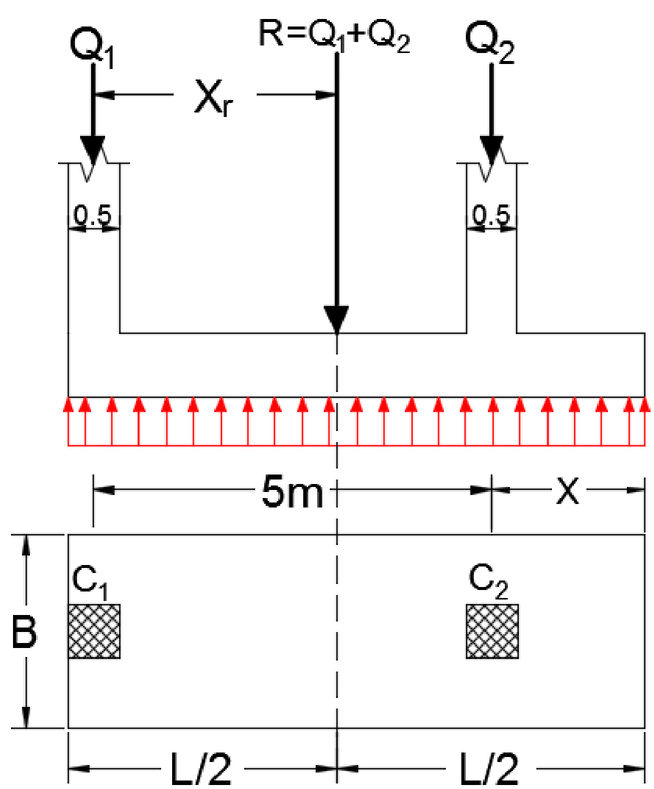
$Q_1 = 500 + 250 = 750$ KN

$Q_2 = 700 + 350 = 1050$ kN

$R = Q_1 + Q_2 = 750 + 1050$

$R = 1800$ kN

The weight of the foundation and the soil is not given, so we neglect it.



$$\sum M_{c_1} = 0.0 \rightarrow 1050 \times 5 = 1800 \times X_r \rightarrow X_r = 2.92\text{ m}$$

To keep uniform pressure: $X_r + \frac{0.5}{2} = \frac{L}{2}$

$$\frac{L}{2} = 2.92 + \frac{0.5}{2} = 3.17\text{m}$$

$$\frac{L}{2} = 3.17\text{m} \rightarrow L = 3.17 \times 2 = 6.34\text{ m} \checkmark.$$

Calculation of B:

$$A_{req} = \frac{Q_{service}}{q_{all,net}} = B \times L \rightarrow \frac{1800}{200} = B \times 6.34 \rightarrow B = 1.42\text{ m} \checkmark.$$

Check for B:

Available value for B:

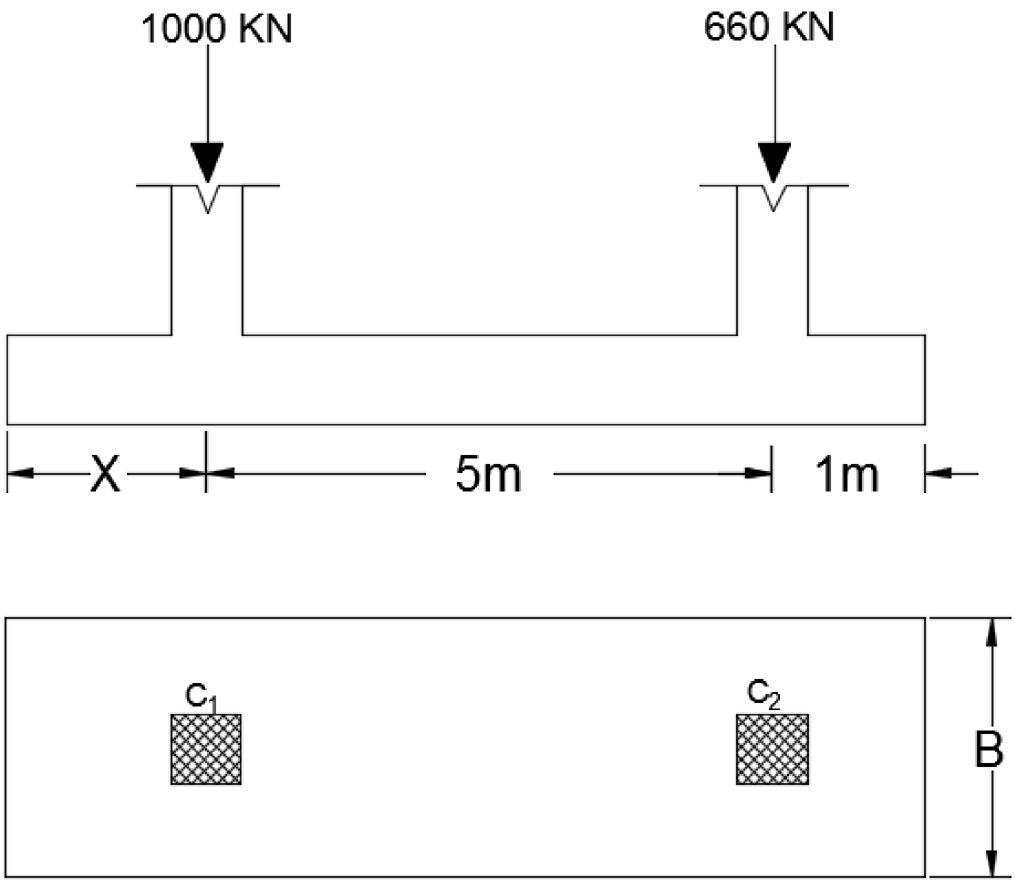
The permitted extension for the width B is 0.5 m, so the available width is 0.5+ (column width)+0.5 = 0.5+0.5+0.5 =1.5 m

$B = 1.42\text{ m} < 1.5\text{m Ok}$

2.

For the combined footing shown below.

- Find distance X so that the contact pressure is uniform.
- If $q_{all,net}$=140 kN/m^2, find B.
- Draw S.F. and B.M diagrams.



Solution

To keep uniform contact pressure under the base, the resultant force R must be at the center of the foundation.

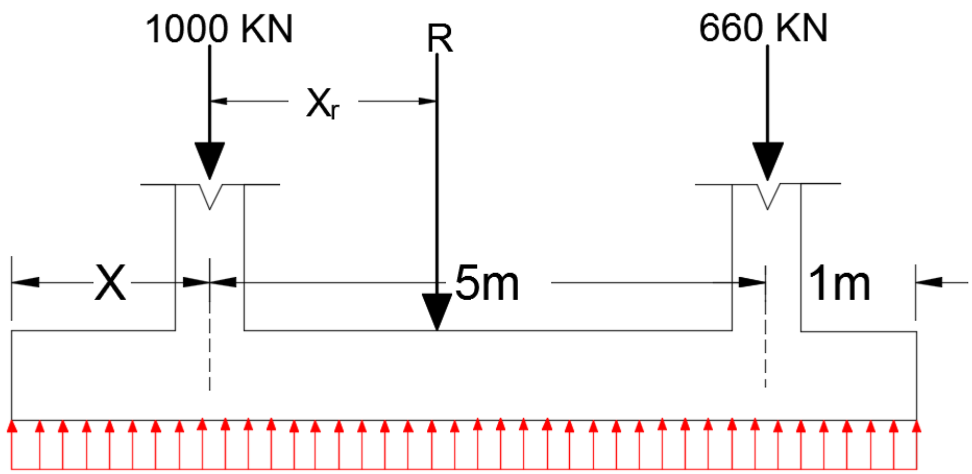
$Q_1 = 1000\ \text{KN}$

$Q_2 = 660\ \text{kN}$

$R = Q_1 + Q_2 = 1000 + 660$

$R = 1660\ \text{kN}$

The weight of the foundation and the soil is not given, so we neglect it.



$$\sum M_{c_1} = 0.0 \rightarrow 660 \times 5 = 1660 \times X_r \rightarrow X_r = 1.98 \text{ m}$$

To keep uniform pressure: $X_r + X = \frac{L}{2}$

$$\frac{L}{2} = (5 - 1.98) + 1 = 4.02 \text{ m}$$

$$\frac{L}{2} = 4.02\text{m} \rightarrow L = 4.02 \times 2 = 8.04 \text{ m}$$

$$X_r + X = \frac{L}{2} \rightarrow 1.98 + X = 4.02 \rightarrow X = 2.04 \text{ m} \checkmark.$$

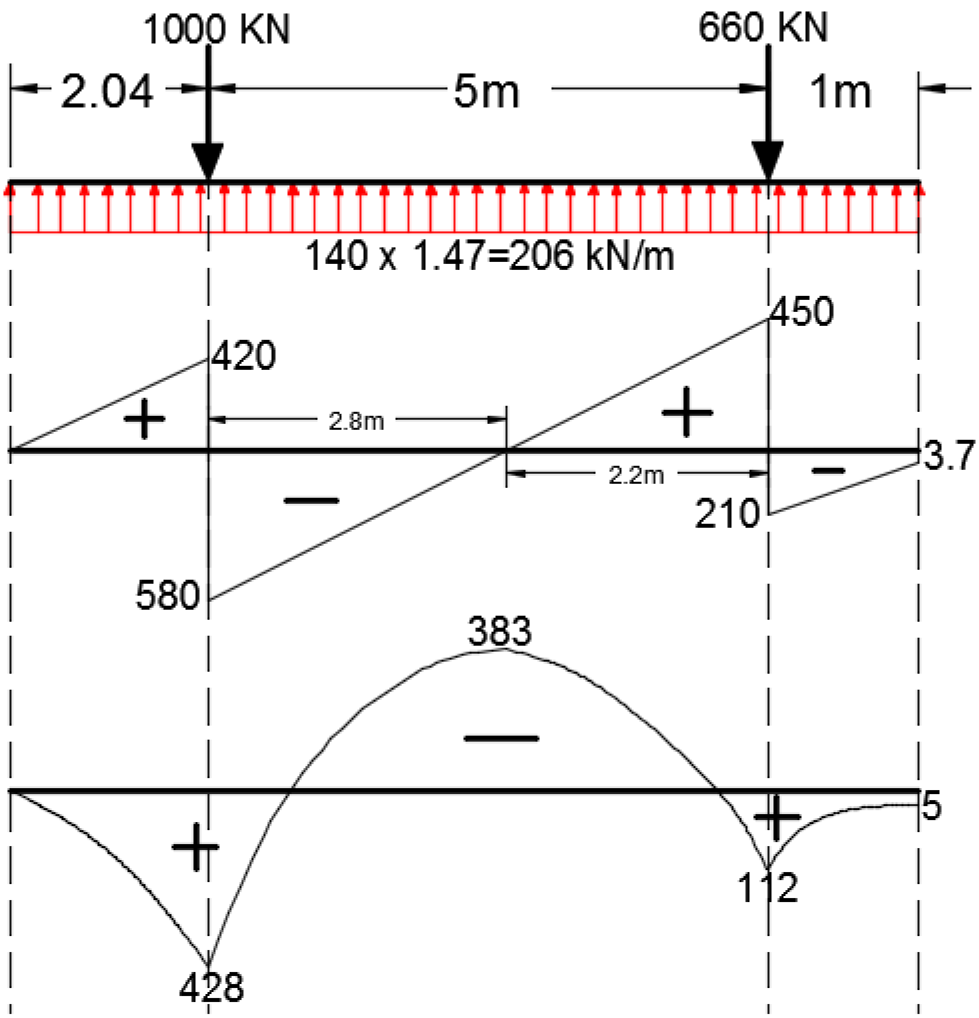
Calculation of B:

$$A_{req} = \frac{Q_{service}}{q_{all,net}} = B \times L \rightarrow \frac{1660}{140} = B \times 8.04 \rightarrow B = 1.47 \text{ m} \checkmark.$$

Drawing SFD and BMD:

To draw SFD and BMD we use factored loads (if givens), but in this problem we given the service loads directly, so we use service loads.

The free body diagram for the footing, SFD and BMD are shown in figure below:



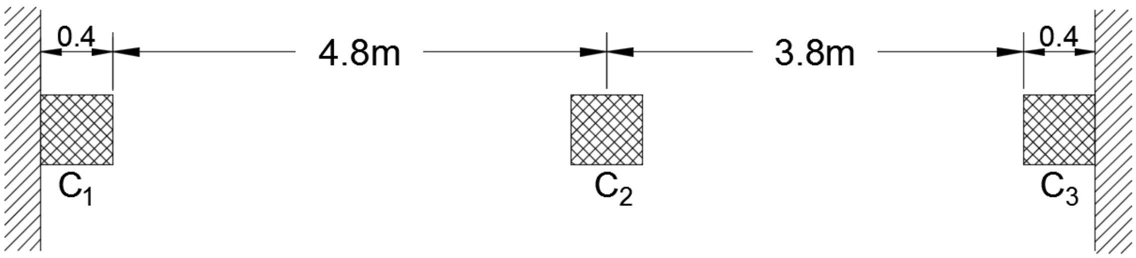
Distance 2.8 $\cong \frac{580}{206}\left(\frac{\text{Shear}}{\text{W}}\right)$

Note that the moment and shear for the footing is the opposite for beams such that the positive moments is at the supports and the minimum moments at the middle of spans, so when reinforced the footing, the bottom reinforcement mustn't cutoff at supports but we can cut it at the middle of the span, also the top reinforcement mustn't cutoff at the middle of the span and we can cut it at the supports (Exactly).

3.

A. For the **rectangular** combined footing shown below**, to make the soil reaction uniform**, determine the live load for the column C_1, knowing that dead load is one and half live load for this column.

Column	DL (tons)	LL (tons)
C_1	DL_1	LL_1
C_2	80	46
C_3	55	35



B. Design a combined footing for the same figure above if C_1 has a dead load of 90 tons and live load of 54 tons, knowing that the extension is not permitted and the soil reaction is uniform. ($q_{all,net}$=20.8 t/m^2).

Solution

A.

For rectangular footing, to keep uniform pressure, the resultant force R must be in the center of the foundation.

$Q_1 = DL + LL$

$DL = 1.5\ LL \rightarrow Q_1 = 1.5\ LL + LL = 2.5\ LL$

$Q_2 = 80 + 46 = 126\ \text{ton}$

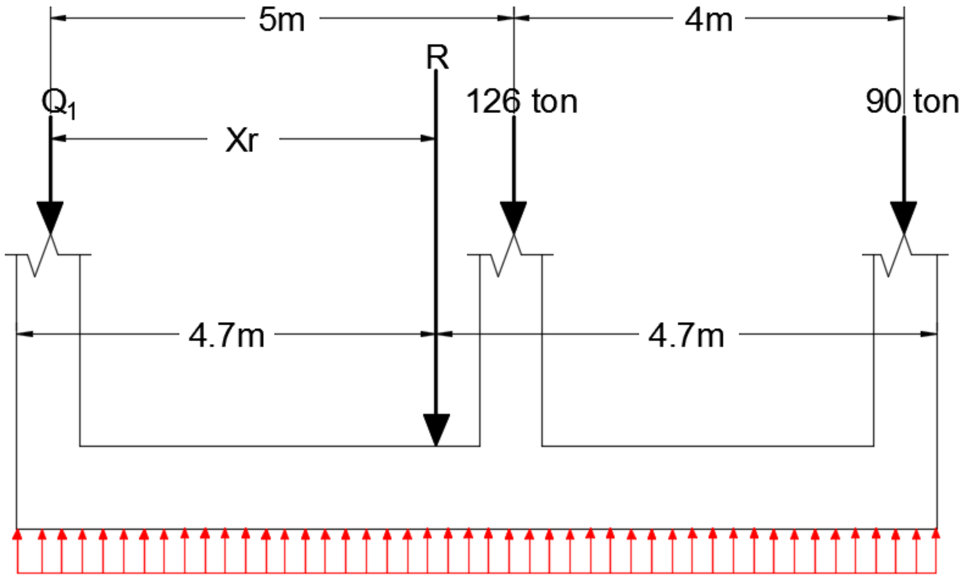
$Q_3 = 55 + 35 = 90\ \text{ton}$

$R = Q_1 + Q_2 + Q_3 = Q_1 + 126 + 90$

$R = Q_1 + 216$

The weight of the foundation and the soil is not given, so we neglect it.

$L = 0.4 + 4.8 + 3.8 + 0.4 = 9.4m \rightarrow 0.5L = 4.7m$



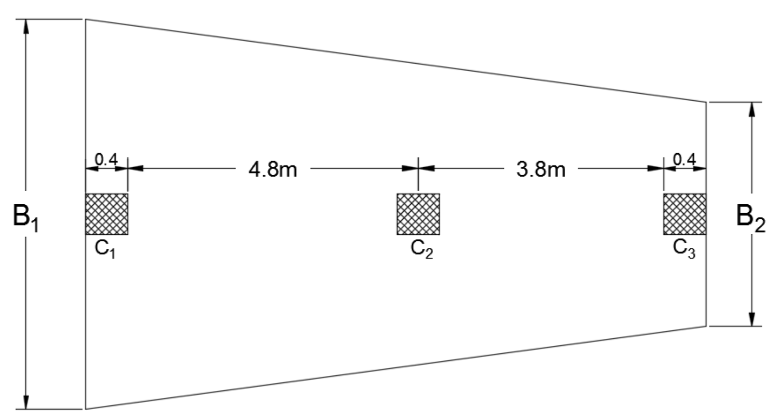
$X_r = 4.7 - 0.2 = 4.5\text{m}$

$\sum M_{c_1} = 0.0 \rightarrow 126 \times 5 + 90 \times 9 = (Q_1 + 216) \times 4.5 \rightarrow Q_1 = 104 \text{ ton}$

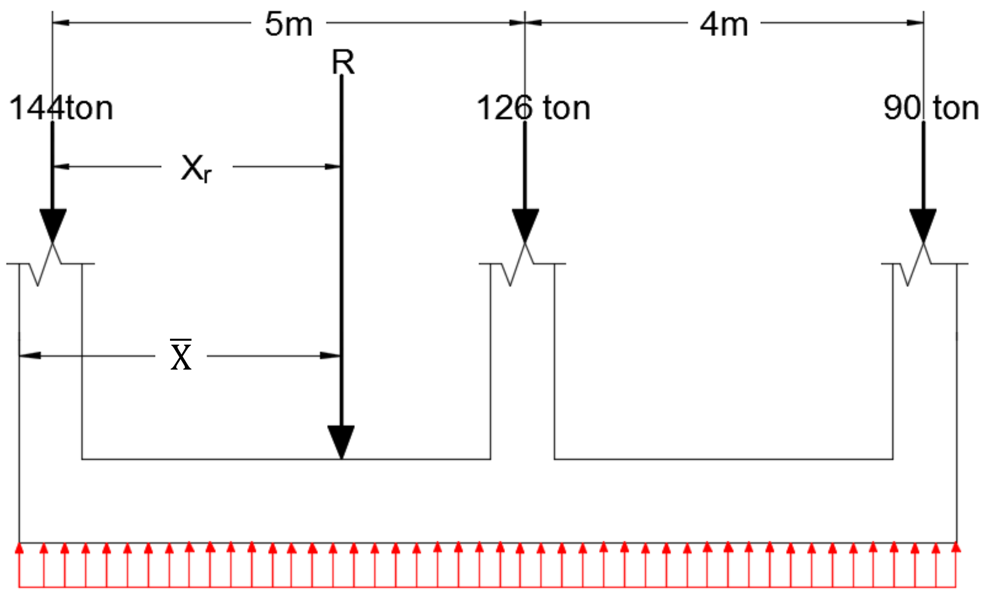
$Q_1 = 104 = 2.5\text{ LL} \rightarrow \text{LL} = 4.6 \text{ ton}$ ✓.

B.

If $Q_1 = 90 + 54 = 144$ ton,
Design the footing to keep uniform contact pressure .
Note that if we want to use rectangular footing, the pressure will be uniform only when $Q_1 = 104\ ton$ otherwise if we want to use rectangular footing the pressure will not be uniform, so to maintain uniform pressure under the given loading we use **trapezoidal combined footing** (The largest width at largest load and smallest width at smallest column load) as shown:



$Q_1 = 144$ ton
$Q_2 = 126$ ton
$Q_3 = 90$ ton
$R = Q_1 + Q_2 + Q_3 = 144 + 126 + 90 = 360$ ton
$L = 0.4 + 4.8 + 3.8 + 0.4 = 9.4\text{m}$



$$A_{req} = \frac{Q_{service}}{q_{all,net}} = \frac{L}{2}(B_1 + B_2) \rightarrow \frac{360}{20.8} = \frac{9.4}{2}(B_1 + B_2)$$

$$\rightarrow (B_1 + B_2) = 3.68 \rightarrow B_1 = 3.68 - B_2 \rightarrow\rightarrow \text{Eq. }(1)$$

$$\sum M_{c_1} = 0.0 \rightarrow 126 \times 5 + 90 \times 9 = 360 \times X_r \rightarrow X_r = 4 \text{ m}$$

$$X_r + 0.2 = \overline{X} \rightarrow 4.2 = \frac{L}{3}\left(\frac{B_1 + 2B_2}{B_1 + B_2}\right) \rightarrow 1.34 = \left(\frac{B_1 + 2B_2}{B_1 + B_2}\right) \rightarrow\rightarrow \text{Eq. }(2)$$

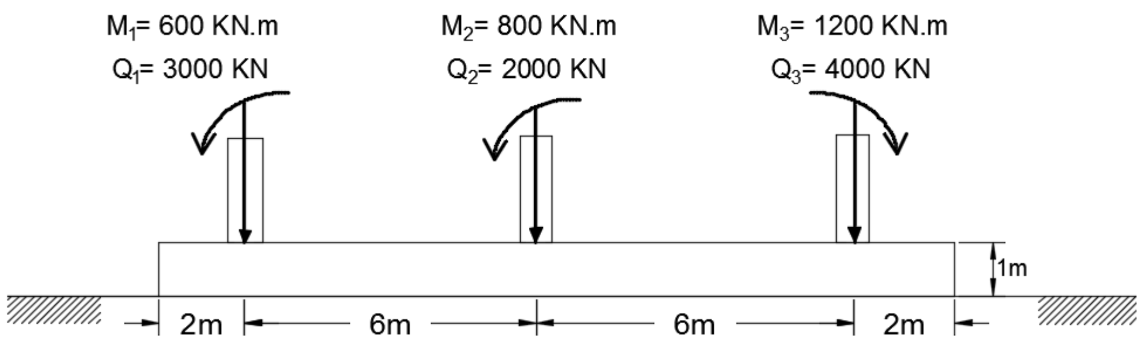
Substitute from Eq.(1) in Eq.(2):

$$1.34 = \left(\frac{3.68 - B_2 + 2B_2}{3.68 - B_2 + B_2}\right) \rightarrow B_2 = 1.25 \text{ m} \checkmark.$$

$$B_1 = 3.68 - 1.25 = 2.43 \text{ m} \checkmark.$$

4.

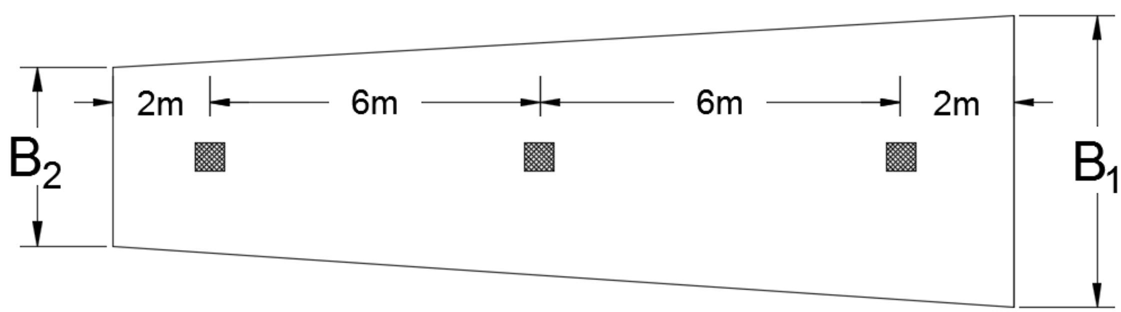
Determine B_1 and B_2 of a trapezoidal footing for a **uniform** soil pressure of 300 kN/m^2. (Consider the weight of the footing "$\gamma_{concrete} = 24\ KN/m^3$").



Solution

Note that we are given the thickness of the footing and the unit weight of concrete (i.e. the weight of the footing) that must be considered.

The largest Width B_1 will be at the largest load (4000) as shown below:



$$R = \sum Q = 9000\ \text{kN}$$

$$L = 2 + 6 + 6 + 2 = 16\text{m}$$

$$\text{Footing Weight} = \text{Footing Volume} \times \gamma_c$$

$$\text{Footing Volume} = \frac{L}{2}(B_1 + B_2) \times 1 = 8(B_1 + B_2)$$

$$\text{Footing Weight} = 8(B_1 + B_2) \times 24 = 192(B_1 + B_2)$$

Note:

$$A_{req} = \frac{Q_{service,net}}{q_{all,net}} = \frac{Q_{service,gross}}{q_{all,gross}}$$

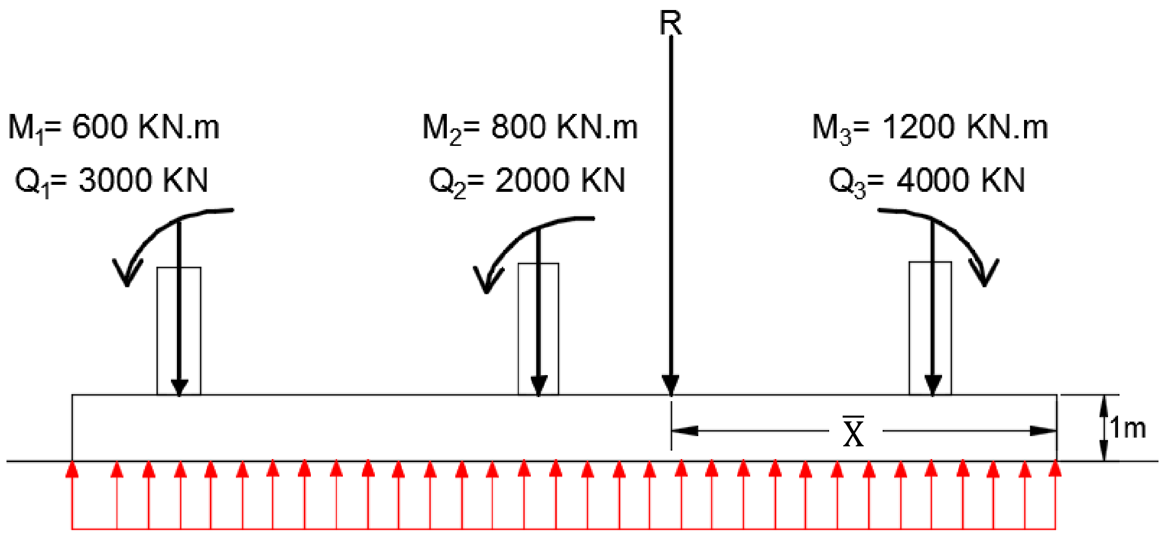
$Q_{service,net}$ = applied column load on the foundation

$Q_{service,gross}$ = Applied column load + footing weight + soil weight

In this problem the footing weight is given and the soil weight is not given, so we neglect it.
The given bearing capacity is allowable gross bearing capacity.

$$A_{req} = \frac{Q_{service,gross}}{q_{all,gross}} = 8(B_1 + B_2) \rightarrow \frac{9000 + 192(B_1 + B_2)}{300}$$

$$B_1 + B_2 = 4.07 \rightarrow B_1 = 4.07 - B_2 \rightarrow\rightarrow \text{Eq.}(1)$$



Now, take summation moments about the center of the footing (to eliminate the moments of the resultant force and the weight of the footing).

$$\sum M_{center} = 0.0 \rightarrow 3000 \times (14 - \bar{X}) + 600 + 2000(8 - \bar{X}) + 800$$
$$-4000 \times (\bar{X} - 2) - 1200 = 0.0 \rightarrow\rightarrow \bar{X} = 7.36 \text{ m}$$

$$\bar{X} = 7.36 = \frac{L}{3}\left(\frac{B_1 + 2B_2}{B_1 + B_2}\right) \rightarrow 7.36 = \frac{16}{3}\left(\frac{B_1 + 2B_2}{B_1 + B_2}\right)$$

$$1.38 = \left(\frac{B_1 + 2B_2}{B_1 + B_2}\right) \rightarrow\rightarrow \text{Eq.}(2) \quad \text{substitute from Eq.}(1)$$

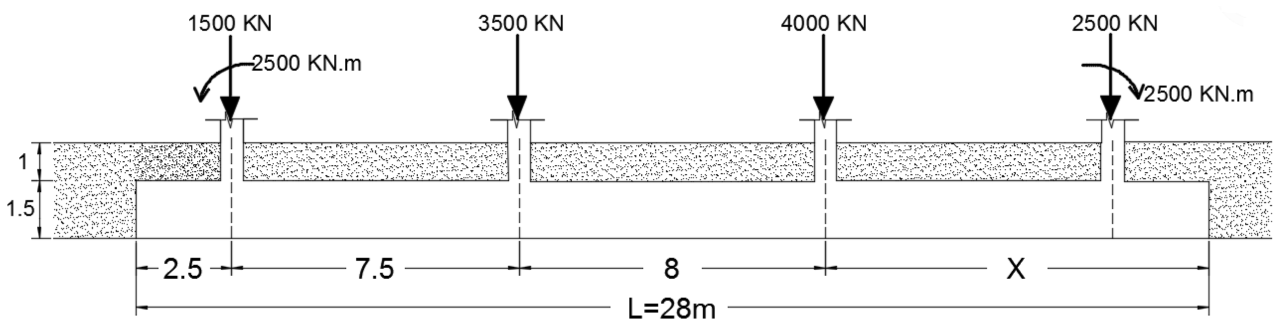
$$1.38 = \left(\frac{4.07 - B_2 + 2B_2}{4.07 - B_2 + B_2}\right) \rightarrow B_2 = 1.55 \text{ m}✓ \rightarrow B_1 = 2.52 \text{ m} ✓.$$

5.

A combined footing consists of four columns as shown in figure, **determine** the width of the rectangular combined footing B.

Allowable gross soil pressure = 255 kN/m^2.

$\gamma_c = 24\ \mathrm{KN/m^3}$ and $\gamma_{soile} = 20\ \mathrm{KN/m^3}$



Solution

Note that the foundation is rectangular (as given) but the pressure in this case will not be uniform, so we will design the footing for eccentric loadings.

$$W_f + W_s = 28 \times B \times (1 \times 20 + 1.5 \times 24) = 1568\ B$$

$$\sum Q = R_{service,gross} = 11500 + 1568\ B$$

Now calculate the moment at the centroid of the footing:

$$\sum M_{@center} = 1500 \times (14 - 2.5) + 2500 + 3500 \times 4 - 4000 \times 4$$
$$-2500(14 - 2.5) - 2500 = -13500\ \mathrm{kN.m}$$

$$e = \frac{\sum M_{@center}}{\sum Q} = \frac{13500}{11500 + 1568\ B}$$

The eccentricity in L direction and since L is so large $e < \frac{L}{6} \rightarrow$

$$q = \frac{Q}{B \times L}\left(1 \pm \frac{6e}{L}\right)$$

$$q_{max} = \frac{Q}{B \times L}\left(1 + \frac{6e}{L}\right)$$ by equating q_{max} and $q_{all,gross} \rightarrow$

$$255 = \frac{11500 + 1568\,B}{B \times 28}\left(1 + \frac{6\left(\frac{13500}{11500 + 1568\,B}\right)}{28}\right)$$

$\rightarrow B = 2.58\ m$ ✓.

Check For B:

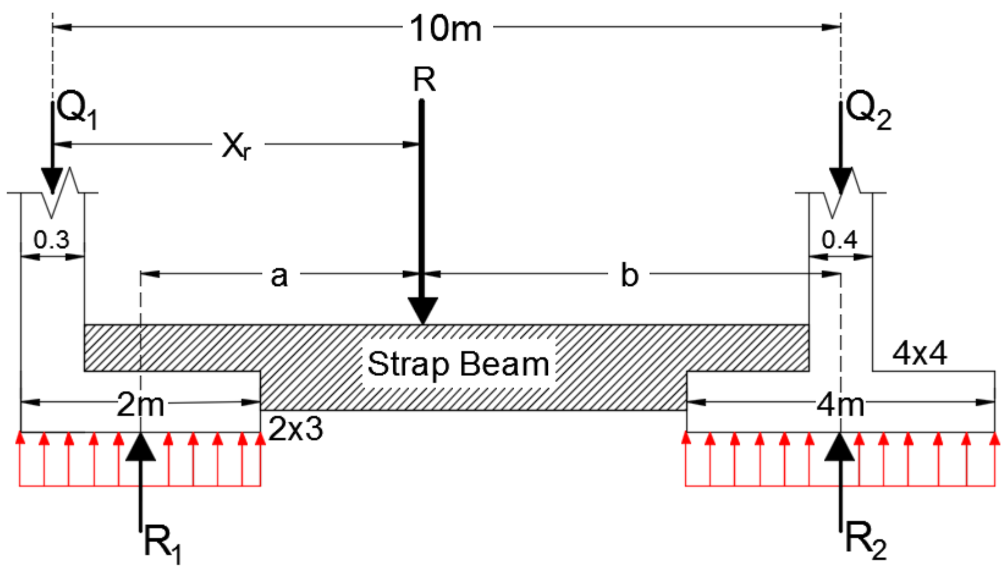
$$e = \frac{13500}{11500 + 1568\ \times 2.58} = 0.868\ m$$

$$\sum Q = 11500 + 1568\ \times 2.58 = 15545.4$$

$$q_{min} = \frac{Q}{B \times L}\left(1 - \frac{6e}{L}\right) = \frac{15545.4}{2.58 \times 28}\left(1 - \frac{6 \times 0.868}{28}\right) = 175.2 kN/m^2 > 0\ Ok$$

6.

For the strap footing shown below, if $q_{all,net}$ =250 kN/m^2, determine Q_1 and Q_2



Solution

$$R_1 = A_1 \times q_{all,net} = (2 \times 3) \times 250 = 1500\ KN$$

$$R_2 = A_2 \times q_{all,net} = (4 \times 4) \times 250 = 4000\ KN$$

$$R = R_1 + R_2 = 1500 + 4000 = 5500\ KN = Q_1 + Q_2$$

$$a + b = 10 + 0.15 - 1 = 9.15m$$

$$\sum M_{R_2} = 0.0 (\text{after use of strap}) \rightarrow 1500 \times 9.15 = 5500 \times b \rightarrow b = 2.5\ m$$

$$\rightarrow a = 9.15 - 2.5 = 6.65\ m \rightarrow X_r = 6.65 + 1 - 0.15 = 7.5m$$

$$\sum M_{@\,Q_1} = 0.0 \rightarrow 10Q_2 = 5500 \times 7.5 \rightarrow Q_2 = 4125 \text{ kN} ✓.$$

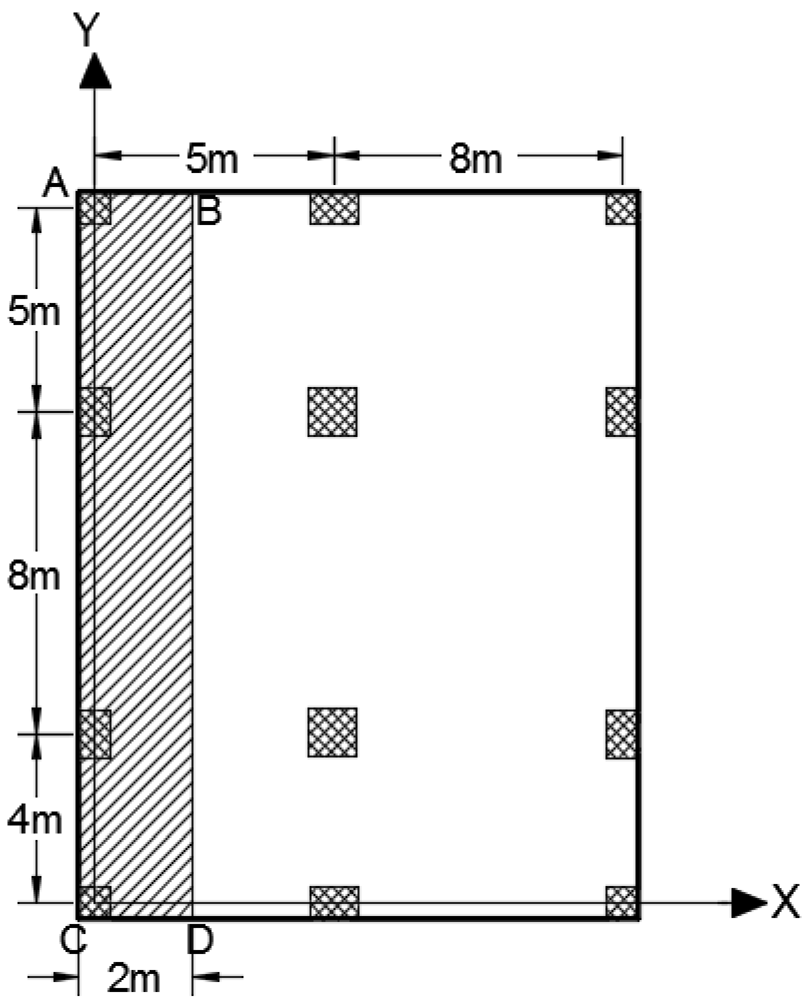
$$\rightarrow Q_1 = R - Q_2 = 5500 - 4125 = 1375 \text{ kN} ✓.$$

7.

For the shown mat foundation, If $q_{all,net}$=150 kN/m^2.

1. Check the adequacy of the foundation dimensions.
2. Draw SFD and BMD for the strip ABCD which is 2m width.

	Interior Columns	Edge Columns	Corner Columns
Columns Dimensions	60cm x 60cm	60 cm x 40 cm	40 cm x 40 cm
Service Loads	1800 kN	1200 kN	600 kN
Factored Loads	2700 kN	1800 kN	900 kN



Solution

Firstly we take the horizontal and vertical axes as shown in figure above.

Calculate the centroid of the footing with respect to these axes:

$B = 5 + 8 + 2 \times 0.2 = 13.4\text{m}$ (horizontal dimension)

$L = 5 + 8 + 4 + 2 \times 0.2 = 17.4\text{m}$ (certical dimension)

$$\overline{X} = \frac{13.4}{2} - \frac{0.4}{2} = 6.5\text{m (from y} - \text{axis)}$$

$$\overline{Y} = \frac{17.4}{2} - \frac{0.4}{2} = 8.5\text{ m (from x} - \text{axis)}$$

Calculate the resultant force R:

$$R = \sum Q_i \text{ (service loads)} = 2 \times 1800 + 6 \times 1200 + 4 \times 600 = 13200\text{ kN}$$

Calculate the location of resultant force R (X_R, Y_R) with respect to X and Y axes:

$$X_R = \frac{\sum Q_i \times X_{ri}}{\sum Q_i}$$

$$= \frac{2 \times 1800 \times 5 + 2 \times 1200 \times 5 + 2 \times 1200 \times 13 + 2 \times 600 \times 13}{13200}$$

$X_R = 5.82\text{ m (from y} - \text{axis)}$

Note that the moments of the first vertical line of columns will equal zero because y-axis is at the centerline of these columns.

$$Y_R = \frac{\sum Q_i \times Y_{ri}}{\sum Q_i}$$

$$= \frac{2 \times 1200 \times 4 + 1800 \times 4 + 2 \times 1200 \times 12 + 1800 \times 12 + 2 \times 600 \times 17 + 1200 \times 17}{13200}$$

$Y_R = 8.2\text{ m (from x} - \text{axis)}$

Note that the moments of the first horizontal line of columns will equal zero because x-axis is at the centerline of these columns.

Calculate the eccentricities:

$e_x = |X_R - \overline{X}| = |5.82 - 6.5| = 0.68$

$e_y = |Y_R - \overline{Y}| = |8.2 - 8.5| = 0.3\text{m}$

Calculate moments in X and Y directions:

$$M_x = e_y \times \sum Q_i = 0.3 \times 13200 = 3960 \text{ kN.m}$$

$$M_y = e_x \times \sum Q_i = 0.68 \times 13200 = 8976 \text{ kN.m}$$

Calculate moment of inertia in X and Y directions:

$$I_x = \frac{B\,L^3}{12} = \frac{13.4 \times 17.4^3}{12} = 5882.6 \text{ m}^4$$

$$I_y = \frac{L\,B^3}{12} = \frac{17.4 \times 13.4^3}{12} = 3488.85 \text{ m}^4$$

Not that the value which **perpendicular** to the required axis will be tripled because it's the value that resist the moment.

Calculate the stresses:

$$q = \frac{\sum Q_i}{A} \pm \frac{M_y}{I_y} X \pm \frac{M_x}{I_x} Y \rightarrow q = \frac{13200}{13.4 \times 17.4} \pm \frac{8976}{3488.85} X \pm \frac{3960}{5882.6} Y$$

$$q = 56.61 \pm 2.57\,X \pm 0.67\,Y$$

If compression, use (+) sign

If Tension, use (−) sign.

But we want to calculate q_{max} and q_{min}

$$q_{max} = 56.61 + 2.57\,X + 0.67\,Y$$

$$q_{min} = 56.61 - 2.57\,X - 0.67\,Y$$

$$X = \text{ maximum horizontal distance from centroid to the edge of mat} = \frac{13.4}{2} = 6.7 \text{ m}$$

$$Y = \text{ maximum vertical distance from centroid to the edge of mat} = \frac{17.4}{2} = 8.7 \text{ m}$$

$$\rightarrow q_{max} = 56.61 + 2.57 \times 6.7 + 0.67 \times 8.7 = 79.66 \text{ kN/m}^2$$

$$\rightarrow q_{min} = 56.61 - 2.57 \times 6.7 - 0.67 \times 8.7 = 33.56 \text{ kN/m}^2$$

$$q_{max} = 79.66 < q_{all,net} = 150 \rightarrow \text{Ok}$$

$$q_{min} = 33.56 > 0.0 \rightarrow \text{Ok}$$

The foundation Dimensions are adequate✓.

Now we want to Draw SFD and BMD for strip ABCD (using factored loads)

Locate point E at the middle of the upper edge of strip (between A and B) and point F at the middle of the lower edge of strip (between C and D).

$$R_u = \sum Q_{ui} = 2 \times 2700 + 6 \times 1800 + 4 \times 900 = 19800 \text{ kN}$$

The eccentricities will not change since we factored all loads by the same factor

$$M_{u,x} = e_y \times \sum Q_{ui} = 0.3 \times 19800 = 5940 \text{ kN.m}$$

$$M_{u,y} = e_x \times \sum Q_{ui} = 0.68 \times 19800 = 13464 \text{ kN.m}$$

$$q_u = \frac{\sum Q_{ui}}{A} \pm \frac{M_{u,y}}{I_y} X \pm \frac{M_{u,x}}{I_x} Y \rightarrow q_u = \frac{19800}{13.4 \times 17.4} \pm \frac{13464}{3488.85} X \pm \frac{5940}{5882.6} Y$$

$$\rightarrow q_u = 84.92 \pm 3.86\, X \pm 1.01\, Y$$

If compression, use (+) sign

If Tension, use (−) sign.

$X =$ horizontal distance from centroid to the points E and F at strip

$$= \frac{13.4}{2} - \frac{2}{2} = 5.7 \text{ m}$$

$Y =$ vertical distance from centroid to points E and F at strip

$$= \frac{17.4}{2} = 8.7 \text{ m}$$

The shown figure explains the moments signs (compression or tension) and the strip ABCD with points E and F:

At point E:

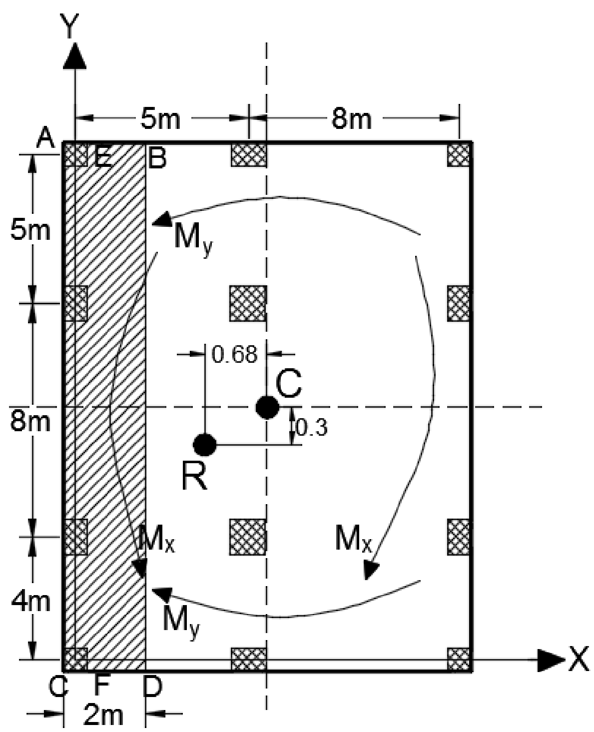
$M_y =$ compression (+)

$M_x =$ tension (−)

At point F:

$M_y =$ compression (+)

$M_x =$ compression (+)



$$q_{u,E} = 84.92 + 3.86 \ \times 5.7 - 1.01 \ \times 8.7 = 98.13 \text{ kN/m}^2$$
$$q_{u,F} = 84.92 + 3.86 \ \times 5.7 + 1.01 \ \times 8.7 = 115.7 \text{ kN/m}^2$$
$$q_{u,avg} = \frac{q_{u,E} + q_{u,F}}{2} = \frac{98.13 + 115.7}{2} = 106.9 \text{ kN/m}^2$$

Now, we check the stability of the strip:

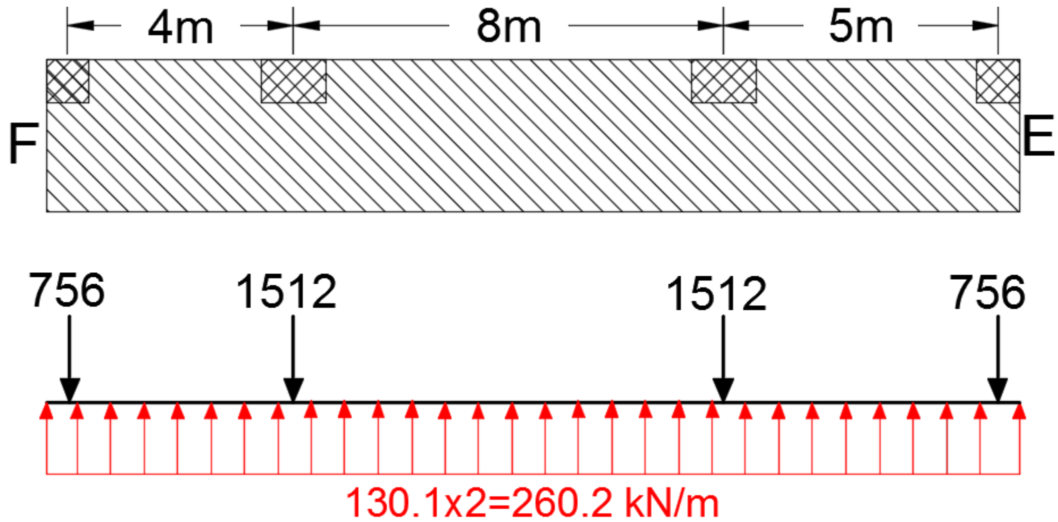
$$\left(\sum Q_{ui}\right)_{strip} = 2 \times 1800 + 2 \times 900 = 5400 \text{ kN}$$
$$q_{u,avg} \times A_{strip} = 106.9 \ \times (17.4 \times 2) = 3720$$
$$\left(\sum Q_{ui}\right)_{strip} \neq q_{u,avg} \times A_{strip} \rightarrow \text{Not stable} \rightarrow \text{Loads must modified}$$

$$\text{Average Load} = \frac{(\sum Q_{ui})_{strip} + q_{u,avg} \times A_{strip}}{2} = \frac{5400 + 3720}{2} = 4560 \text{ kN}$$
$$(Q_{ui})_{mod} = Q_{ui} \times \frac{\text{Average Load}}{(\sum Q_{ui})_{strip}} = Q_{ui} \times \frac{4560}{5400} = 0.84 Q_{ui}$$
$$(Q_{u,\text{corner columns}})_{mod} = 0.84 \times 900 = 756 \text{ Kn}$$
$$(Q_{u,\text{edge columns}})_{mod} = 0.84 \times 1800 = 1512 \text{ kN}$$
$$(q_{u,avg})_{mod} = q_{u,avg} \times \frac{\text{Average Load}}{q_{u,avg} \times A_{strip}} = 106.9 \times \frac{4560}{3720} = 131 \text{kN/m}^2$$

Now, we check the stability of the strip (after modification):

$$\left(\sum Q_{ui}\right)_{strip} = 2 \times 1512 + 2 \times 756 = 4536 \text{ kN}$$
$$q_{u,avg} \times A_{strip} = 131 \ \times (17.4 \times 2) = 4558.8 \text{ kN}$$
$$\left(\sum Q_{ui}\right)_{strip} \cong q_{u,avg} \times A_{strip} \ \rightarrow \text{can be considered stable}$$

The final loads on the strip are shown below:



Now you can draw SFD and BMD but you will observe that the SFD and BMD will not enclosed to zero (at the end) and this because the vertical loads not exactly the same $4536 \cong 4558.8$.
I try to say " don't confused if SFD and BMD" are not enclosed to zero (i.e. trust yourself ☺). (see problem No.2).

8.

For the mat foundation shown in the figure below:

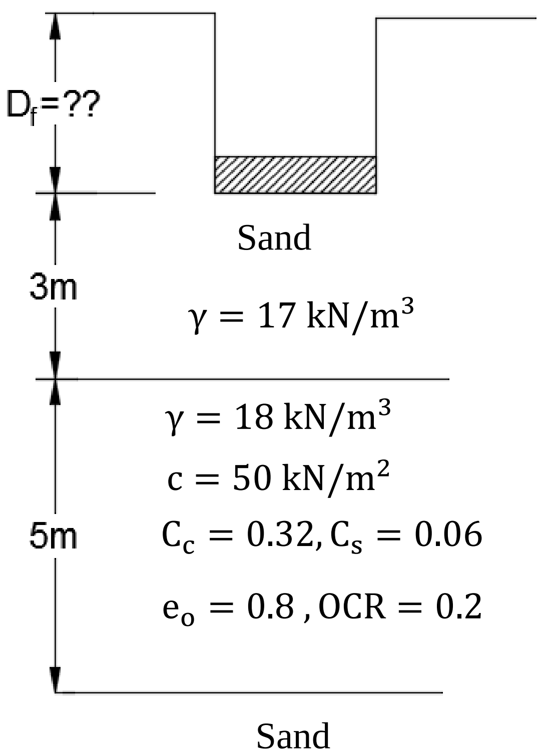
A. Calculate the depth of embedment for fully compensated foundation (ignore the mar self weight).

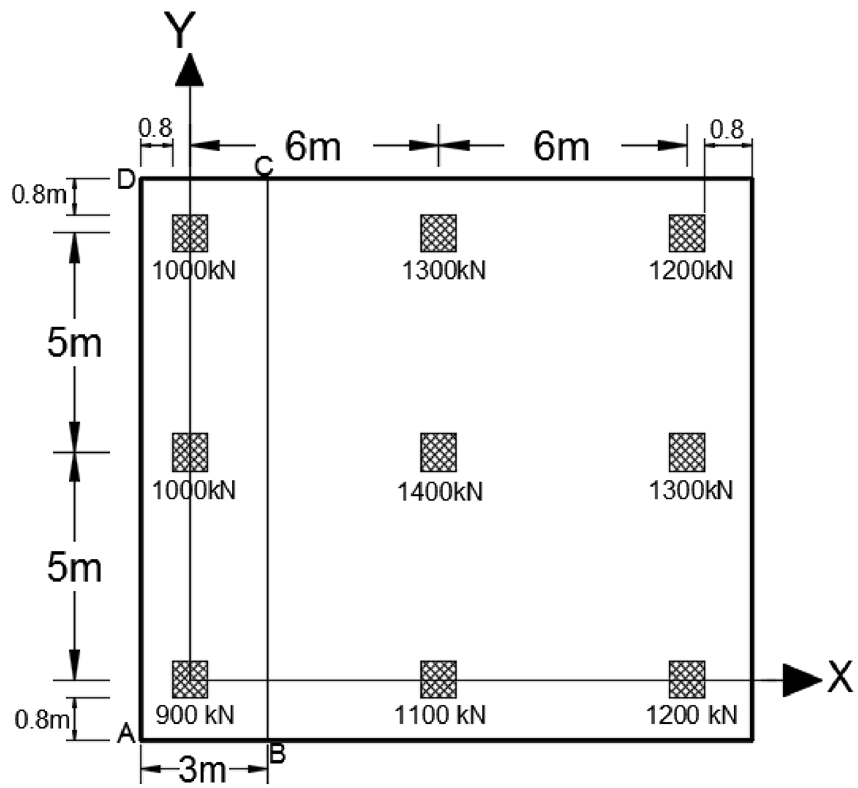
B. Find the pressure below points A, B, C and D.

C.Draw the SFD fir the strip ABCD.

D. For the soil profile shown below, calculate the primary consolidation settlement for clay layer using D_f =1.5m.

Hint: Use 2:1 method and all columns are 80 cm x 80 cm.





Solution

A-

The net applied pressure from the structure is:

$$q = \frac{Q}{A} - \gamma D_f \quad \text{but for fully compensated footing } \rightarrow q = 0.0 \rightarrow \frac{Q}{A} = \gamma D_f$$

$$\rightarrow D_f = \frac{Q}{A \times \gamma}$$

$$Q = \sum Q_i = 1000 + 1000 + 1300 + 900 + 1300 + 1400 + 1100$$
$$+1200 + 1300 + 1200 = 10400 \text{ KN}$$

$A = \text{area of mat foundation} = B \times L$

$$B = 5 + 5 + 2 \times 0.8 + 2 \times \frac{0.8}{2} = 12.4 \text{ m}$$

$$L = 6 + 6 + 2 \times 0.8 + 2 \times \frac{0.8}{2} = 14.4 \text{ m}$$

$A = 12.4 \times 14.4 = 178.56 \text{ m}^2$

$\gamma = \text{unit weight of the soil that will be excavated} = 17 \text{ kN/m}^3$

$$\rightarrow D_f = \frac{10400}{178.56 \times 17} = 3.426 \text{ m} \checkmark.$$

B-

Firstly we take the horizontal and vertical axes as shown in figure above.

Calculate the centroid of the footing with respect to these axes:

$B = 12.4\text{m (vertical dimension)}$

$L = 14.4\text{m (horizontal dimension)}$

$$\overline{X} = \frac{14.4}{2} - 0.8 - \frac{0.8}{2} = 6 \text{ m (from y} - \text{axis)}$$

$$\overline{Y} = \frac{12.4}{2} - 0.8 - \frac{0.8}{2} = 5 \text{ m (from x} - \text{axis)}$$

Calculate the resultant force R:

$$R = \sum Q_i = 10400 \text{ kN (as calculated above)}$$

Calculate the location of resultant force R (X_R, Y_R) with respect to X and Y axes:

$$X_R = \frac{\sum Q_i \times X_{ri}}{\sum Q_i}$$
$$= \frac{(1300 + 1400 + 1100) \times 6 + (2 \times 1200 + 1300) \times 12}{10400}$$

$X_R = 6.461 \text{ m (from y} - \text{axis)}$

Note that the moments of the first vertical line of columns will equal zero because y-axis is at the centerline of these columns.

$$Y_R = \frac{\sum Q_i \times Y_{ri}}{\sum Q_i}$$
$$= \frac{(1000 + 1400 + 1300) \times 5 + (1000 + 1300 + 1200) \times 10}{10400}$$
$$Y_R = 5.14 \text{ m (from x} - \text{axis)}$$

Note that the moments of the first horizontal line of columns will equal zero because x-axis is at the centerline of these columns.

Calculate the eccentricities:

$$e_x = |X_R - \overline{X}| = |6.461 - 6| = 0.461\text{m}$$
$$e_y = |Y_R - \overline{Y}| = |5.144 - 5| = 0.144 \text{ m}$$

Calculate moments in X and Y directions:

$$M_x = e_y \times \sum Q_i = 0.144 \times 10400 = 1497.6 \text{ kN.m}$$
$$M_y = e_x \times \sum Q_i = 0.461 \times 10400 = 4794.4 \text{ kN.m}$$

Calculate moment of inertia in X and Y directions:

$$I_x = \frac{L\,B^3}{12} = \frac{14.4 \times 12.4^3}{12} = 2287.95 \text{ m}^4$$
$$I_y = \frac{B\,L^3}{12} = \frac{12.4 \times 14.4^3}{12} = 3085.5 \text{ m}^4$$

Not that the value which **perpendicular** to the required axis will be tripled because it's the value that resist the moment.

Calculate the stresses:

$$q = \frac{\sum Q_i}{A} \pm \frac{M_y}{I_y} X \pm \frac{M_x}{I_x} Y \rightarrow q = \frac{10400}{12.4 \times 14.4} \pm \frac{4794.4}{3085.5} X \pm \frac{1497.6}{2287.95} Y$$
$$q = 58.24 \pm 1.55X \pm 0.6545\,Y$$

If compression, use (+) sign

If Tension, use (–) sign.

At point A:

$$M_y = \text{tension } (-)$$
$$M_x = \text{tension } (-)$$
$$X_A = \frac{L}{2} = \frac{14.4}{2} = 7.2 \text{ m} \qquad Y_A = \frac{B}{2} = \frac{12.4}{2} = 6.2 \text{ m}$$

$q_A = 58.24 - 1.55 \times 7.2 - 0.6545 \times 6.2 = 43.02 \text{ kN/m}^2 \checkmark.$

At point B:

$M_y = \text{tension } (-)$

$M_x = \text{tension } (-)$

$X_B = \frac{L}{2} - 3 = \frac{14.4}{2} - 3 = 4.2 \text{ m} \qquad Y_B = \frac{B}{2} = \frac{12.4}{2} = 6.2 \text{ m}$

$q_B = 58.24 - 1.55 \times 4.2 - 0.6545 \times 6.2 = 47.67 \text{ kN/m}^2 \checkmark.$

At point C:

$M_y = \text{tension } (-)$

$M_x = \text{tension } (+)$

$X_C = \frac{L}{2} - 3 = \frac{14.4}{2} - 3 = 4.2 \text{ m} \qquad Y_C = \frac{B}{2} = \frac{12.4}{2} = 6.2 \text{ m}$

$q_C = 58.24 - 1.55 \times 4.2 + 0.6545 \times 6.2 = 55.78 \text{ kN/m}^2 \checkmark.$

At point D:

$M_y = \text{tension } (-)$

$M_x = \text{tension } (+)$

$X_D = \frac{L}{2} = \frac{14.4}{2} = 7.2 \text{ m} \qquad Y_D = \frac{B}{2} = \frac{12.4}{2} = 6.2 \text{ m}$

$q_D = 58.24 - 1.55 \times 7.2 + 0.6545 \times 6.2 = 51.14 \text{ kN/m}^2 \checkmark.$

C-

Locate point E at the middle of the upper edge of strip (between C and D) and point F at the middle of the lower edge of strip (between A and B).

Note that we are not given factored load, so we use the given loads to draw SFD and BMD.

The relation for calculating the stress (q) will remains unchanged because the loads on columns doesn't changed.

$q = 58.24 \pm 1.55X \pm 0.6545\ Y$

At point E:

$M_y = \text{tension } (-)$

$M_x = \text{tension } (+)$

$X_E = \frac{L}{2} - \frac{3}{2} = \frac{14.4}{2} - \frac{3}{2} = 5.7 \text{ m} \qquad Y_E = \frac{B}{2} = \frac{12.4}{2} = 6.2 \text{ m}$

$q_E = 58.24 - 1.55 \times 5.7 + 0.6545 \times 6.2 = 53.46 \text{ kN/m}^2$

At point F:

$M_y = \text{tension}\ (-)$

$M_x = \text{tension}\ (-)$

$X_F = \frac{L}{2} - \frac{3}{2} = \frac{14.4}{2} - \frac{3}{2} = 5.7\ \text{m} \qquad Y_F = \frac{B}{2} = \frac{12.4}{2} = 6.2\ \text{m}$

$q_F = 58.24 - 1.55 \times 5.7 - 0.6545\ \times 6.2 = 45.35\ \text{kN/m}^2 \checkmark.$

$q_{avg} = \frac{q_E + q_F}{2} = \frac{53.46 + 45.35}{2} = 49.4\ \text{kN/m}^2$

Now, we check the stability of the strip:

$\left(\sum Q_i\right)_{strip} = 2 \times 1000 + 900 = 2900\ \text{kN}$

$q_{u,avg} \times A_{strip} = 49.4\ \times (12.4 \times 3) = 1837.68\ \text{kN}$

$\left(\sum Q_i\right)_{strip} \neq q_{avg} \times A_{strip} \rightarrow \text{Not stable} \rightarrow \text{Loads must modified}$

$\text{Average Load} = \frac{(\sum Q_i)_{strip} + q_{avg} \times A_{strip}}{2} = \frac{2900 + 1837.68}{2} = 2368.8\ \text{kN}$

$(Q_i)_{mod} = Q_i \times \frac{\text{Average Load}}{(\sum Q_i)_{strip}} = Q_i \times \frac{2368.8}{2900} = 0.816 Q_i$

$(Q_1)_{mod} = 0.816 \times 1000 = 816\ \text{KN}$

$(Q_2)_{mod} = 0.816 \times 900 = 734.4\ \text{kN}$

$(q_{avg})_{mod} = q_{avg} \times \frac{\text{Average Load}}{q_{avg} \times A_{strip}} = 49.4 \times \frac{2368.8}{1837.68} = 63.67 \text{kN/m}^2$

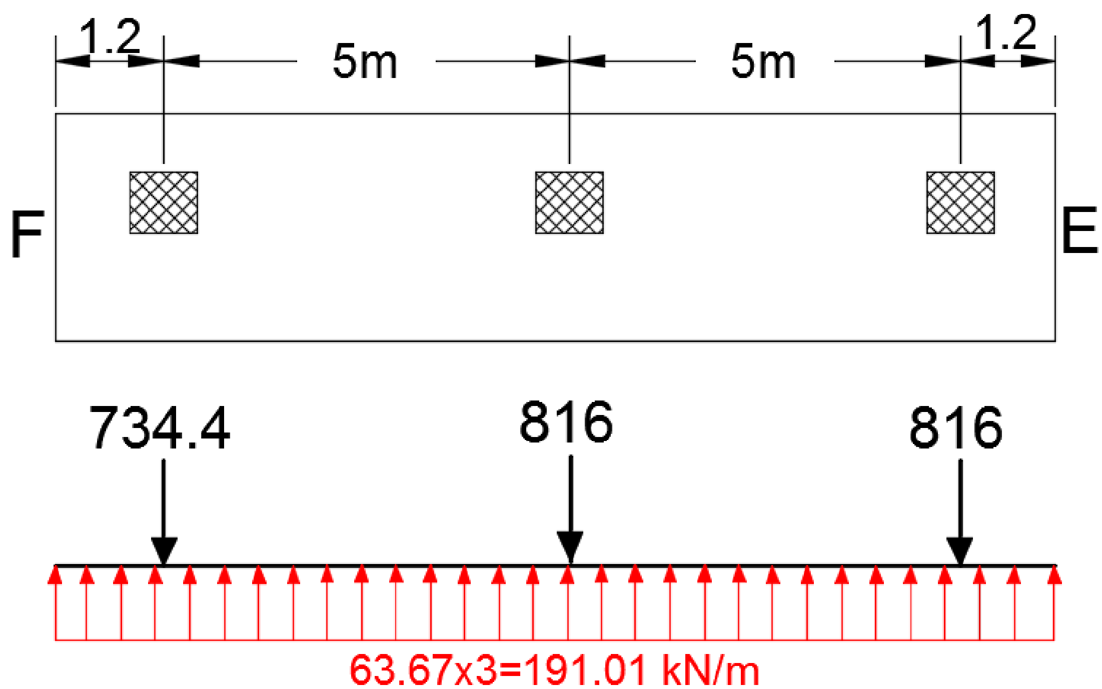
Now, we check the stability of the strip (after modification):

$\left(\sum Q_i\right)_{strip} = 2 \times 816 + 734.4 = 2366.4\ \text{kN}$

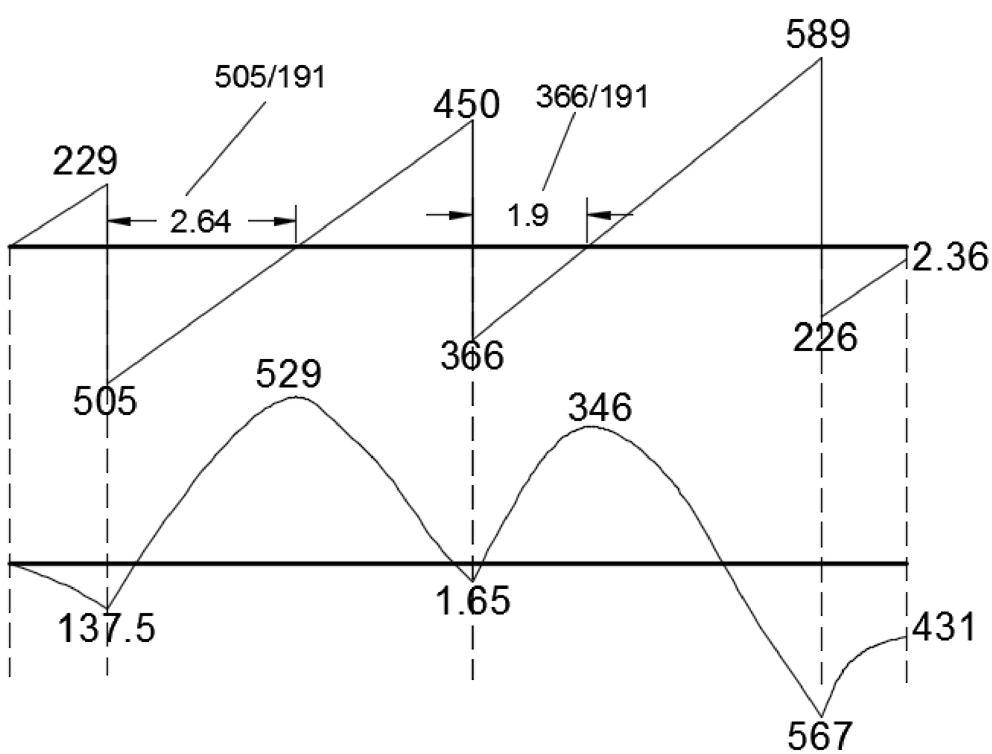
$q_{u,avg} \times A_{strip} = 63.67\ \times (12.4 \times 3) = 2368.52\ \text{kN}$

$\left(\sum Q_{ui}\right)_{strip} \cong q_{u,avg} \times A_{strip} \rightarrow \text{can be considered stable}$

The final loads on the strip are shown below:



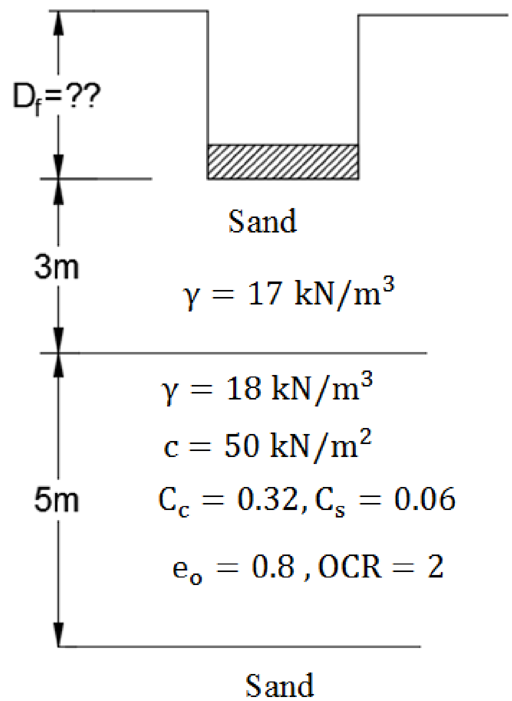
The SFD and BMD are shown below:



Note that the BMD not enclosed to zero, because we assume that the pressure under the strip is uniform and this assumption is not accurate.

D-

To calculate consolidation settlement, the following parameters should be calculated firstly:



Calculation of present effective stress at the **middle** of clay layer (σ'_o):

$D_f = 1.5$ m (as given)

$$\sigma'_o = 17 \times 4.5 + 18 \times \frac{5}{2} = 121.5\ \mathrm{kN/m^2}$$

Calculation of preconsolidation pressure (σ'_c):

$$\mathrm{OCR} = \frac{\sigma'_c}{\sigma'_o} \rightarrow 2 = \frac{\sigma'_c}{121.5} \rightarrow \sigma'_c = 243\ \mathrm{kN/m^2}$$

Calculation of average stress increase on the clay layer ($\Delta\sigma'_{av}$):

$$\Delta\sigma'_{av} = \frac{\Delta\sigma'_t + 4\Delta\sigma'_m + \Delta\sigma'_b}{6}$$

We calculate the stress increase at the top, middle and bottom of clay layer by using (2:1) method (as given):

$$\Delta\sigma'_z = \frac{P}{(B+z)\times(L+z)}\ , P = \sum Q_i = 10400\ \mathrm{kN}, B = 12.4\mathrm{m}, L = 14.4\mathrm{m}$$

At the top of the clay layer ($\Delta\sigma'_t$):

z = 3m (distance from mat foundation to the required point)

$$\Delta\sigma'_t = \frac{10400}{(12.4+3)\times(14.4+3)} = 38.8\ \mathrm{kN/m^2}$$

At the middle of the clay layer ($\Delta\sigma'_m$):

$z = 5.5$

$$\Delta\sigma'_m = \frac{10400}{(12.4+5.5)\times(14.4+5.5)} = 29.2\ \mathrm{kN/m^2}$$

At the bottom of the clay layer ($\Delta\sigma'_b$):

$z = 5.5$

$$\Delta\sigma'_b = \frac{10400}{(12.4+8)\times(14.4+8)} = 22.75\ \mathrm{kN/m^2}$$

$$\rightarrow \Delta\sigma'_{av} = \frac{38.8 + 4 \times 29.2 + 22.75}{6} = 29.72\ \mathrm{kN/m^2}$$

$\sigma'_o + \Delta\sigma'_{av} = 121.5 + 29.72 = 151.22 < \sigma'_c = 243 \rightarrow\rightarrow$

Use the following equation for calculating consolidation settlement:

For $(\sigma'_c \geq \sigma'_o + \Delta\sigma'_{av})$:

$$S_c = \frac{C_s \times H}{1 + e_o} \times \log\left(\frac{\sigma'_o + \Delta\sigma'_{av}}{\sigma'_o}\right)$$

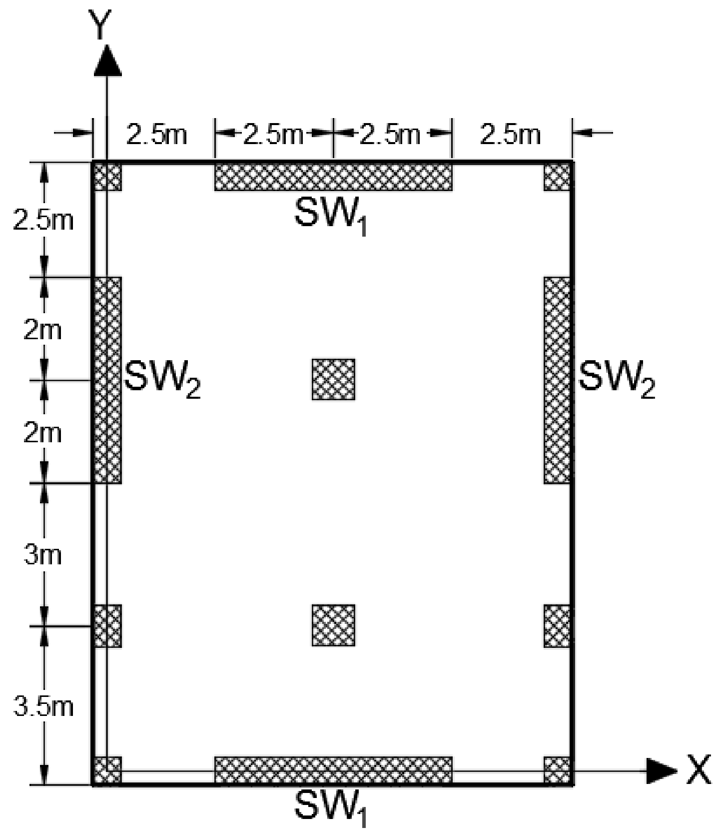
$C_s = 0.06\ ,\ \ e_o = 0.8\ ,\ \ H = 5\mathrm{m} \rightarrow$

$$S_c = \frac{0.06 \times 5}{1 + 0.8} \times \log\left(\frac{151.22}{121.5}\right) = 0.01583\ \mathrm{m} = 15.83\ \mathrm{mm}\ ✓.$$

9.

For the shown mat foundation, If $q_{all,net}$=150 kN/m^2, check the adequacy of the foundation dimensions.

	Interior Columns	Edge Columns	Corner Columns	SW_1	SW_2
Dimensions	60cm x 60cm	60cm x 40cm	40cm x 40cm	40cm x 5m	40cm x 4m
Loads	1800 kN	1200 kN	600 kN	1500 kN	1300 kN
Moment	0.0	0.0	0.0	800 KN.m	600 KN.m



Solution

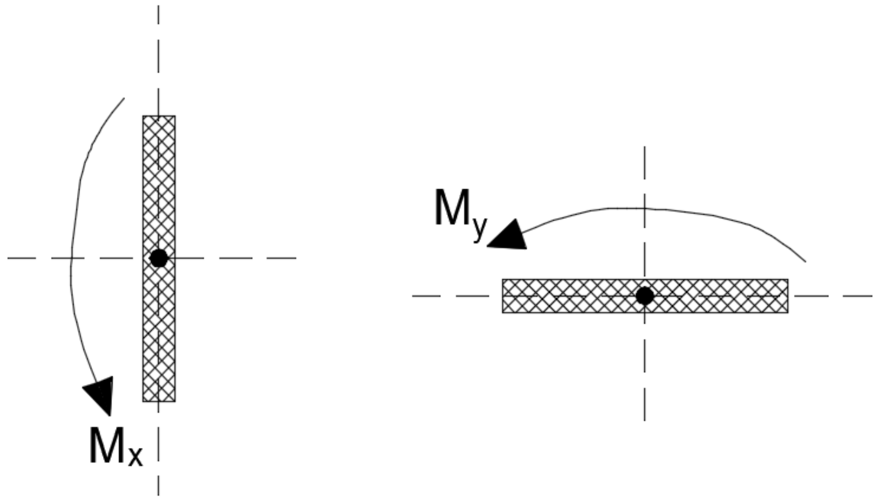
What is shear walls??

It's a structural element that used when the number of stories of the building became more than 7 stories to resist the horizontal forces (earthquake and wind).

Theses horizontal forces will exert a moments on shear walls, and thereby shear walls will resist these moment about its **strongest axis only.**

Also shear walls can resist **vertical loads** as a column.

The following figure explains how shear walls resist the applied moment (about its strongest axis):



Now, return to problem above, the procedures of solution will not differ from the other problem discussed above except in calculating the center of resultant force (X_R , Y_R) and for this problem will be calculated as following:

$$R = \sum Q = 4 \times 600 + 2 \times 1800 + 2 \times 1200 + 2 \times 1500 + 2 \times 1300$$
$$= 12802 \text{ kN}$$

To calculate X_R we take summation moments about y-axis:
Note that the moment of SW_1 is about y-axis so will be considered here, but the moment of SW_2 is about x-axis and will not be considered here.
The loads of all shear walls will exert moment exactly as columns.
Always take the moments on shear walls in the same direction of the moments from columns loads because this will exerts the worst case (maximum eccentricity).

$$\sum M_{@y-axis} = 0.0 \rightarrow 2 \times 800 + 2 \times 1500 \times (5 - 0.2)$$
$$+2 \times 1800 \times (5 - 0.2) + 2 \times 600 \times (10 - 0.2 - 0.2)$$
$$+1300 \times (10 - 0.2 - 0.2) + 1200 \times (10 - 0.2 - 0.2) = 12802X_R$$
$$\rightarrow X_R = 5.374 \text{ m}$$

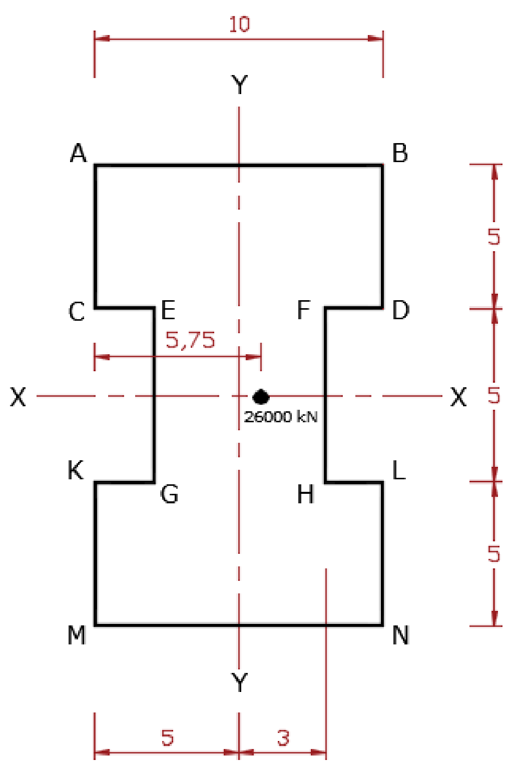
To calculate Y_R we take summation moments about X-axis:
Note that the moment of SW_2 is about X-axis so will be considered here, but the moment of SW_1 is about Y-axis and will not be considered here.
The loads of all shear walls will exert moment exactly as columns.

$$\sum M_{@X-axis} = 0.0 \rightarrow 2 \times 1200 \times (3.5 - 0.2) + 1800 \times (3.5 - 0.2)$$
$$+2 \times 600 + 2 \times 1300 \times (8.5 - 0.2) + 1800 \times (8.5 - 0.2)$$
$$+2 \times 600 \times (13 - 0.2 - 0.2) + 1500 \times (13 - 0.2 - 0.2) = 12802 Y_R$$
$$\rightarrow Y_R = 6.686 \text{ m}$$

Note:
The loads on the shear wall may be given as distributed load (kN/m) in this case multiply this load by the length of shear wall to be a concentrated load.
Now you can complete the problem exactly with the same procedures discussed in previous problems.

10.

Calculate the base pressure at the points indicated below the mat foundation shown. Total vertical load acting on the foundation is 26000 kN and it's location as shown in the figure.



Solution

Any shape you given it the first step is to calculate the centroid of this shape, but because the shape of the mat shown above is symmetry, the centroid will be at the middle of horizontal and vertical distance.

The idea in this problem is how to calculate moments of inertia in x and y directions as following:

$$I_{yy} = \frac{10^3 \times 5}{12} + \frac{6^3 \times 5}{12} + \frac{10^3 \times 5}{12} = 923.33\ m^4$$

$$I_{xx} = I_{box} - I_{two\ rectangles}$$

$$= \frac{15^3 \times 10}{12} - \frac{5^3 \times 2}{12} - \frac{5^3 \times 2}{12} = 2770.83\ m^4$$

Also we can calculate I_{xx} by using parallel axis theorem:

$$I_{xx} = \frac{5^3 \times 6}{12} + 2 \times \left(\frac{5^3 \times 10}{12} + Ad^2\right)$$

2: because the upper and the lower rectangles are the same.

$A =$ area of upper and lower rectangles $= 10 \times 5 = 50$

$d =$ distance from the center of the rectangle to $x-$axis $= 5m$

$$\rightarrow I_{xx} = \frac{5^3 \times 6}{12} + 2 \times \left(\frac{5^3 \times 10}{12} + 50 \times 5^2\right) = 2770.83$$

$$q = \frac{\sum Q_i}{A} \pm \frac{M_y}{I_y} X \pm \frac{M_x}{I_x} Y$$

But, as shown in the given figure, no eccentricity in y-direction, so the moment about x-axis equal zero and the equation will be:

$$q = \frac{\sum Q_i}{A} \pm \frac{M_y}{I_y} X$$

$$\sum Q_i = 26000\ Kn$$

$A =$ mat area $= 15 \times 10 - 2 \times (5 \times 2) = 130\ m^2$

$$M_y = \sum Q_i \times e_x$$

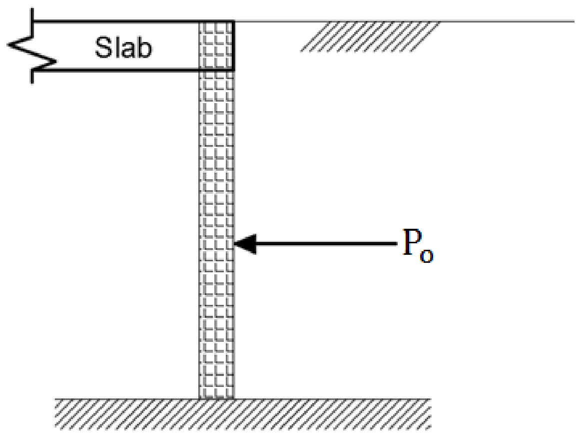
$e_x = 5.75 - 5 = 0.75\ m \rightarrow M_y = 26000 \times 0.75 = 19500\ kN.m$

Now, compute the stress at any point you want with sign convention discussed previously.

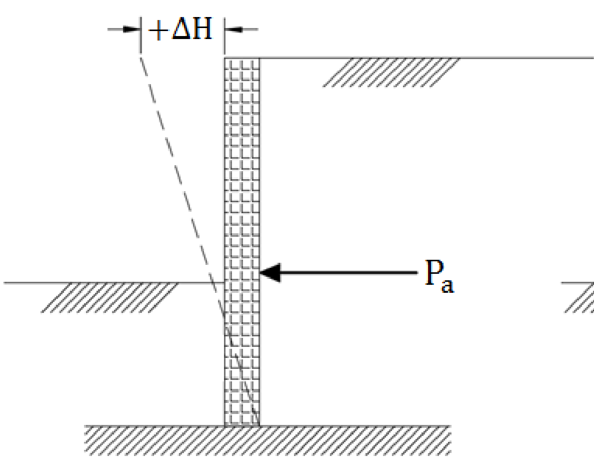
Chapter (7)
Lateral Earth Pressure

Introduction

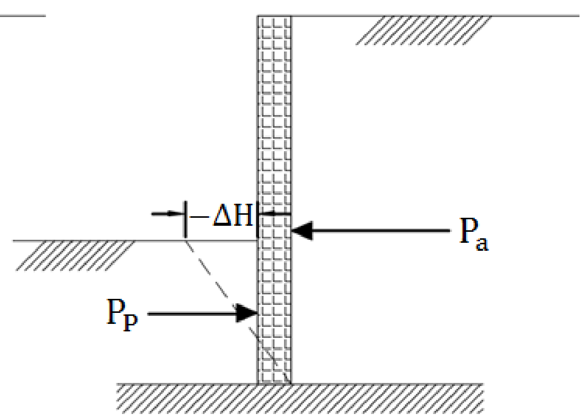
Vertical or near vertical slopes of soil are supported by retaining walls, cantilever sheet-pile walls, sheet-pile bulkheads, braced cuts, and other similar structures. The proper design of those structures required **estimation of lateral earth pressure**, which is a function of several factors, such as **(a)** type and amount of wall movement, **(b)** shear strength parameters of the soil, **(c)** unit weight of the soil, and **(d)** drainage conditions in the backfill. The following figures shows a retaining wall of height H. For similar types of backfill.



At rest LEP



Active LEP



Passive LEP

As shown in figure above, there are three types of Lateral Earth Pressure (LEP):

1. At Rest Lateral Earth Pressure:
The wall may be restrained from moving, for example; basement wall is restrained to move due to slab of the basement and the lateral earth force in this case can be termed as" P_o".

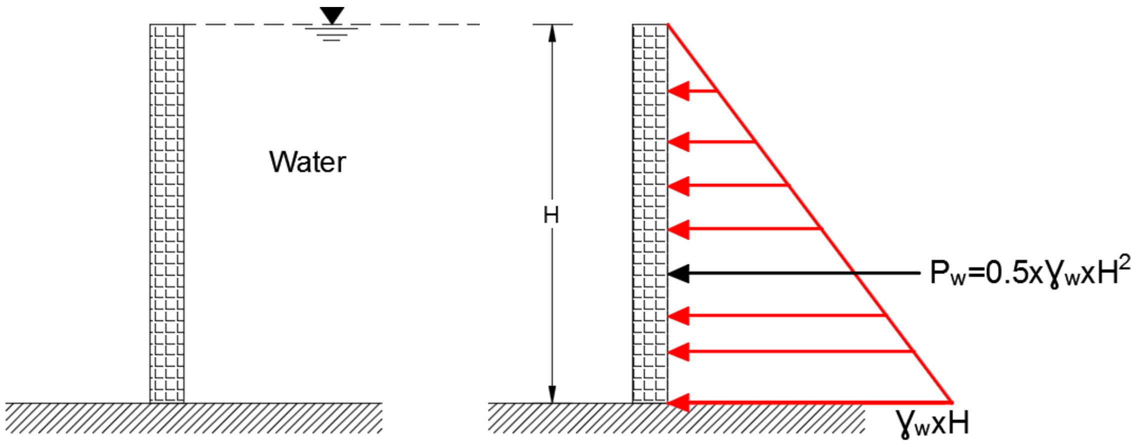
2. Active Lateral Earth Pressure:
In case of the wall is free from its upper edge (retaining wall), the wall may move away from the soil that is retained with distance " $+\Delta H$ " (i.e. the soil pushes the wall away) this means the soil is active and the force of this pushing is called active force and termed by " P_a".

3. Passive Lateral Earth Pressure:
For the wall shown above (retaining wall) in the left side there exist a soil with height less than the soil in the right and as mentioned above the right soil will pushes the wall away, so the wall will be **pushed into** the left soil (i.e. soil compresses the left soil) this means the soil has a passive effect and the force in this case is called passive force and termed by " P_P".

Now, we want to calculate the lateral pressure from water firstly and from earth (3 cases mentioned above) secondly.

Lateral Pressure from Water

As we learned previously in fluid mechanics course, the pressure of static fluid at a specific point is the same in all directions "Pascal's Law". So if there exist water in the soil (saturated soil) **we must** calculate the vertical stress for soil alone (effective stress) and calculate the vertical pressure for water alone (because the horizontal pressure of water is the same as vertical pressure), but for soil, each one –according soil parameters- having different transformation factor from vertical to horizontal pressure as we will discuss later. The following figure showing that the horizontal pressure of water against a wall is the same as vertical pressure:



At Rest Lateral Earth Pressure:

As stated above, the soil in this case is static and can't pushes the wall with any movement, the transformation factor of vertical pressure to horizontal pressure in this case is "K_o" and the lateral earth force is termed by "P_o"

Calculation of at rest lateral earth force "P_o" for different cases:

Firstly the value of K_o can be calculated as following:

$K_o = 1 - \sin\phi$

Always, (at rest) lateral earth pressure at any depth (z) may be calculated as following:

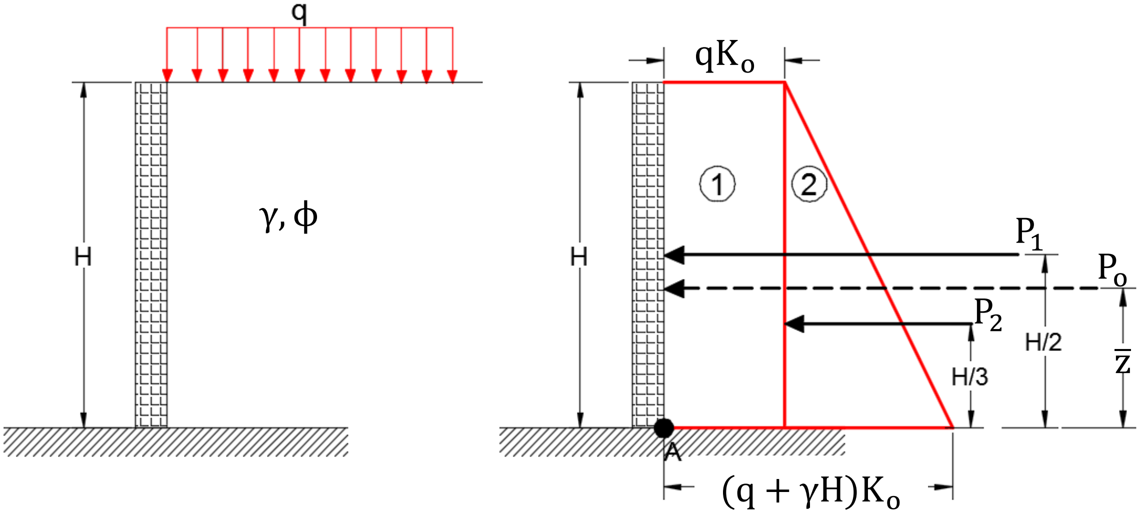
$\sigma_{h,o} = \text{vertical effective pressure } (\sigma'_v) \times K_o + \text{Pore water pressure } (u)$

$\sigma'_v = q + \gamma' \times z \quad , \quad u = \gamma_w \times z$

$\rightarrow \sigma_{h,o} = \sigma'_v \times K_o + u$

Note that the value of (u) doesn't multiplied by any factor since the horizontal pressure of water is the same as vertical pressure.

As shown there is no water table.



Firstly we calculate the vertical stress at each depth (each change):

At depth $z = 0.0$:

$\sigma'_v = q$

At depth $z = H$:

$\sigma'_v = q + \gamma \times H$

So, lateral at rest pressure at each depth now can be calculated:

At depth $z = 0.0$:

$\sigma_{h,o} = qK_o$

At depth $z = H$:

$\sigma_{h,o} = (q + \gamma \times H) \times K_o$

Now calculate the lateral forces P_1 and P_2 and then calculate P_o:

$P_1 = \text{Area of rectangle } (1) = qK_o \times H$ (per unit lenght)

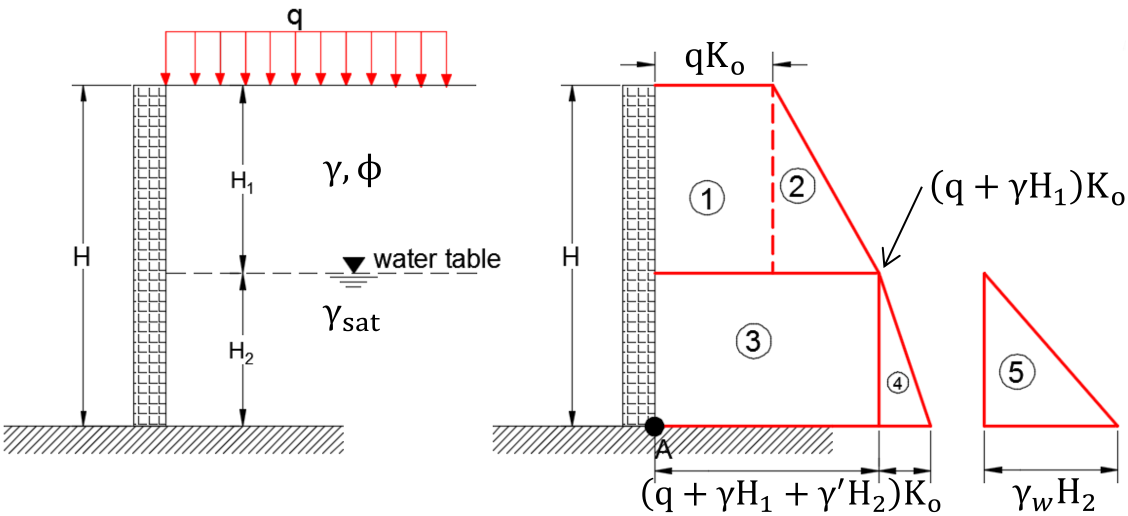
$$P_2 = \text{Area of triangle } (2) = \frac{1}{2} \times (\gamma \times H \times K_o) \times H = \frac{1}{2} \times \gamma \times H^2 \times K_o$$

$P_o = P_1 + P_2$

To find the location of P_o, take summation moments at point A (for ex.):

$$P_1 \times \frac{H}{2} + P_2 \times \frac{H}{3} = P_o \times \bar{z} \rightarrow\rightarrow \bar{z} = \checkmark.$$

In case of water table (if exist):



Firstly we calculate the vertical stress at each depth (each change):

At depth $z = 0.0$:

$\sigma'_v = q$

At depth $z = H_1$:

$\sigma'_v = q + \gamma \times H_1 \qquad u = 0.0$

At depth $z = H_2$:

$\sigma'_v = q + \gamma \times H_1 + \gamma' H_2 \quad (\gamma' = \gamma_{sat} - \gamma_w)$

$u = \gamma_w H_2$

Now calculate the lateral forces P_1, P_2, P_3, P_4 and P_5 then calculate P_o:

$P_1 = \text{area of recangle}(1) = qK_oH_1$

$P_2 = \text{area of triangle}(2) = \frac{1}{2}\gamma H_1^2 K_o$

$P_3 = \text{area of recangle}(3) = (q + \gamma H_1)K_o \times H_2$

$P_4 = \text{area of triangle}(4) = \frac{1}{2}\gamma' H_2^2 K_o$

$P_5 = \text{area of triangle}(5) = \frac{1}{2}\gamma_w H_2^2 \quad (\text{factor for water} = 1)$

$P_o = P_1 + P_2 + P_3 + P_4 + P_5$

To find the location of P_o, take summation moments at point A (for ex.):
(**Note**, P_5 must be included in moment equation)

(See example 7.1 Page 327)

Lateral Earth PressureTheories

Two theories are used to calculate lateral earth pressure (active and passive): **Rankine Earth Pressure theory and Coulomb's Earth Pressure theory.**
Firstly we will learn rankine earth pressure theory (the most important) and then coulomb earth pressure theory.

Rankine Active Lateral Earth Pressure

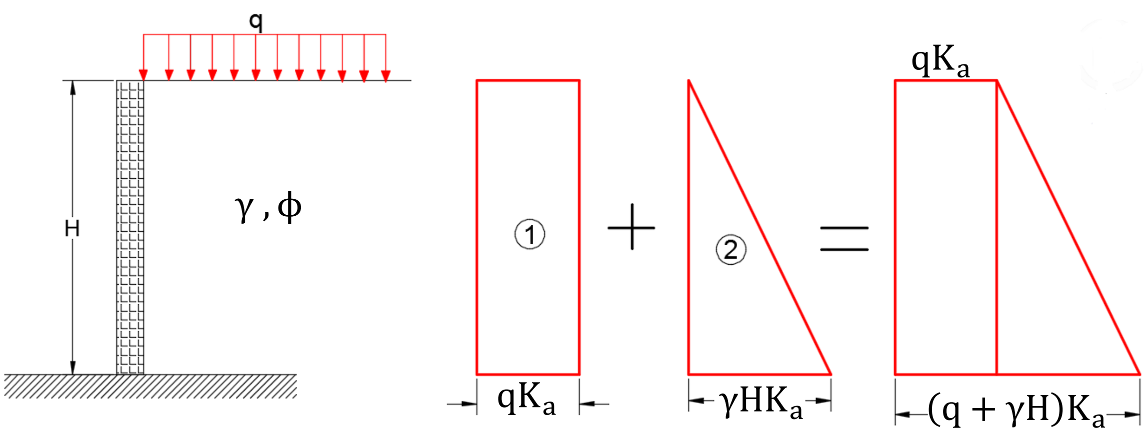
This theory is based mainly on the assumption of neglecting friction between the soil and the wall, so no shear forces are developed on soil particles.
As previously introduced, the soil in this case pushes the wall far away.
The transformation factor of vertical pressure to horizontal pressure in this case is "K_a" and the lateral earth force is termed by "P_a"

Firstly the value of K_a can be calculated as following:

$$K_a = \tan^2\left(45 - \frac{\phi}{2}\right)$$

There are different cases:

In case of granular soil (pure sand):



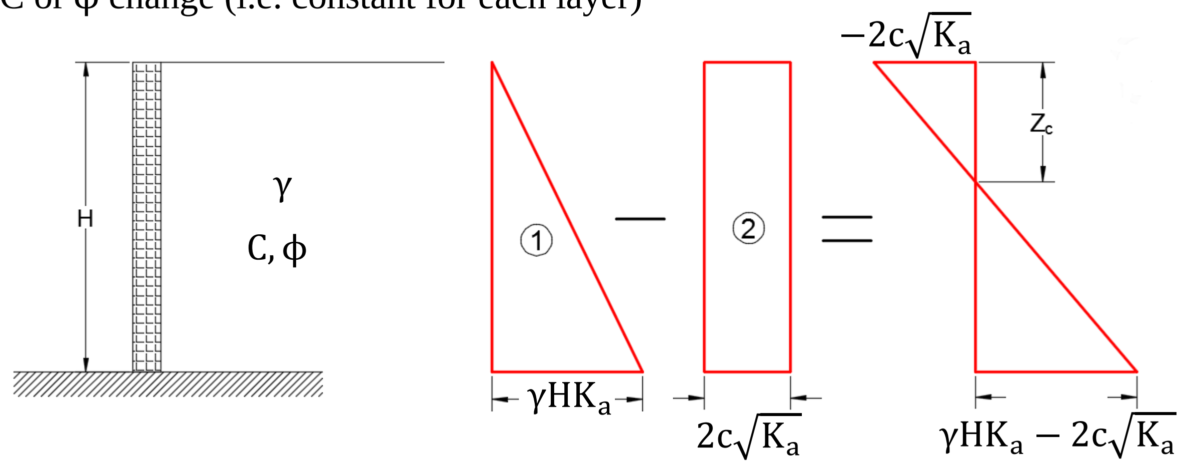
Exactly as the case of at rest LEP but here the transformation factor is K_a

$$P_a = P_1 + P_2$$

If the soil is $\mathbf{C - \phi}$ soil:

The clay exerts a lateral earth pressure with value of $2c\sqrt{K}$ (in general).
In case of active earth pressure the value of K is K_a, and when the wall moves away from soil, the soil particles will disturbed and the cohesion of soil will decreased, so in case of active earth pressure we subtract the lateral earth pressure of clay because the cohesion of clay decreased.

The value of $2c\sqrt{K}$ is constant along the layer, and differ when the value of C or ϕ change (i.e. constant for each layer)



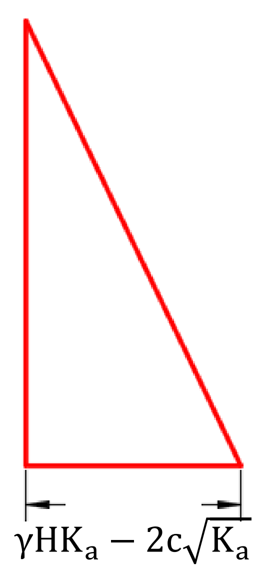
As we see, at depth z=0.0, the lateral earth pressure is $-2c\sqrt{K_a}$ this negative value (i.e. the soil will exerts a tensile stress on the wall) and this tensile stress will causes cracking on the wall from depth z=0.0 to depth = z_c.

Calculation of z_c:

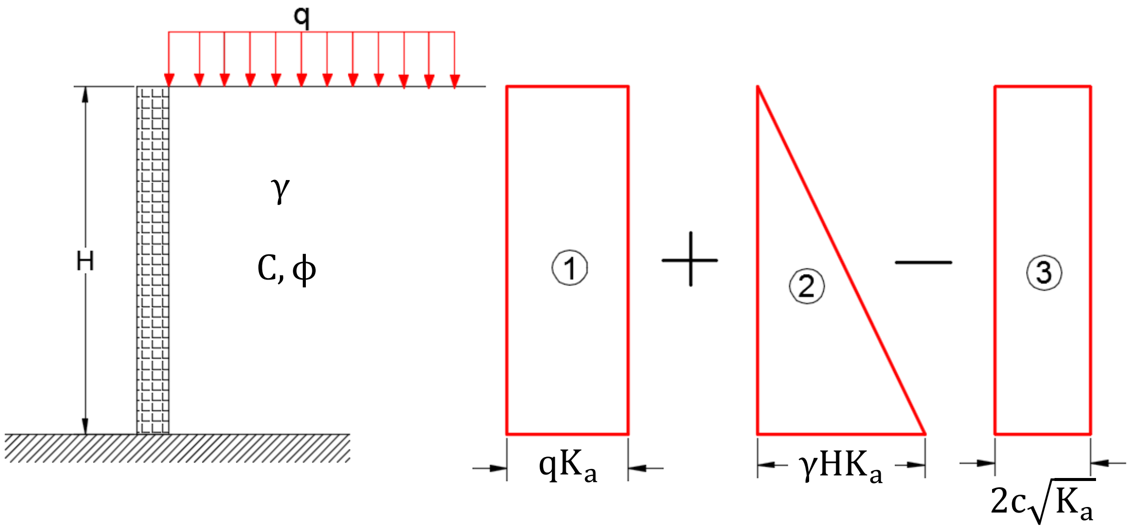
The lateral pressure will be zero at depth z_c:

$$\gamma\, z_c K_a - 2c\sqrt{K_a} = 0.0 \rightarrow\ z_c = \frac{2c\sqrt{K_a}}{\gamma K_a} = \frac{2c\sqrt{K_a}}{\gamma\, \sqrt{K_a} \times \sqrt{K_a}} = \frac{2c}{\gamma\, \sqrt{K_a}}$$

But, we know that soil can't develop any tension, so in design (in practice) we modify negative pressure to be zero and design for it (more safe because we enlarge lateral pressure) as shown:



If there exist surcharge:



So the final equation for active lateral earth pressure at and depth z can be calculated as following:

$\sigma_{h,a} = (q + \gamma H)K_a - 2c\sqrt{K_a}$

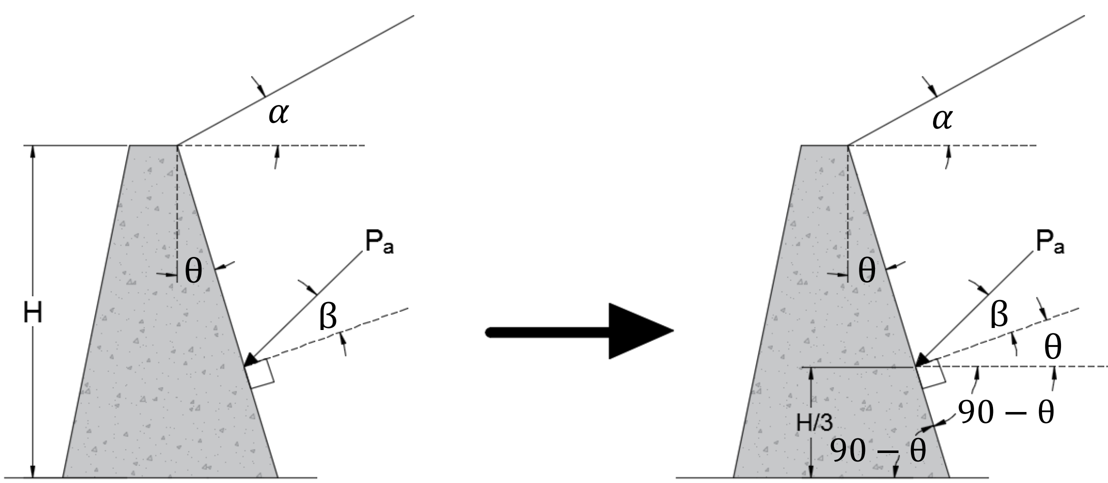
Note:

If there exist a water table, calculate the lateral force from water alone and then added it to the lateral force from soil to get total active force.

Generalized Case for Rankine Active Pressure:

In previous section, we calculate rankine active lateral pressure for the **vertical wall and horizontal backfill**, but here we will calculate the lateral earth pressure for general case (inclined wall and inclined backfill):

This General case for Granular soil only (pure sand)



$\alpha =$ inclination of backfill with horizontal
$\theta =$ inclination of wall with vertical
$\beta =$ inclination of P_a with the normal to the wall

From trigonometry, the angle between the normal to the wall and horizontal is θ.

Calculation of P_a:

$P_a = \text{Vertical force} \times K_a$

$\text{Vertical force} = \text{area of vertical pressure digram} = \frac{1}{2}\gamma H^2$

The value of K_a in this case is calculated from the following equation:

$$K_a = \frac{\cos(\alpha - \theta)\sqrt{1 + \sin^2\phi - 2\sin\phi\cos\psi_a}}{\cos^2\theta\left(\cos\alpha + \sqrt{\sin^2\phi - \sin^2\alpha}\right)}$$

$$\psi_a = \sin^{-1}\left(\frac{\sin\alpha}{\sin\phi}\right) - \alpha + 2\theta$$

The angle β is:

$$\beta = \tan^{-1}\left(\frac{\sin\phi\ \sin\psi_a}{1 - \sin\phi\ \cos\psi_a}\right)$$

$$P_a = \frac{1}{2}\gamma H^2 \times K_a$$

The location of P_a is $\frac{H}{3}$ from base as shown above

If we need the horizontal and vertical components of P_a:

$P_{a,h} = P_a\cos(\beta + \theta)$

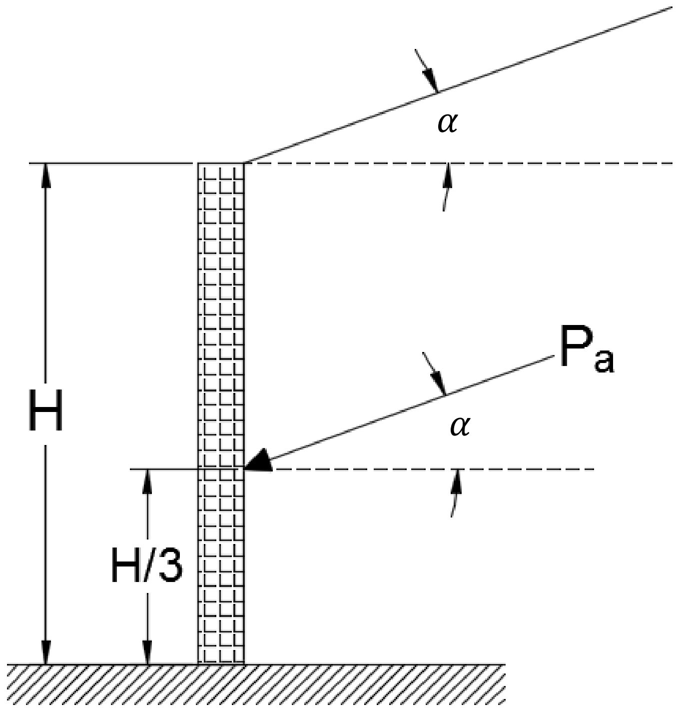
$P_{a,v} = P_a\sin(\beta + \theta)$

Case of vertical wall and inclined backfill:

Two cases:

1. **For pure sand**
2. **For $C - \phi$ soil**

For Pure sand:



Here P_a is inclined with angle α with horizontal.

Effective vertical pressure $= \gamma H$

$$P_a = \frac{1}{2}\gamma H^2 K_a$$

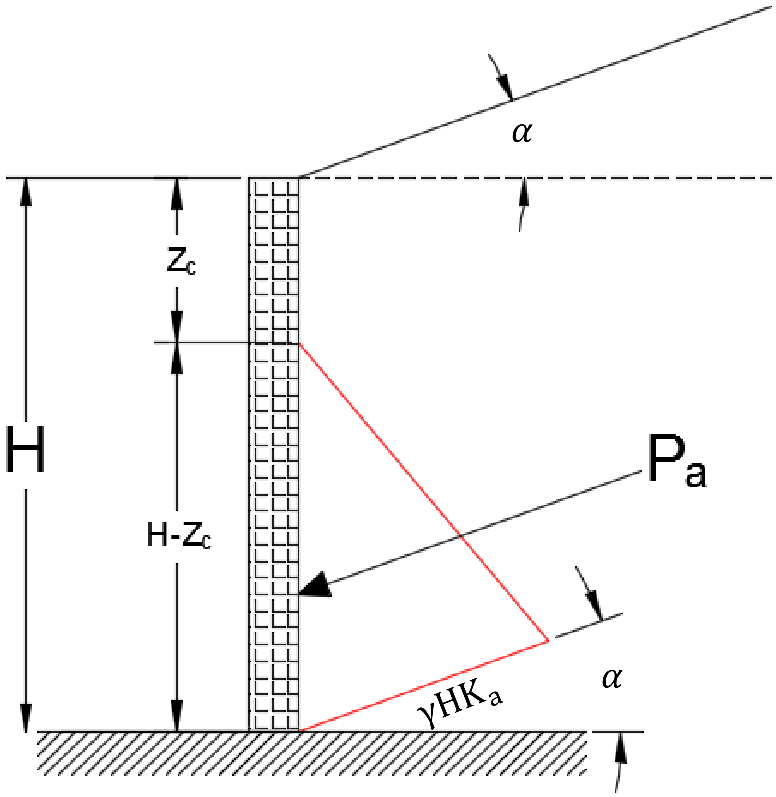
K_a in this case is calculated from the following equation:

$$K_a = \cos\alpha \frac{\cos\alpha - \sqrt{\cos^2\alpha - \cos^2\phi}}{\cos\alpha + \sqrt{\cos^2\alpha - \cos^2\phi}}$$

Or, by using (**Table 7.1Page 337)** easier than the equation above.

For $C-\phi$ soil

Here the force P_a is inclined with angle α with horizontal, but the calculation of P_a will differ because there exist clay and there is a tensile stress from clay at depth Z_c as shown:



The value of Z_c in this case is calculated as following:

$$Z_c=\frac{2c}{\gamma}\sqrt{\frac{1+\sin\phi}{1-\sin\phi}}$$

Calculation of P_a:

Vertical pressure$=\gamma H$

Horizontal pressure $=\gamma HK_a$

$K_a=K'_a\cos\alpha \rightarrow$ Horizontal pressure $=\gamma HK'_a\cos\alpha$

$$P_a=\frac{1}{2}\times(\gamma HK'_a\cos\alpha)\times(H-Z_c)$$

K'_a can be calculated from (**Table 7.2 Page 338**)

Note

The calculated value of P_a is inclined by angle α with horizontal, so:

$P_{a,h} = P_a \cos(\alpha)$
$P_{a,v} = P_a \sin(\alpha)$

Rankine Passive Lateral Earth Pressure

As previously introduced, the wall in this case pushed into the soil.
The transformation factor of vertical pressure to horizontal pressure in this case is "K_P" and the lateral earth force is termed by "P_P"

Firstly the value of K_P can be calculated as following:

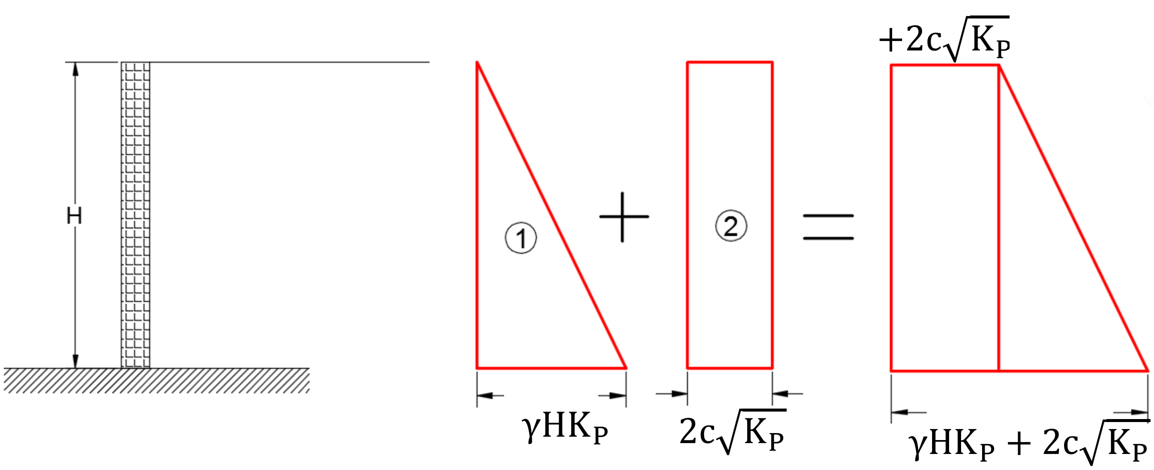
$$K_P = \tan^2\left(45 + \frac{\phi}{2}\right)$$

The only difference between passive and active is in the formula of calculating K.

If the soil is $\mathbf{C - \phi}$ soil:

In case of passive earth pressure the value of K is K_P, and when the wall moves into the soil, the soil particles will converges and the cohesion of soil will increased, so in case of passive earth pressure we **add** the lateral earth pressure of clay because the cohesion of clay increased.

The value of $2c\sqrt{K}$ is constant along the layer, and differ when the value of C or ϕ change (i.e. constant for each layer)



If there exist surcharge, will be added to vertical pressure and the final form for passive rankine pressure will be:

$$\sigma_{h,P} = (q + \gamma H)K_P + 2c\sqrt{K_P}$$

Rankine Passive Pressure (Vertical wall and inclined backfill):

The same as rankine active pressure for this case, the only difference is in the equation of calculating K_P (negative sign in active transformed to positive sign in passive). **(Page 363).**

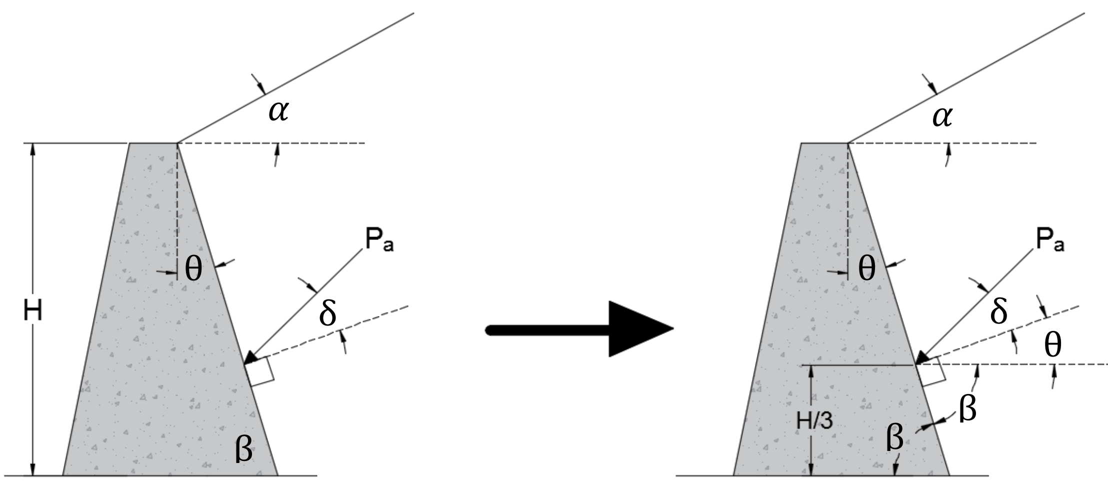
Coulomb's Lateral Earth Pressure Theory

The main assumption of this theory is considering the friction between the wall and the soil, this friction angle between the soil and the wall is (δ). So there exist shear stresses on the soil particles and the equations for calculating passive lateral earth coefficient will differ from equations of active lateral earth coefficient.

This theory deal only with granular soil (pure sand).

Coulomb's Active Lateral Earth Pressure

General case (inclined wall and inclined backfill):



$\alpha =$ inclination of backfill with horizontal
$\theta =$ inclination of wall with vertical
$\beta =$ inclination of wall with the horizontal
$\delta =$ friction angle between soil and wall
From trigonometry, the angle between the normal to the wall and horizontal is θ.

Calculation of P_a:

$P_a = \text{Vertical force} \times K_a$

$\text{Vertical force} = \text{area of vertical pressure digram} = \frac{1}{2}\gamma H^2$

The value of K_a in this case is calculated from **equation 7.26 Page 342.**

The value of δ is less than ϕ since δ is friction angle between wall and soil, but ϕ is friction angle between soil itself.

In general, the value of $\delta = \left(\frac{1}{2} \rightarrow \frac{2}{3}\right)\phi$ so there are tables for calculating K_a for $\delta = \frac{2}{3}\phi$ and for $\delta = \frac{1}{2}\phi$

(Table 7.4 Page 343) for $\delta = \frac{2}{3}\phi \rightarrow$

K_a is obtained according the following angles:

α, β and ϕ

(Table 7.5 Page 344) for $\delta = \frac{1}{2}\phi$

Now, we can easily calculate the value of P_a:

$$P_a = \frac{1}{2}\gamma H^2 \times K_a$$

$$P_{a,h} = P_a \cos(\delta + \theta) \qquad P_{a,v} = P_a \sin(\delta + \theta)$$

Special case (when the wall is vertical "$\beta = 90$" and the backfill is horizontal "$\alpha = 0$") .

$$P_a = \frac{1}{2}\gamma H^2 \times K_a \quad \text{(Inclined by angle } \delta \text{ with horizontal)}$$

K_a can be calculated from(**Table 7.3 P342**)according the values of δ and ϕ, so for this special case:

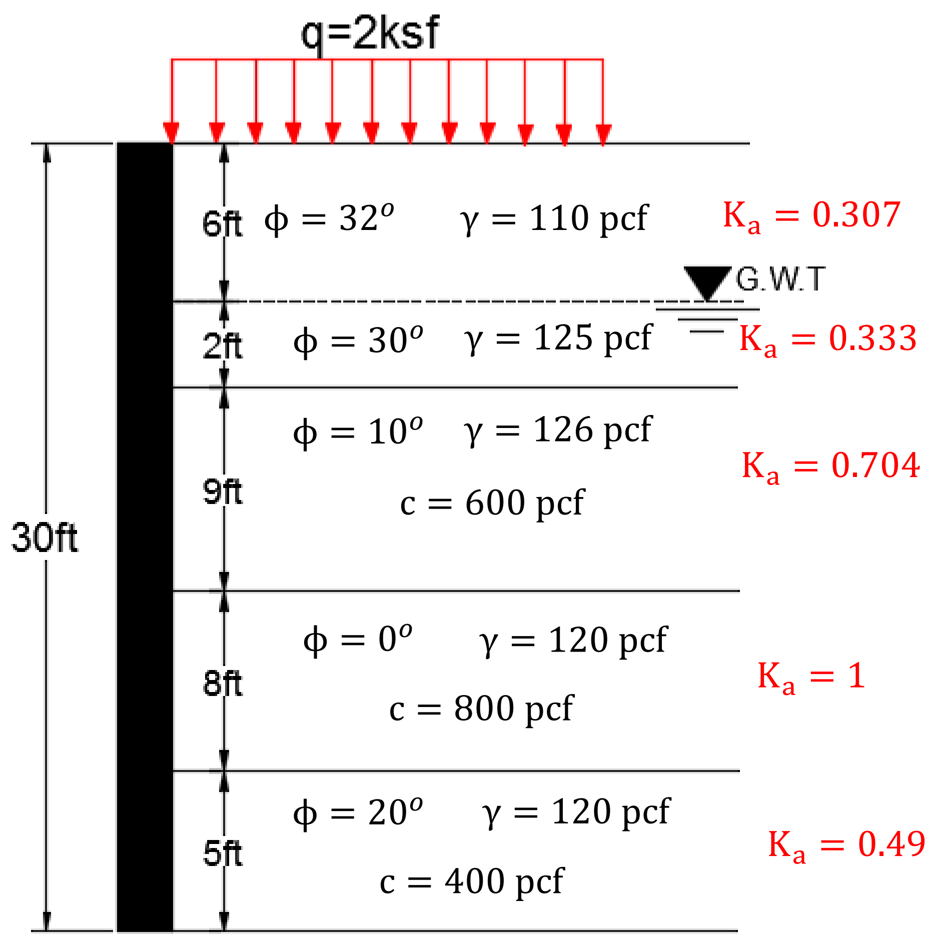
$$P_{a,h} = P_a \cos\delta \qquad P_{a,v} = P_a \sin\delta$$

Coulomb's Passive Lateral Earth Pressure

The same as coulomb active pressure for this case, the only difference is in the equation of calculating K_P (negative sign in active transformed to positive sign in passive). **(Page 365).**

Problems:

For the shown figure below. Plot the pressure diagram and find the resultant force F and its location under **active conditions.**



Solution

Note that the value of ϕ is differ for each layer, so the value of K_a will differ for each layer, thus the first step is to calculate the value of K_a for each layer.

$$K_a = \tan^2\left(45 - \frac{\phi}{2}\right)$$

For $\phi = 32 \rightarrow K_a = \tan^2\left(45 - \frac{32}{2}\right) = 0.307$

Calculate K_a for each layer by the same way.

The values of K_a are written on each layer on the figure above.

Now, calculate the lateral earth pressure at each depth (each change) from soil alone (i.e. vertical effective pressure x K_a), then the water will considered alone.
Before calculating the pressure, at each change we calculate the lateral earth pressure **just before** and **just after** the layer because the value of K_a is differ before and after the layer.
The general formula for active lateral earth pressure at any depth is:

$\sigma_{h,a} = (q + \gamma H)K_a - 2c\sqrt{K_a} \qquad \gamma_w = 62.4\text{pcf}$

$q = 2\text{ksf} = 2000\text{psf}$

$\textbf{@z = 0.0}$:

$\sigma_{h,a} = (2000 + 0) \times 0.307 - 0 = 614 \text{ psf}$

$\textbf{@z = 6ft}$:

Just before ($K_a = 0.307, c = 0$):

$\sigma_{h,a} = (2000 + 110 \times 6) \times 0.307 - 0 = 816.62 \text{ psf}$

Just after ($K_a = 0.333 , c = 0$):

$\sigma_{h,a} = (2000 + 110 \times 6) \times 0.333 - 0 = 885.8 \text{ psf}$

$\textbf{@z = 8ft}$:

Just before ($K_a = 0.333, c = 0$):

$\sigma_{h,a} = (2000 + 110 \times 6 + (125 - 62.4) \times 2) \times 0.333 - 0 = 927.5 \text{ psf}$

Just after ($K_a = 0.704 , c = 600$):

$$\sigma_{h,a} = (2000 + 110 \times 6 + (125 - 62.4) \times 2) \times 0.704 - 2 \times 600 \times \sqrt{0.704} = 953.9 \text{ psf}$$

$\textbf{@z = 17ft}$:

Just before ($K_a = 0.704, c = 600$):

$$\sigma_{h,a} = (2000 + 110 \times 6 + (125 - 62.4) \times 2 + (126 - 62.4) \times 9) \times 0.704 - 2 \times 600 \times \sqrt{0.704} = 1356.9 \text{ psf}$$

Just after ($K_a = 1 , c = 800$):

$$\sigma_{h,a} = (2000 + 110 \times 6 + (125 - 62.4) \times 2 + (126 - 62.4) \times 9) \times 1 - 2 \times 800 \times \sqrt{1} = 1757.6 \text{ psf}$$

@z = 25ft:

Just before ($K_a = 1, c = 800$):

$$\sigma_{h,a} = \begin{pmatrix} 2000 + 110 \times 6 + (125 - 62.4) \times 2 + (126 - 62.4) \times 9 \\ +(120 - 62.4) \times 8 \end{pmatrix} \times 1$$
$$-2 \times 800 \times \sqrt{1} = 2218.4 \text{ psf}$$

Just before ($K_a = 0.49, c = 400$):

$$\sigma_{h,a} = \begin{pmatrix} 2000 + 110 \times 6 + (125 - 62.4) \times 2 + (126 - 62.4) \times 9 \\ +(120 - 62.4) \times 8 \end{pmatrix} \times 0.49$$
$$-2 \times 400 \times \sqrt{0.49} = 1311 \text{ psf}$$

@z = 30ft:

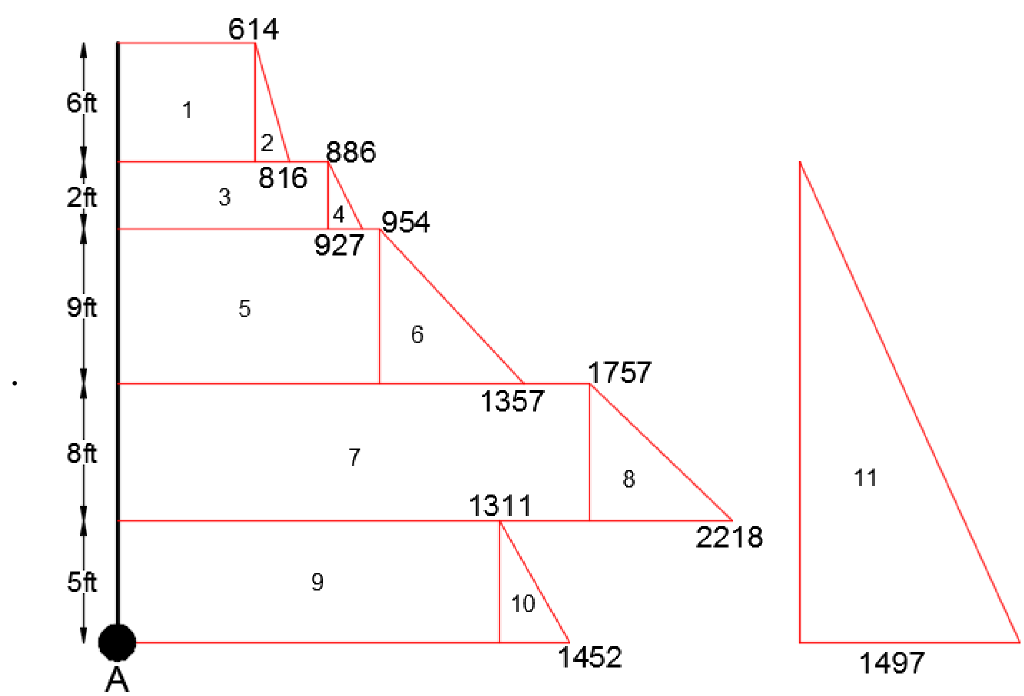
$$\sigma_{h,a} = \begin{pmatrix} 2000 + 110 \times 6 + (125 - 62.4) \times 2 + (126 - 62.4) \times 9 \\ +(120 - 62.4) \times 8 + (120 - 62.4) \times 5 \end{pmatrix} \times 0.49$$
$$-2 \times 400 \times \sqrt{0.49} = 1452 \text{ psf}$$

Now we calculate the pore water pressure:

Water starts at depth 6ft to the end (i.e. depth of water is 24 ft)

$u = 62.4 \times 24 = 1497.6$ psf

Now we draw the pressure diagram as following:



Now calculate the force for each shape (1 to 11) i.e. area of each shape and then sum all of these forces to get total active lateral force P_a.

$P_a \cong 57214\ \mathrm{Ib/ft'}$ ✓.

To calculate the location of P_a, take summation moments at point A (Include the moment from water force "don't forget it")

The location of P_a is $\cong 10.7$ ft (above point A) ✓.

Chapter (8)
Retaining Walls

Introduction

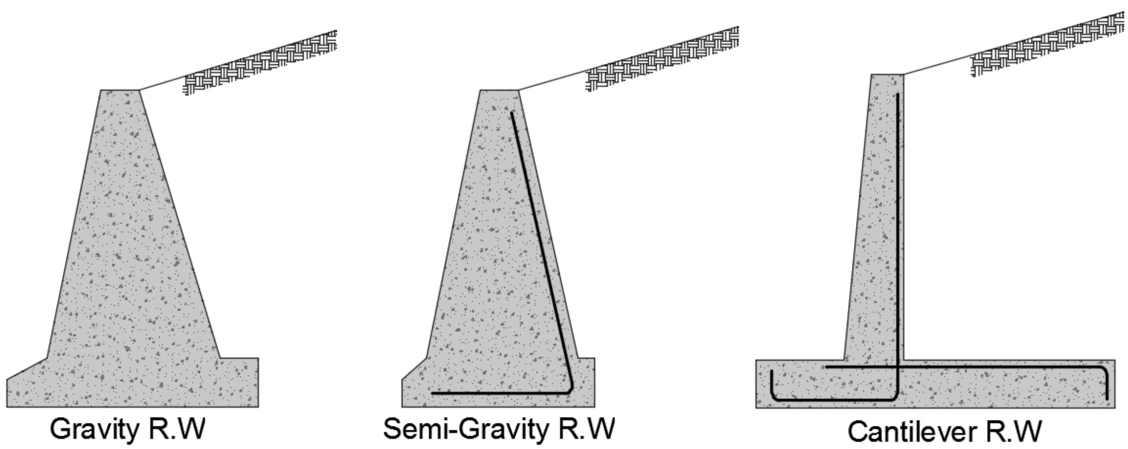
Retaining walls must be designed for lateral earth pressure. The procedures of calculating lateral earth pressure was discussed previously in Chapter7. Different types of retaining walls are used to retain soil in different places. Three main types of retaining walls:

1. Gravity retaining wall (depends on its weight for resisting lateral earth force because it have a large weigh)
2. Semi-Gravity retaining wall (reduce the dimensions of the gravity retaining wall by using some reinforcement).
3. Cantilever retaining wall (reinforced concrete wall with small dimensions and it is the most economical type and the most common)

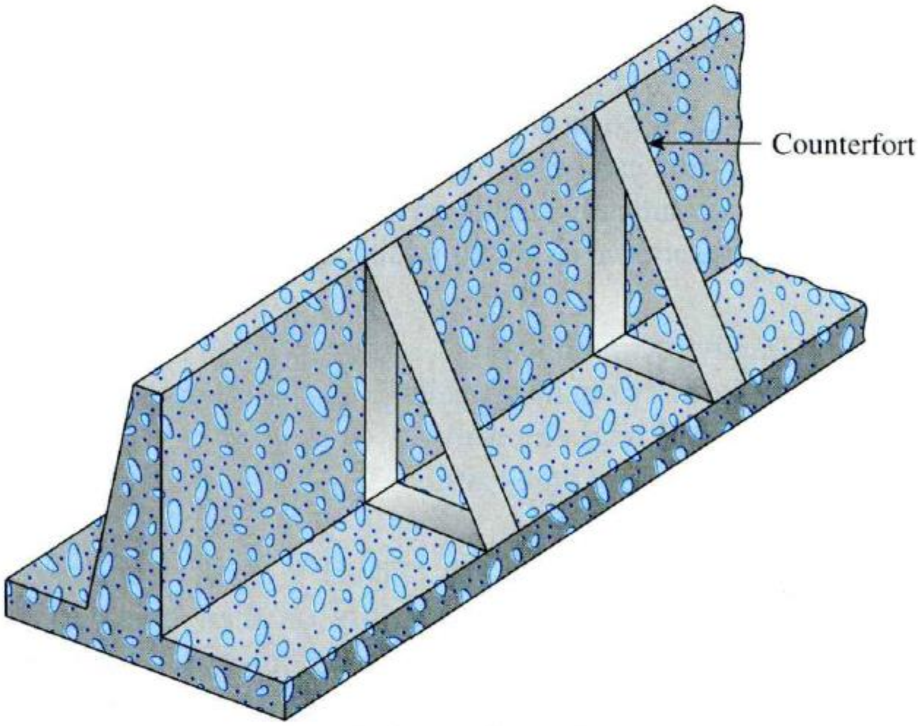
Note:

Structural design of cantilever retaining wall is depend on separating each part of wall and design it as a cantilever, so it's called cantilever R.W.

The following figure shows theses different types of retaining walls:

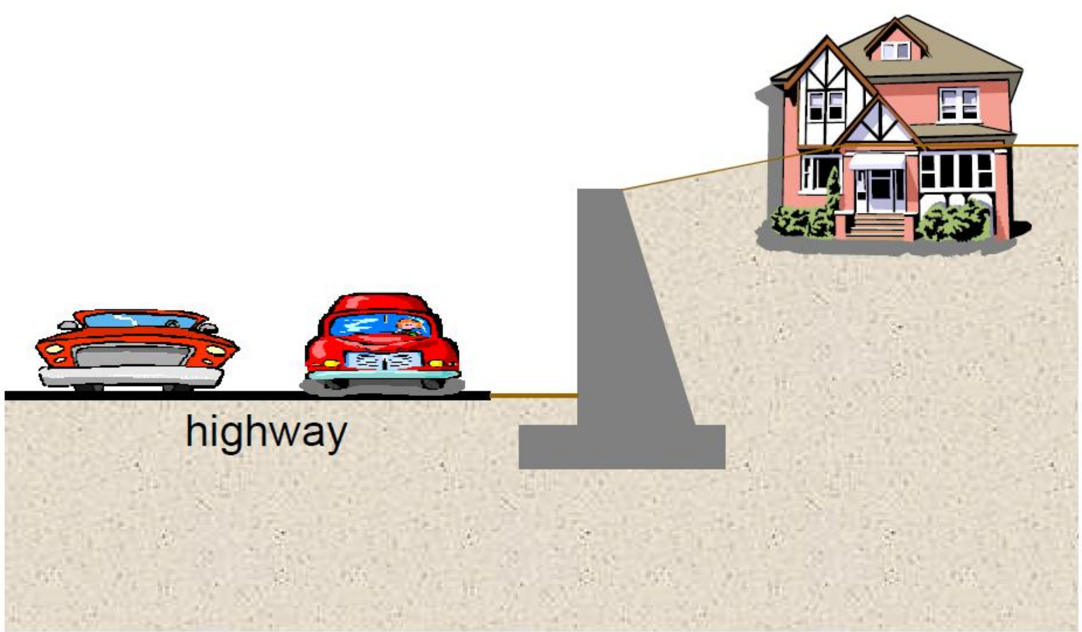


There are another type of retaining wall called "counterfort RW" and is a special type of cantilever RW used when the height of RW became larger than 6m, the moment applied on the wall will be large so we use spaced counterforts every a specified distance to reduce the moment RW.



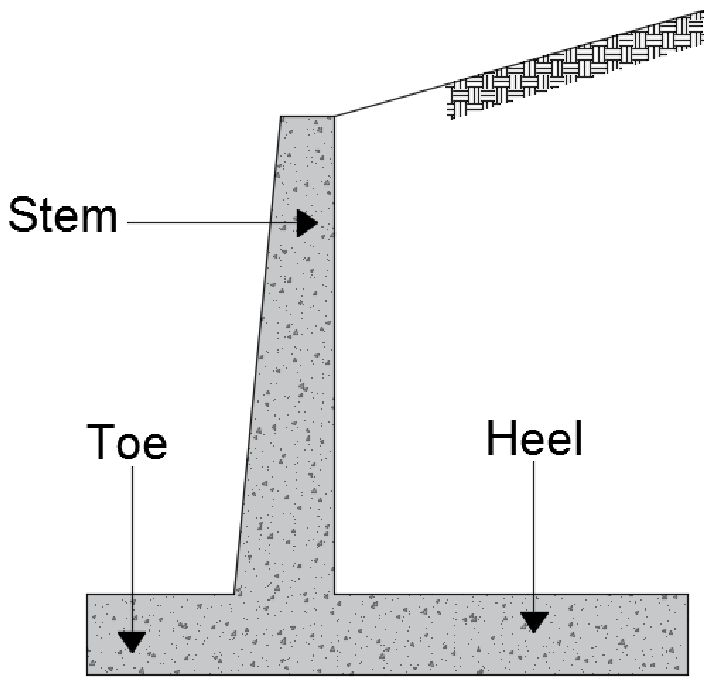
Where we use Retaining Walls

Retaining walls are used in many places, such as retaining a soil of high elevation (if we want to construct a building in lowest elevation) or retaining a soil to save a highways from soil collapse and for several applications.
The following figure explain the function of retaining walls:



Elements of Retaining Walls

Each retaining wall divided into three parts; stem, heel, and toe as shown for the following cantilever footing (as example):



Application of Lateral Earth Pressure Theories to Design

Rankine Theory:

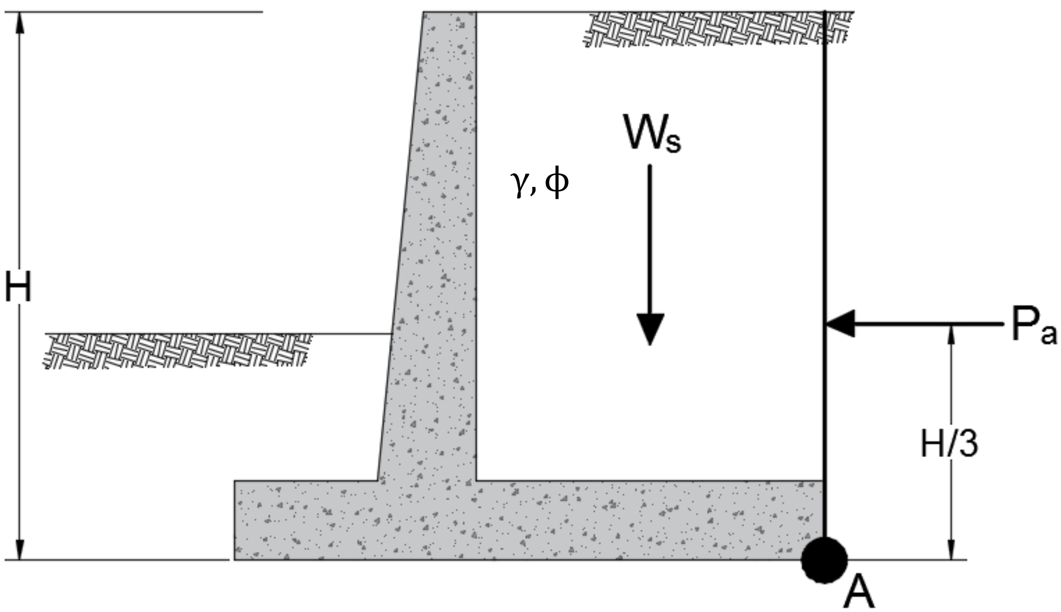
Rankine theory discussed in Ch.7 was modified to be suitable for designing a retaining walls.

This modification is drawing a vertical line from the lowest-right corner till intersection with the line of backfill, and then considering the force of soil acting on this vertical line.

The soil between the wall and vertical line is not considered in the value of P_a, so we take this soil in consideration as a vertical weight applied on the stem of the retaining wall as will explained later.

The following are all cases of rankine theory in designing a retaining wall:

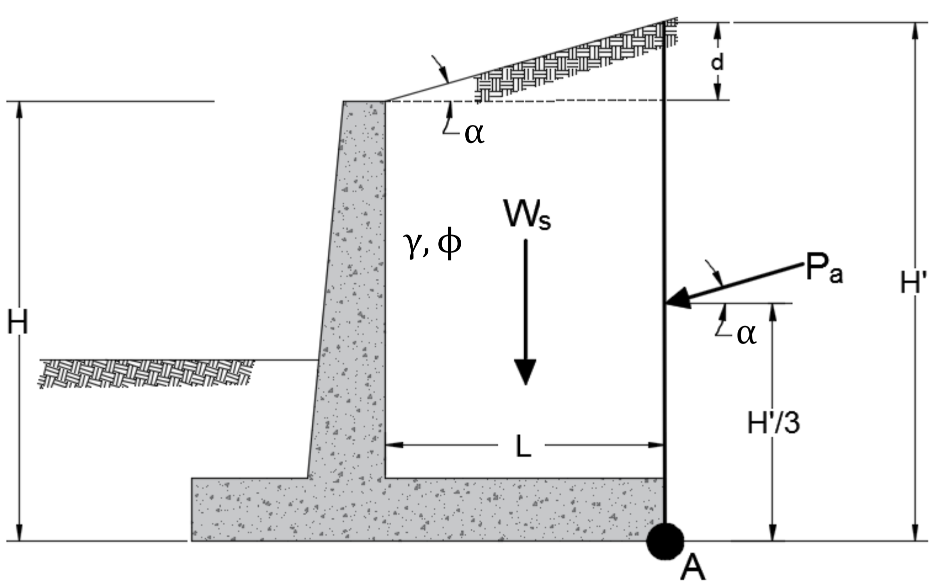
1. The wall is vertical and backfill is horizontal:



Here the active force P_a is horizontal and can be calculated as following:

$$P_a = \frac{1}{2}\gamma H^2 K_a \qquad , \qquad K_a = \tan^2\left(45 - \frac{\phi}{2}\right)$$

2. The wall is vertical and the backfill is inclined with horizontal by angle $(\boldsymbol{\alpha})$:



Here the active force P_a is inclined with angle (α) and can be calculated as following:

$$P_a = \frac{1}{2}\gamma H'^2 K_a$$

Why H'? → Because the pressure is applied on the vertical line (according active theory) not on the wall, so we need the height of this vertical line H'

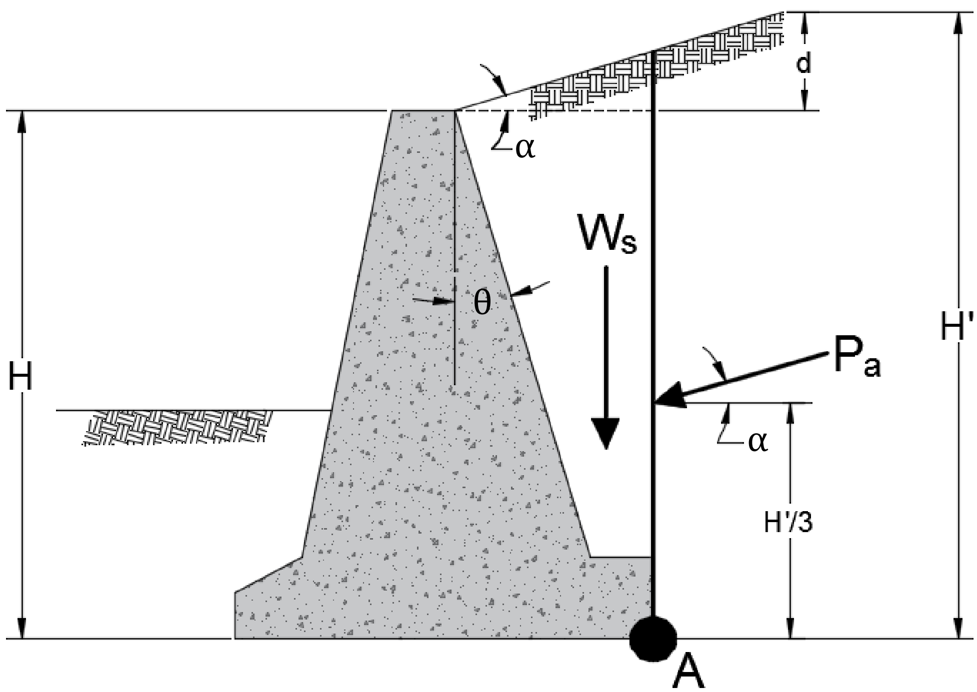
$H' = H + d \rightarrow\rightarrow d = L\tan\alpha$

K_a is calculating from (**Table 7.1 Page 337**)

Now the calculated value of P_a is inclined with an angle (α), so its analyzed in horizontal and vertical axes and then we use the horizontal and vertical components in design as will explained later.

$P_{a,h} = P_a\cos(\alpha)$, $P_{a,v} = P_a\sin(\alpha)$

3. The wall is inclined by angle $(\theta)$with vertical and the backfill is inclined with horizontal by angle (α):



Note that the force P_a is inclined with angle (α) and not depend on the inclination of the wall because the force applied on the vertical line and can be calculated as following:

$$P_a = \frac{1}{2}\gamma H'^2 K_a$$

What about K_a ???

K_a is depend on the inclination of the wall and inclination of the backfill because it's related to the soil itself and the angle of contact surface with this soil, so K_a can be calculated from the following equation (Ch.7):

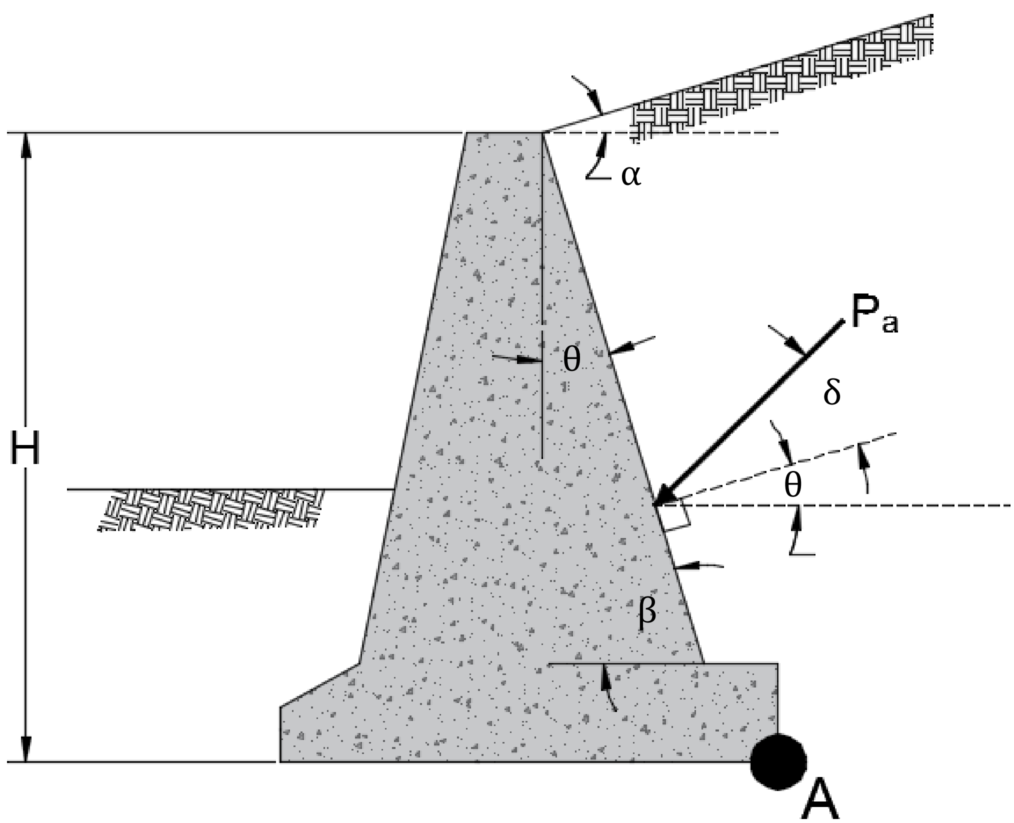
$$K_a = \frac{\cos(\alpha - \theta)\sqrt{1 + \sin^2\phi - 2\sin\phi\cos\psi_a}}{\cos^2\theta\left(\cos\alpha + \sqrt{\sin^2\phi - \sin^2\alpha}\right)}$$

$$\psi_a = \sin^{-1}\left(\frac{\sin\alpha}{\sin\phi}\right) - \alpha + 2\theta$$

$$P_{a,h} = P_a\cos(\alpha) \qquad , \qquad P_{a,v} = P_a\sin(\alpha)$$

Coulomb's Theory:

Coulomb's theory discussed in Ch.7 will remains unchanged (without any modifications) in this chapter. The force P_a is applied directly on the wall, so whole soil retained by the wall will be considered in P_a and thereby the weight of soil will not apply on the heel of the wall.



$$P_a = \frac{1}{2}\gamma H^2 K_a$$

Why H??? Here the force P_a is applied directly on the wall, so the lateral pressure of the soil is applied on the wall from start to end, so we only take the height of the wall (in coulomb theory).

K_a is calculated from **(Table 7.4 and 7.5 Page 343)** according the following angles:

ϕ, α, β and δ

As shown, the force P_a is inclined with angle $(\delta + \theta)$ with horizontal, so:

$$P_{a,h} = P_a \cos(\delta + \theta) \quad , \quad P_{a,v} = P_a \sin(\delta + \theta)$$

What about Passive Force

You can always calculate passive force from rankine theory even if its require to solve the problem based on coulomb's theory, because we concerned about rankine and coulomb's theories in active lateral pressure.

Important Note:

Coulomb's theory can't be used in the following cases:

1. If the soil retained by the wall is $C - \phi$ soil, because coulomb's theory deal only with granular soil (pure sand).
2. If wall friction angle between retained soil and the wall is equal zero.
3. If we asked to solve the problem using rankine theory ☺.

Stability of Retaining Wall

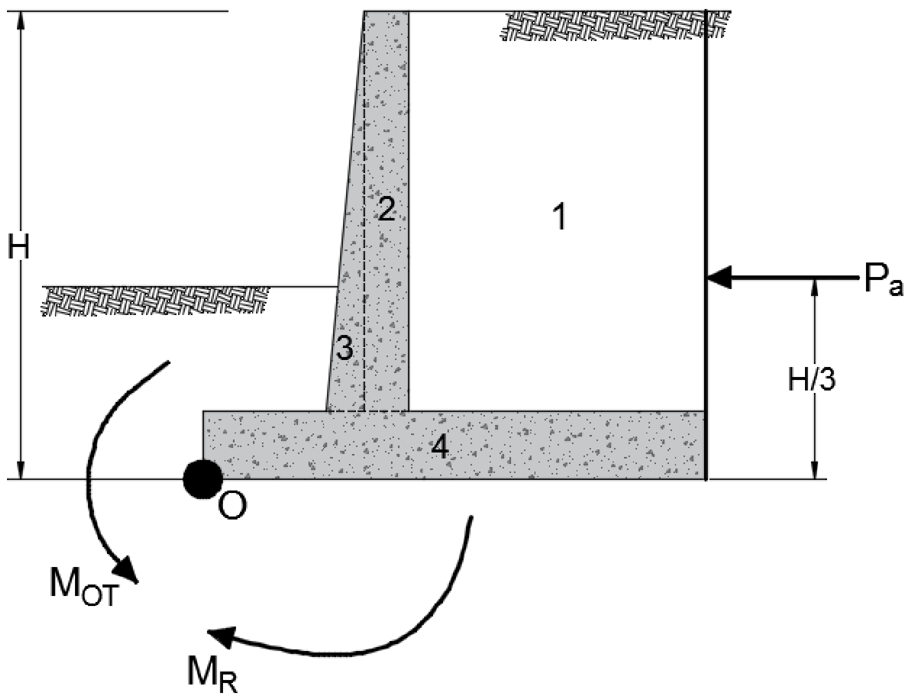
A retaining wall may be fail in any of the following:

1. It may overturn about its toe.
2. It may slide along its base.
3. It may fail due to the loss of bearing capacity of the soil supporting the base.
4. It may undergo deep-seated shear failure.
5. It may go through excessive settlement.

We will discuss the stability of retaining wall for the first three types of failure (overturning, sliding and bearing capacity failures).

We will use rankine theory to discusses the stability of these types of failures. Coulomb's theory will be the same with only difference mentioned above (active force applied directly on the wall).

Stability for Overturning



The **horizontal** component of active force will causes overturning on retaining wall about point O by moment called "overturning moment"

$$M_{OT} = P_{a,h} \times \frac{H}{3}$$

This overturning moment will resisted by all vertical forces applied on the base of retaining wall:

1. Vertical component of active force $P_{a,v}$(if exist).
2. Weight of all soil above the heel of the retaining wall.
3. Weight of each element of retaining wall.
4. Passive force (we neglect it in this check for more safety).

Now, to calculate the moment from these all forces (resisting moment) we prepare the following table:

$\text{Force} = \text{Volume} \times \text{unit weight} \qquad \text{but, we take a strip of 1 mlength}$

$\rightarrow \text{Force} = \text{Area} \times \text{unit weight}$

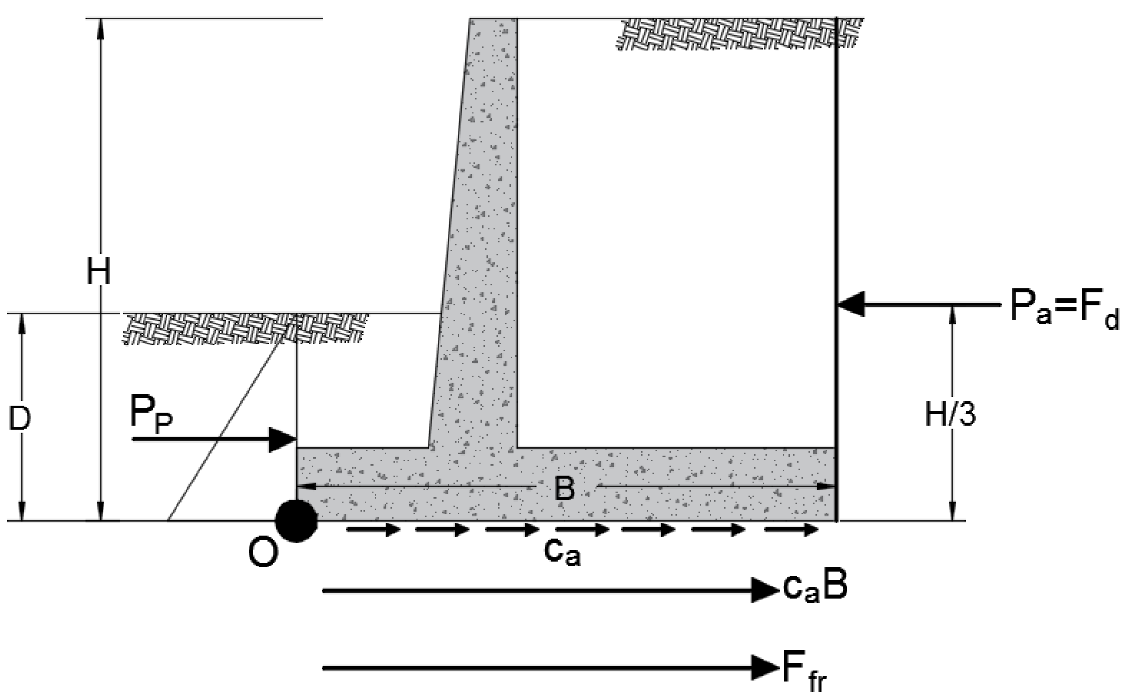
Section	Area	Weight/unit length of the wall	Moment arm measured from O	Moment about O
1	A_1	$W_1 = A_2 \times \gamma_1$	X_1	M_1
2	A_2	$W_2 = A_2 \times \gamma_c$	X_2	M_2
3	A_3	$W_3 = A_2 \times \gamma_c$	X_3	M_3
4	A_4	$W_4 = A_2 \times \gamma_c$	X_4	M_4
		$P_{a,v}$(if exist).	B	M_V
$\sum$		$\sum V$		$\sum M = M_R$

$\gamma_1 =$ unit weight of the soil above the heel of RW

$$FS_{OT} = \frac{M_R}{M_{OT}} \geq 2$$

Note:
If you asked to consider passive force→ consider it in the resisting moment and the factor of safety remains 2. (So we neglect it here for safety).

Stability for Sliding along the Base



Also, the horizontal component of active force may causes movement of the wall in horizontal direction (i.e. causes sliding for the wall), this force is called driving force $F_d = P_{a,h}$.

This driving force will be resisted by the following forces:

1. Adhesion between the soil (under the base) and the base of retaining wall:

$c_a =$ adhesion along the base of RW (KN/m)

$C_a = c_a \times B =$ adhesion force under the base of RW (KN)

c_a can be calculated from the following relation:

$c_a = K_2 c_2$ $\quad\quad c_2 =$ cohesion of soil under the base

So adhesion force is:

$C_a = K_2 c_2 B$

2. Friction force due to the friction between the soil and the base of RW:

Always friction force is calculated from the following relation:

$F_{fr} = \mu_s N$

Here N is the sum of vertical forces calculated in the table of the first check (overturning)

$\rightarrow N = \sum V$ (including the vertical component of active force)

$\mu_s =$ coefficient of friction (related to the friction between soil and base)

$\mu_s = \tan(\delta_2) \quad\quad \delta_2 = K_1 \phi_2 \rightarrow\rightarrow \mu_s = \tan(K_1 \phi_2)$

$\phi_2 =$ friction angle of the soil under the base.

$$\rightarrow F_{fr} = \sum V \times \tan(K_1 \phi_2)$$

Note:

$$K_1 = K_2 = \left(\frac{1}{2} \rightarrow \frac{2}{3}\right) \text{ if you are not given them} \rightarrow \text{take } K_1 = K_2 = \frac{2}{3}$$

3. Passive force P_P.(Calculated using rankine theory).

So the total resisting force F_R can be calculated as following:

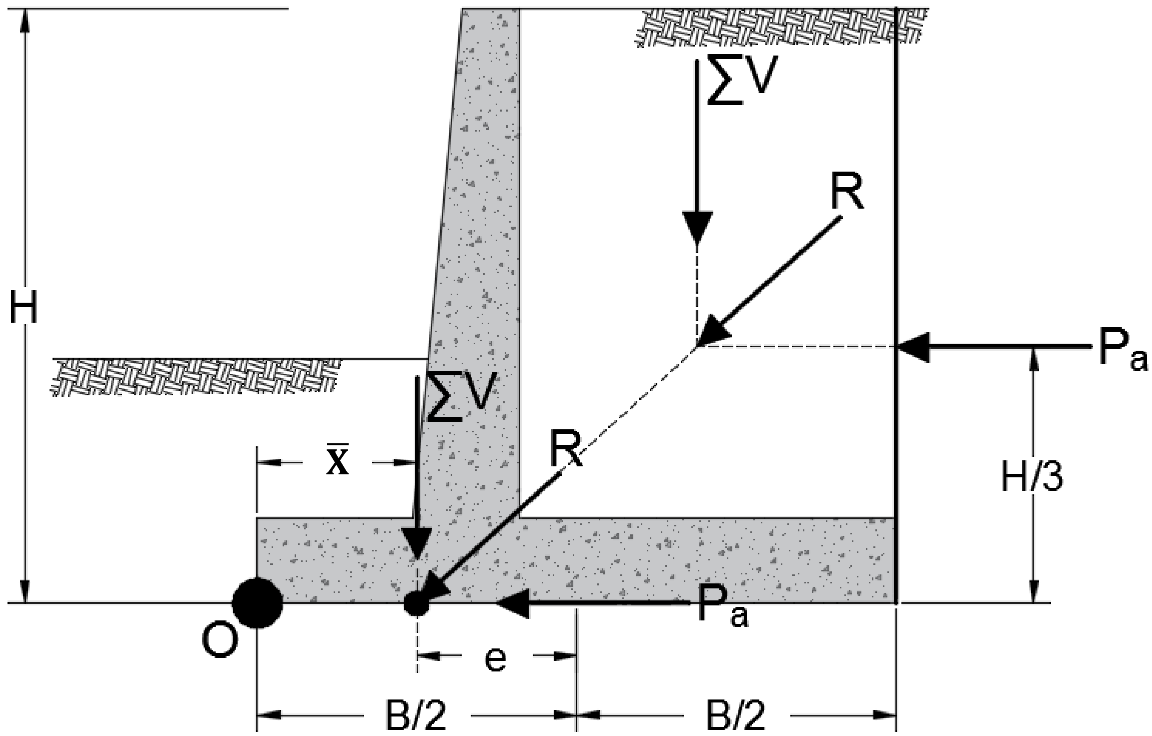
$$F_R = \sum V \times \tan(K_1 \phi_2) + K_2 c_2 B + P_P$$

Factor of safety against sliding:

$$FS_S = \frac{F_R}{F_d} \geq 2 \text{ (if we consider } P_P \text{ in } F_R)$$

$$FS_S = \frac{F_R}{F_d} \geq 1.5 \text{ (if we dont consider } P_P \text{ in } F_R)$$

Check Stability for Bearing Capacity Failure



As we see, the resultant force (R) is not applied on the center of the base of retaining wall, so there is an eccentricity between the location of resultant force and the center of the base, this eccentricity **may be calculated as following:**

From the figure above, take summation moments about point O:

$$M_O = \sum V \times \bar{X}$$

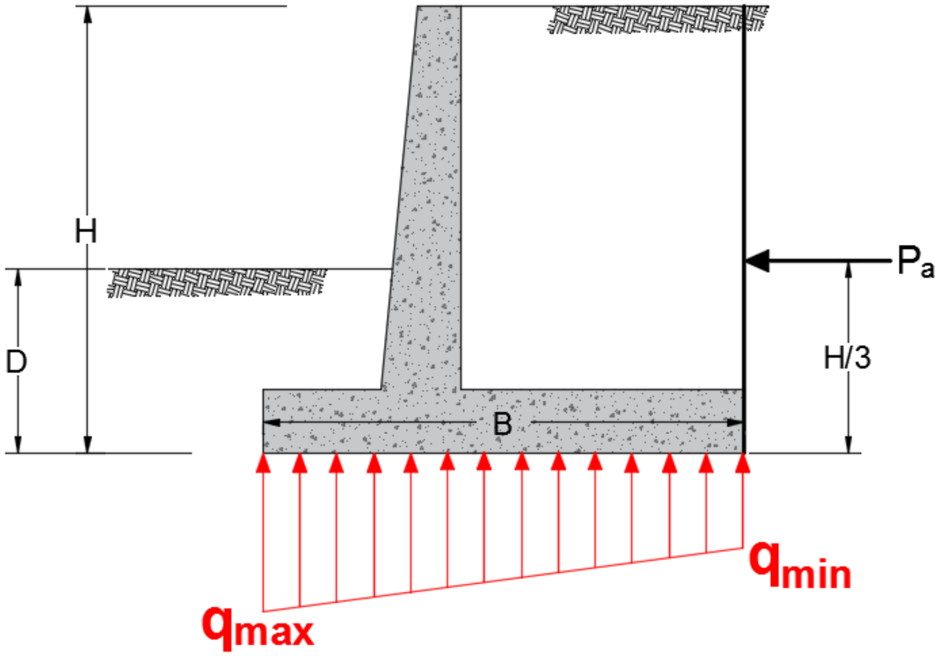
From the first check (overturning) we calculate the overturning moment and resisting moment about point O, so the difference between these two moments gives the net moment at O.

$$M_O = M_R - M_{OT}$$

$$\rightarrow M_R - M_{OT} = \sum V \times \bar{X} \rightarrow\rightarrow \bar{X} = \frac{M_R - M_{OT}}{\sum V}$$

$$e = \frac{B}{2} - \bar{X} = \checkmark \text{ (see the above figure).}$$

Since there exist eccentricity, the pressure under the base of retaining wall is not uniform (there exist maximum and minimum values for pressure).



We calculate q_{max} and q_{min} as stated in chapter 3:

Eccentricity in B-direction and retaining wall can be considered strip footing

If $\mathbf{e < \frac{B}{6}}$

$$q_{max} = \frac{\sum V}{B \times 1}\left(1 + \frac{6e}{B}\right)$$

$$q_{min} = \frac{\sum V}{B \times 1}\left(1 - \frac{6e}{B}\right)$$

If $\mathbf{e > \frac{B}{6}}$

$$q_{max,new} = \frac{4\sum V}{3 \times 1 \times (B - 2e)}$$

Now, we must check for q_{max}:

$q_{max} \leq q_{all} \rightarrow q_{max} = q_{all}$ (at critical case)

$$FS_{B.C} = \frac{q_u}{q_{max}} \geq 3$$

Calculation of $\mathbf{q_u}$:

q_u is calculated using Meyerhof equation as following:

$$q_u = cN_cF_{cs}F_{cd}F_{ci} + qN_qF_{qs}F_{qd}F_{qi} + 0.5B\gamma N_\gamma F_{\gamma s}F_{\gamma d}F_{\gamma i}$$

Where

$c =$ Cohesion of soil under the base

$q =$ Effective stress at the level of the base of retaining wall.

$q = \gamma_2 \times D_f$

D_f here is the depth of soil above the toe $= D$ (above figure)

$\rightarrow q = \gamma_2 \times D$

$\gamma =$ unit weight of the soil under the base of the RW.

Important Note:
May be a water table under the base or at the base or above the base (three cases discussed in chapter 3) is the same here, so be careful don't forget Ch.3.

$B = B' = B - 2e$

$N_c, N_q, N_\gamma =$ Myerhof bearing capacity factors (Table3.3)according the friction angle for the soil under the base

$F_{cs} = F_{qs} = F_{\gamma s} = 1$ (since RW is considered a strip footing)

Depth factors: (We use B not B')
Here since the depth D is restively small to width of the base B, in most **cases** $\frac{D}{B} \leq 1 \rightarrow\rightarrow$

1. **For $\phi = 0.0$**

$$F_{cd} = 1 + 0.4\left(\frac{D_f}{B}\right)$$

$$F_{qd} = 1$$

$$F_{\gamma d} = 1$$

2. **For $\phi > 0.0$**

$$F_{cd} = F_{qd} - \frac{1 - F_{qd}}{N_c \tan\phi}$$

$$F_{qd} = 1 + 2\tan\phi\,(1 - \sin\phi)^2\left(\frac{D_f}{B}\right)$$

$$F_{\gamma d} = 1$$

Inclination Factors:

Note that the resultant force applied on the base of the foundation is not vertical, but it is inclined with angle $\beta = \Psi$ (with vertical), this angle can be calculated as following:

$$\beta = \Psi = \tan^{-1}\left(\frac{P_{a,h}}{\sum V}\right)$$

$$F_{ci} = F_{qi} = \left(1 - \frac{\beta^{\circ}}{90}\right)^2$$

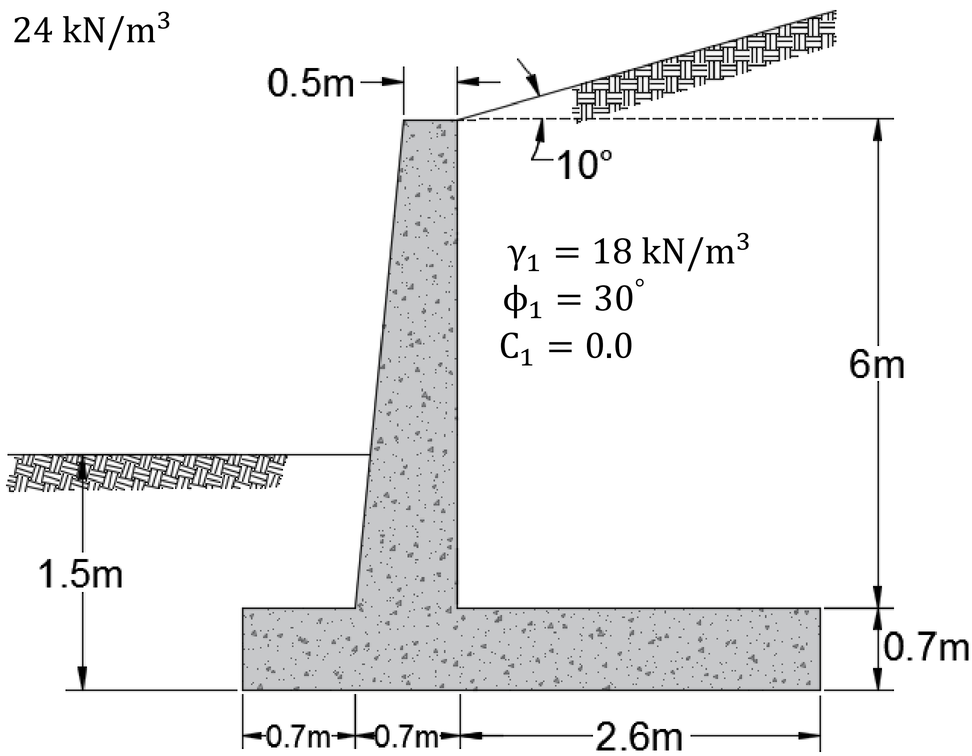
$$F_{\gamma i} = \left(1 - \frac{\beta^{\circ}}{\phi^{\circ}}\right)$$

Problems:

1.

The cross section of the cantilever retaining wall shown below. Calculate the factor of safety with respect to overturning, sliding, and bearing capacity.

$\gamma_c = 24\ \text{kN/m}^3$



$\gamma_2 = 19\ \text{kN/m}^3$

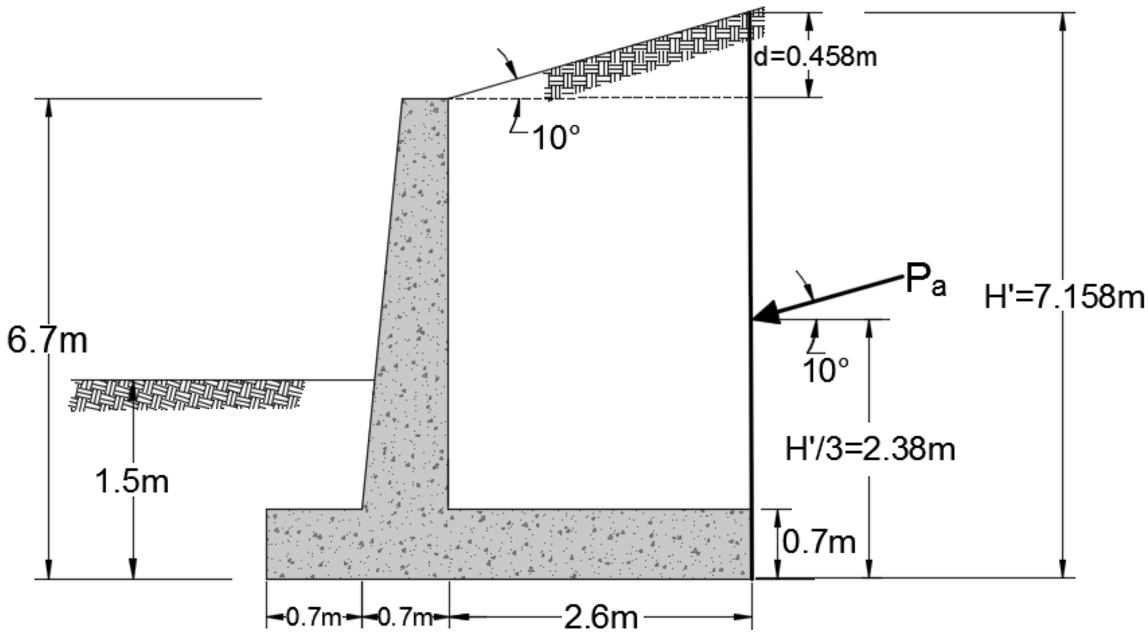
$\phi_2 = 24^{\circ}$

$C_2 = 40\text{kN/m}^2$

Solution

Since it is not specified a method for solving the problem, directly we use Rankine theory.

Now draw a vertical line starts from the right-down corner till reaching the backfill line and then calculate active force (P_a):



$$\tan 10 = \frac{d}{2.6} \rightarrow d = 2.6 \times \tan 10 = 0.458\text{m}$$

$$H' = 6.7 + d = 6.7 + 0.458 = 7.158\text{m}$$

Now we calculate P_a:

$$P_a = \frac{1}{2} \times \gamma_1 \times H'^2 \times K_a$$

Since the backfill is inclined and the wall is vertical, K_a is calculated from **Table 7.1** according the values of $\alpha = 10$ and $\phi_1 = 30$:

$K_a = 0.3495$

$$\rightarrow P_a = \frac{1}{2} \times 18 \times 7.158^2 \times 0.3495 = 161.2\text{ kN}$$

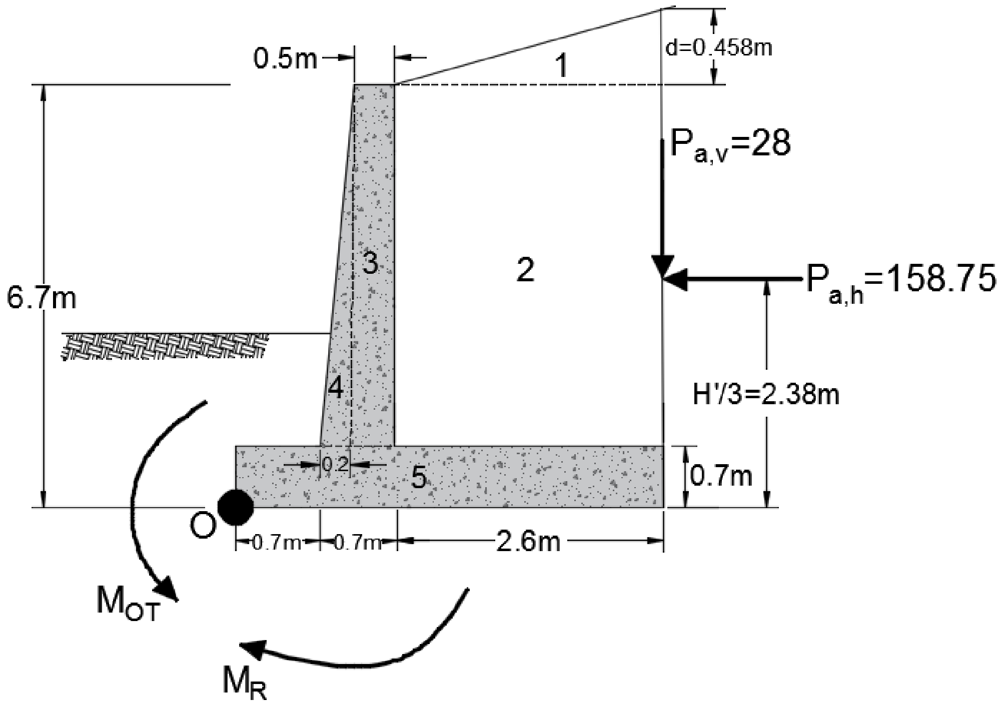
Location of P_a:

Location= $\frac{H'}{3} = \frac{7.158}{3} = 2.38$

The force P_a is inclined with angle $\alpha = 10$ with horizontal:

$P_{a,h} = 161.2\cos(10) = 158.75$, $P_{a,v} = 161.2\sin(10) = 28$

Check for Overturning:



$M_{OT} = 158.75 \times 2.38 = 337.8 \text{ KN.m}$

Now to calculate M_R we divided the soil and the concrete into rectangles and triangles to find the area easily (as shown above) **and to find the arm from the center of each area to point O** as prepared in the following table:

Section	Area	Weight/unit length of the wall	Moment arm measured from O	Moment about O
1	0.595	$0.595 \times 18 = 10.71$	$4 - \frac{2.6}{3} = 3.13$	33.52
2	15.6	$15.6 \times 18 = 280.8$	$1.4 + 1.3 = 2.7$	758.16
3	3	$3 \times 24 = 72$	$1.4 - 0.25 = 1.15$	82.8
4	0.6	$0.6 \times 24 = 14.4$	$0.9 - \frac{0.2}{3} = 0.833$	12
5	2.8	$2.8 \times 24 = 67.2$	$\frac{4}{2} = 2$	134.4
		$P_{a,v} = 28$	B=4	112
$\sum$		$\sum V = 470.11$		$M_R = 1132.88$

Note that we neglect passive force because it is not obligatory.

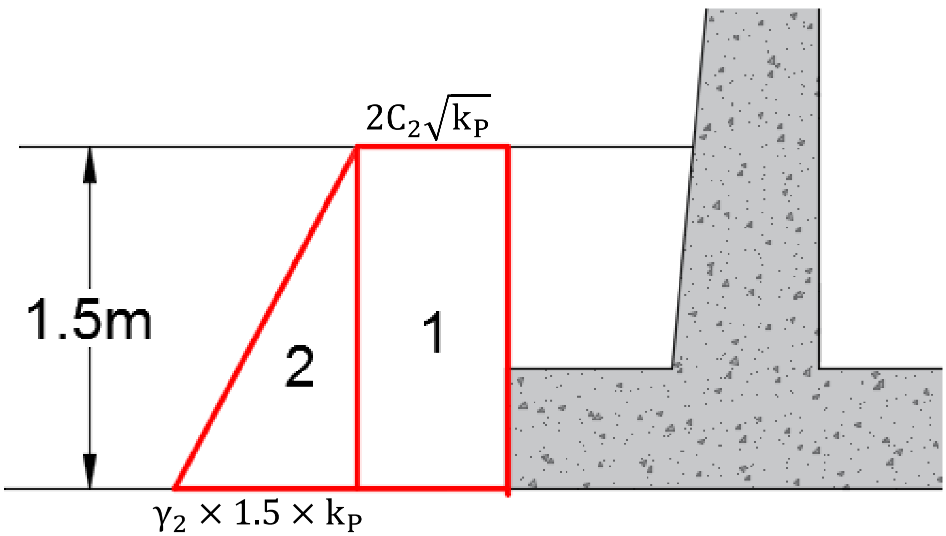
$$FS_{OT} = \frac{M_R}{M_{OT}} = \frac{1132.88}{377.8} = 2.99 > 2 \rightarrow \textbf{OK} \checkmark.$$

Check for Sliding:

$FS_S = \dfrac{F_R}{F_d} \geq 2$ (if we consider P_P in F_R)

It is preferable to consider passive force in this check.

Applying rankine theory on the soil in the left (draw vertical line till reaching the soil surface).



k_P is calculated for the soil using rankine theory without considering any iniclination of the wall, because it is calculated for the soil below the bas

$$k_P = \tan^2\left(45 + \frac{\phi_2}{2}\right) = \tan^2\left(45 + \frac{20}{2}\right) = 2.04$$

$$P_1 = (\text{rectangle area}) = \left(2 \times 40 \times \sqrt{2.04}\right) \times 1.5 = 171.4\ \text{kN}$$

$$P_2 = (\text{triangle area}) = \frac{1}{2} \times (19 \times 1.5 \times 2.04) \times 1.5 = 43.6\ \text{kN}$$

$$P_P = P_1 + P_2 = 171.4 + 43.6 = 215\ \text{kN}$$

$$F_d = P_{a,h} = 158.75\ \text{Kn}$$

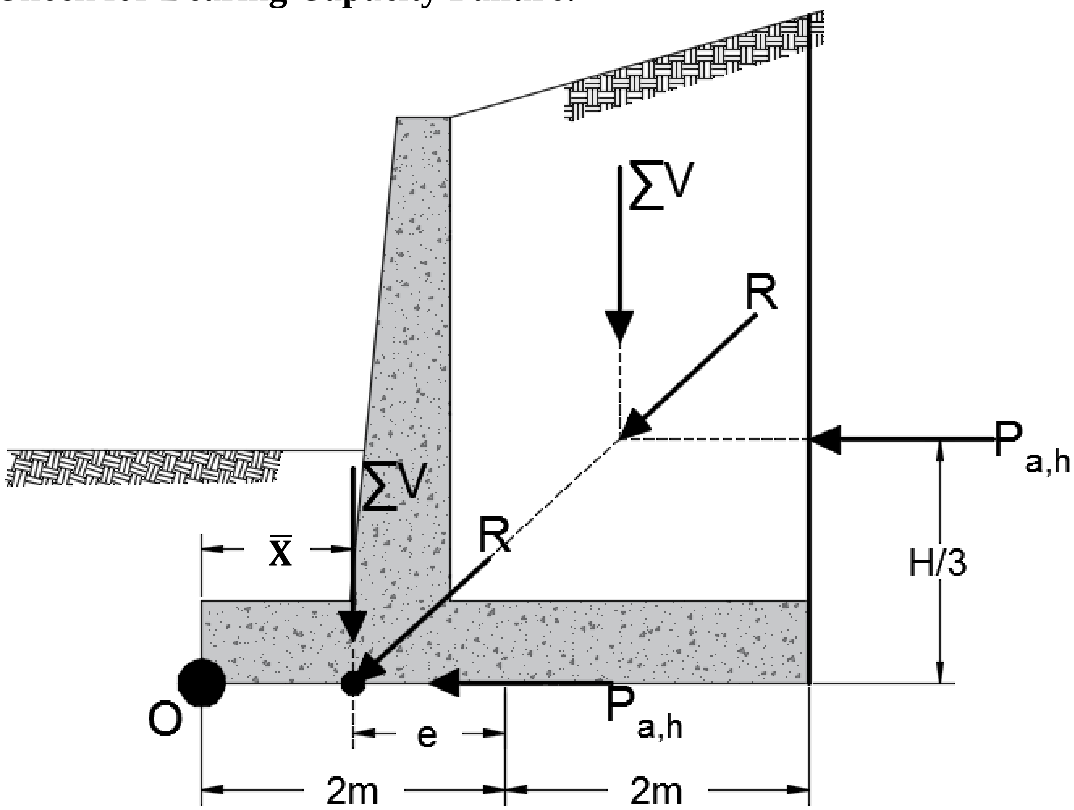
$$F_R = \sum V \times \tan(K_1\phi_2) + K_2 c_2 B + P_P$$

Take $K_2 = K_2 = 2/3$ $\quad \sum V = 470.11$ (from table of first check)

$$F_R = 470.11 \times \tan\left(\frac{2}{3} \times 20\right) + \frac{2}{3} \times 40 \times 4 + 215 = 433.1\ \text{kN}$$

$$\rightarrow FS_S = \frac{433.1}{158.75} = 2.72 > 2 \rightarrow \textbf{OK} \checkmark.$$

Check for Bearing Capacity Failure:



As stated previously, $\bar{X}$ can be calculated as following:

$$\bar{X} = \frac{M_R - M_{OT}}{\sum V} = \frac{1132.88 - 377.8}{470.11} = 1.6\ \text{m}$$

$$e = \frac{B}{2} - \bar{X} = 2 - 1.6 = 0.4\text{m}$$

$$\frac{B}{6} = \frac{4}{6} = 0.667 \rightarrow e = 0.4 < \frac{B}{6} \rightarrow\rightarrow\rightarrow$$

$$q_{max} = \frac{\sum V}{B \times 1}\left(1 + \frac{6e}{B}\right) = \frac{470.11}{4 \times 1}\left(1 + \frac{6 \times 0.4}{4}\right) = 188.04\ \text{kN/m}^2$$

$$q_{min} = \frac{\sum V}{B \times 1}\left(1 - \frac{6e}{B}\right) = \frac{470.11}{4 \times 1}\left(1 - \frac{6 \times 0.4}{4}\right) = 47\ \text{kN/m}^2$$

Calculation of q_u (for the soil below the bas):

$q_u = cN_cF_{cs}F_{cd}F_{ci} + qN_qF_{qs}F_{qd}F_{qi} + 0.5B\gamma N_\gamma F_{\gamma s}F_{\gamma d}F_{\gamma i}$

$c = 40 \quad , \quad q = 1.5 \times 19 = 28.5 \quad , \quad \gamma = 19$

$B = B' = B - 2e = 4 - 2(0.4) = 3.2m$

Shape factors = 1 (RW can be considered strip footing).

For $\phi = 20 \rightarrow N_c = 14.83\ , N_q = 6.4\ , N_\gamma = 5.39$ (from table 3.3)

Depth factors: (We use B not B')

$$\frac{D}{B} = \frac{1.5}{4} = 0.375 < 1 \text{ and } \phi = 20 > 0.0 \rightarrow\rightarrow$$

$$F_{qd} = 1 + 2\tan\phi\,(1 - \sin\phi)^2\left(\frac{D_f}{B}\right)$$

$$= 1 + 2\tan 20\,(1 - \sin 20)^2\,(0.375) = 1.12$$

$$F_{cd} = F_{qd} - \frac{1 - F_{qd}}{N_c \tan\phi} = 1.12 - \frac{1 - 1.12}{14.83 \times \tan 20} = 1.14$$

$$F_{\gamma d} = 1$$

Inclination Factors:

$$\beta = \Psi = \tan^{-1}\left(\frac{P_{a,h}}{\sum V}\right) = \tan^{-1}\left(\frac{158.75}{470.11}\right) = 18.6 \text{ (with vertical)}$$

$$F_{ci} = F_{qi} = \left(1 - \frac{\beta^\circ}{90}\right)^2 \left(1 - \frac{18.6}{90}\right)^2 = 0.63$$

$$F_{\gamma i} = \left(1 - \frac{\beta^\circ}{\phi^\circ}\right) = \left(1 - \frac{18.6}{20}\right) = 0.07$$

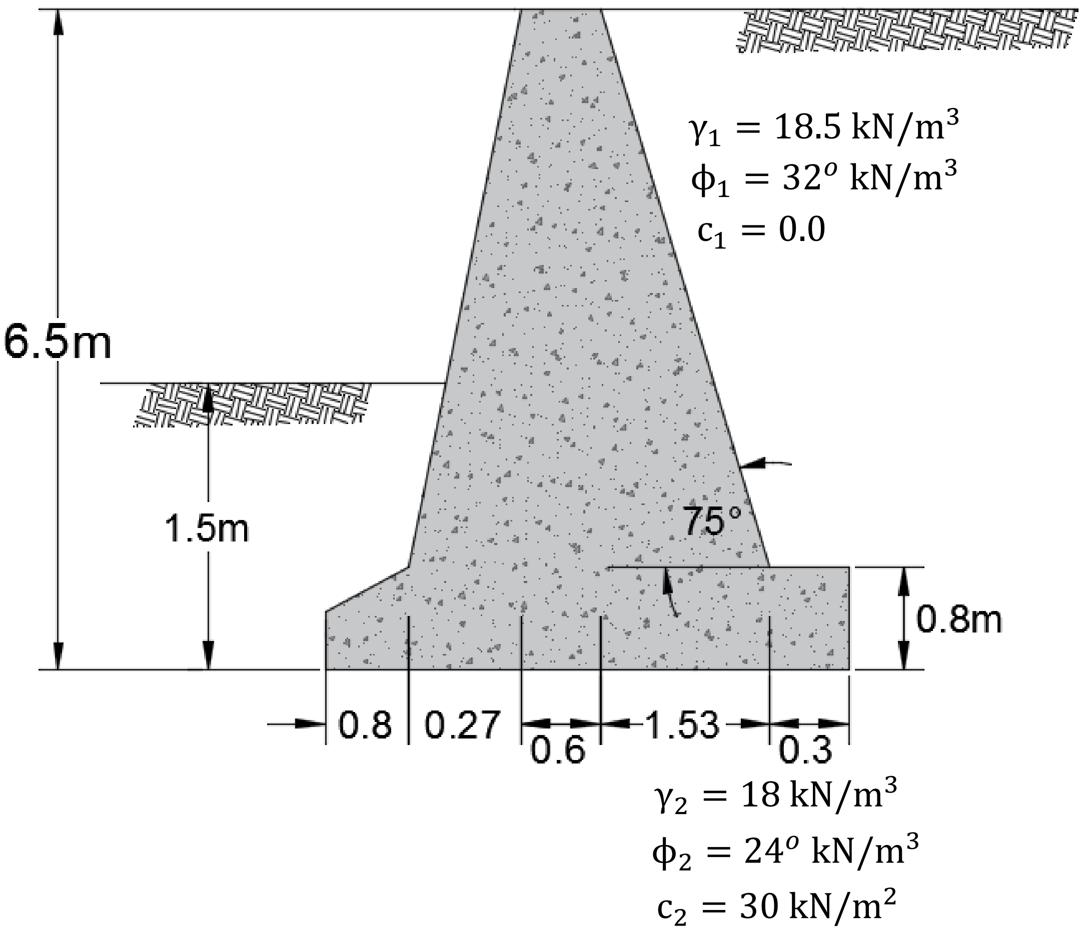
$$\rightarrow q_u = 40 \times 14.83 \times 1.14 \times 0.63 + 28.5 \times 6.4 \times 1.12 \times 0.63 + 0.5 \times 3.2 \times 19 \times 5.39 \times 1 \times 0.07$$

$$\rightarrow q_u = 566.2\ kN/m^2$$

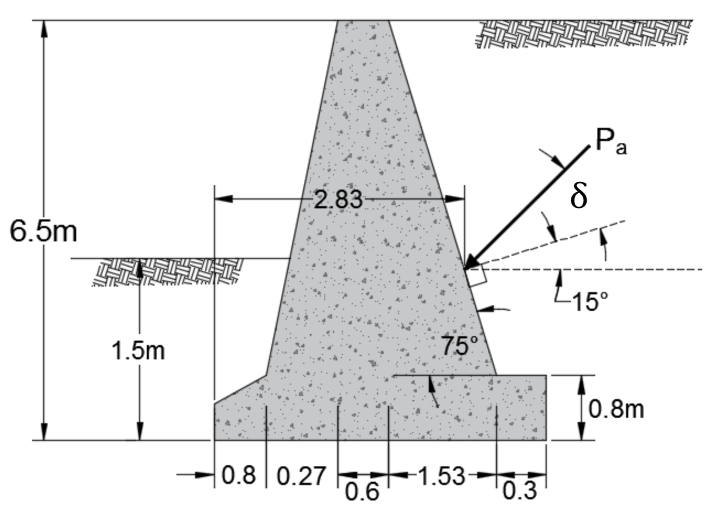
$$FS_{B.C} = \frac{q_u}{q_{max}} = \frac{566.2}{188.04} = 3.01 > 3 \text{ (slightly satisfied)}\textbf{OK} ✓.$$

2. **(Example 8.2)**

A gravity retaining wall is shown in figure below. Use $\delta = \frac{2}{3}\phi_1$ and coulomb's active earth pressure theory.



(See the solution from textbook) with the following notes:



1. As seen the force P_a is applied directly on the wall.
2. The force P_a is inclined with angle δ with the normal to the wall and inclined with angle $(\delta + 15)$ with horizontal.
3. The distance 2.83 (arm of vertical component of P_a from toe corner) is not given and can be calculated as following:

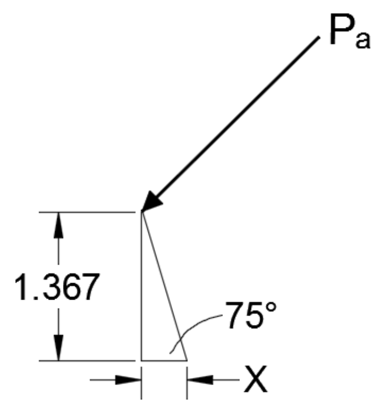
The location of force P_a is $\frac{H}{3} = \frac{6.5}{3} = 2.167$

$2.167 - 0.8 = 1.367$

$$\tan(75) = \frac{1.367}{X} \rightarrow X = \frac{1.367}{\tan(75)}$$

$\rightarrow X = 0.366 \text{ m}$

$2.83 = (3.5 - 0.3) - 0.366$



4.
Note that when we want to calculate passive force (in overturning pressure) we use rankine theory for the following two reasons:

✓ The soil below the base is $C - \phi$ soil, so we can't use coulomb's theory because it deals only with granular soil.

✓ It is required to use coulomb's theory in calculating of active force, however in calculating passive force we can always use rankine theory.

5. In calculating summation of vertical forces, the weighs if soil above the heel are not taken in consideration because the force is applied directly on the wall.
6. Always when calculating P_a the height of the wall (H) is always taken even if the backfill is inclined because the force applied directly on the wall.
7. (After these notes, solve the problem by yourself ☺).

3.

For the retaining wall shown below.

a. Find the lateral earth pressure distribution.

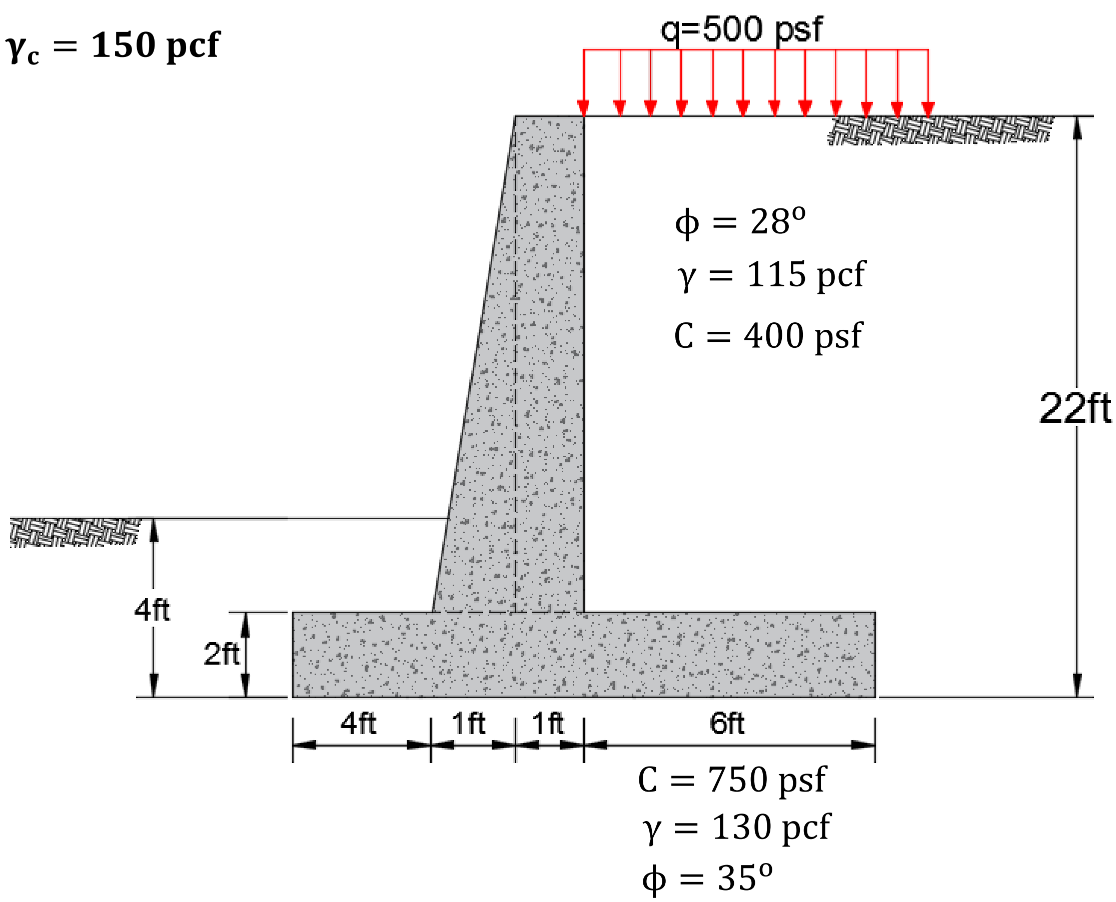
b. Compute P_a (Rankine).

c. Calculate Overturning stability.

d. Compute Sliding safety factor.

e. Locate the resultant on the base of the footing and determine the eccentricity.

f. Calculate the factor of safety against bearing capacity failure.



Solution

We use rankine theory because there is no friction between the soil and the wall.

a.

Calculation of active lateral earth pressure distribution:

$$\sigma_{h,a} = (q + \gamma H)K_a - 2c\sqrt{K_a} \qquad K_a = \tan^2\left(45 - \frac{\phi}{2}\right)$$

$$K_a = \tan^2\left(45 - \frac{28}{2}\right) = 0.361 \quad \text{(for the soil retained by the wall)}$$

$@z = 0.0$ (right side)

$$\sigma_{h,a} = (500 + 0) \times 0.361 - 2 \times 400 \times \sqrt{0.361} = -300.2 \text{ psf}$$

$@z = H = 22\text{ft}$ (right side)

$$\sigma_{h,a} = (500 + 115 \times 22) \times 0.361 - 2 \times 400 \times \sqrt{0.361} = 613.2 \text{ psf}$$

Since the pressure at the top is negative, so there are some depth causing cracking on the wall and may be calculated as following:

$$(500 + 115 \times Z_c) \times 0.361 - 2 \times 400 \times \sqrt{0.361} = 0.0 \rightarrow Z_c = 7.23 \text{ ft}$$

Calculation of passive lateral earth pressure distribution:

$$\sigma_{h,P} = (q + \gamma H)K_P + 2c\sqrt{K_P} \qquad K_P = \tan^2\left(45 + \frac{\phi}{2}\right)$$

$$K_P = \tan^2\left(45 + \frac{35}{2}\right) = 3.69 \text{ (for the soil below the wall "left soil")}$$

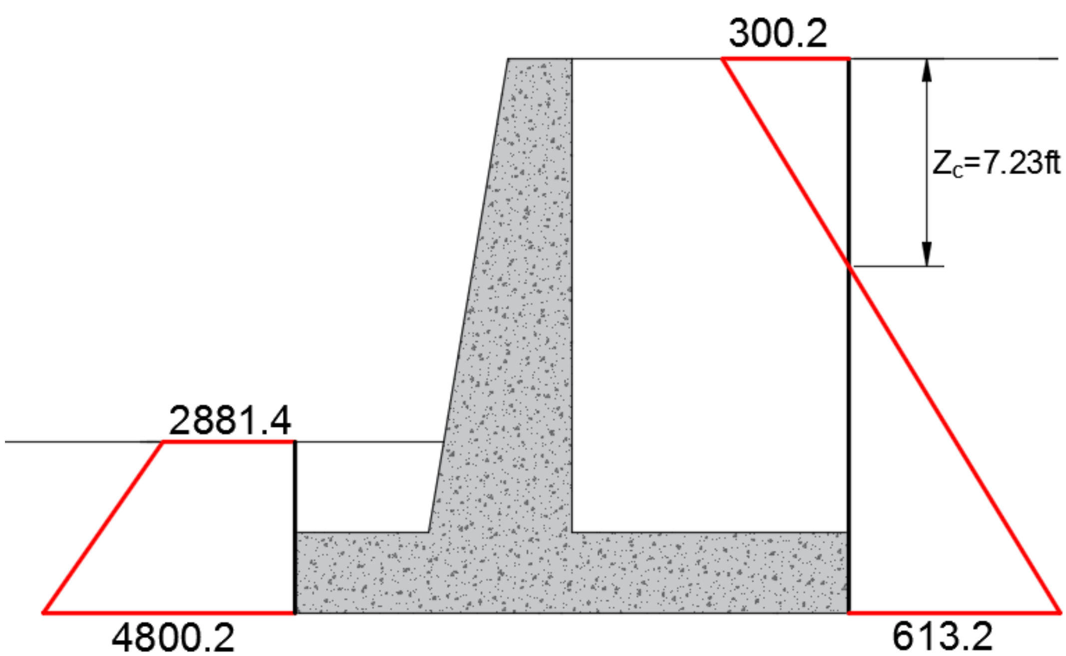
$@z = 0.0$ (left side)

$$\sigma_{h,P} = (0 + 0) \times 3.69 + 2 \times 750 \times \sqrt{3.69} = 2881.4 \text{ psf}$$

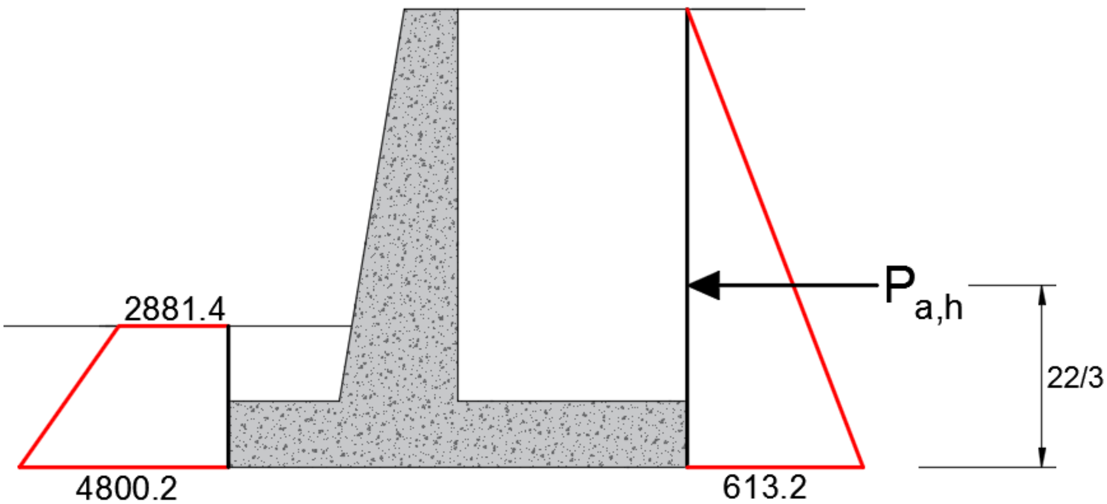
$@z = 4\text{ft}$ (left side)

$$\sigma_{h,P} = (0 + 130 \times 4) \times 3.69 + 2 \times 750 \times \sqrt{3.69} = 4800.2 \text{ psf}$$

Now the LEP distribution on the wall is as following:



Now, for design purposes, we modify this pressure to be zero at the top of the wall (for more safety), so the LEP distribution after modification is:



b.

We calculate the active rankine force using the modified LEP because we use it in designing process.

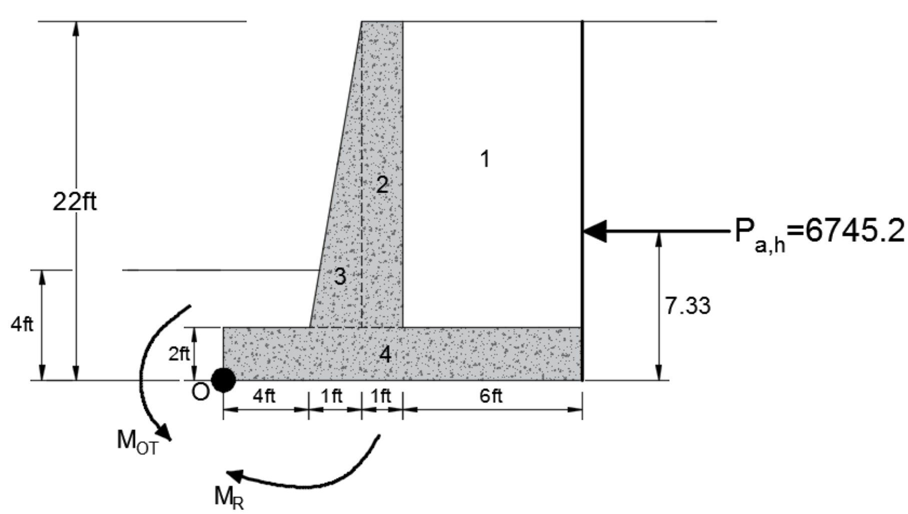
$$P_a = (\text{area of modified triangle}) = \frac{1}{2} \times 613.2 \times 22 = 6745.2\ \text{Ib.}$$

Location of P_a is at $\frac{22}{3} = 7.33$ ft (from the base)

If we want to calculate passive force (for sliding check):

$$P_P = (\text{area of trapezoidal}) = \frac{1}{2} \times (2881.4 + 4800.2) \times 4 = 15363.2\ \text{Ib.}$$

c. Overturning Stability:



Note that there is no vertical component of active force.

$M_{OT} = 6745.2 \times 7.33 = 49442.3 \text{ Ib. ft.}$

Now to calculate M_R we divided the soil and the concrete into rectangles and triangles to find the area easily (as shown above) **and to find the arm from the center of each area to point O** as prepared in the following table:

Section	Area	Weight/unit length of the wall	Moment arm measured from O	Moment about O
1	120	$120 \times 115 = 13800$	$12 - 3 = 9$	124200
2	20	$20 \times 150 = 3000$	$6 - 0.5 = 5.5$	16500
3	10	$10 \times 150 = 1500$	$5 - \frac{1}{3} = 4.67$	7005
4	24	$24 \times 150 = 3600$	$\frac{12}{2} = 6$	21600
$\sum$		$\sum V = 21900$		$M_R = 169305$

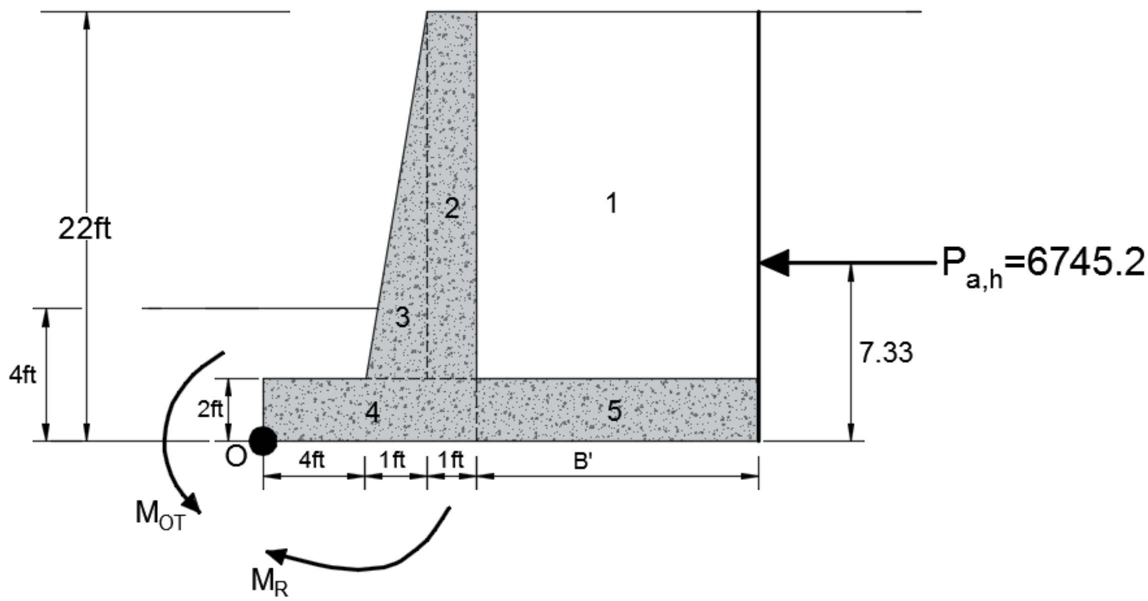
$$FS_{OT} = \frac{M_R}{M_{OT}} = \frac{169305}{49442.3} = 3.42 > 2 \rightarrow \textbf{OK} \checkmark.$$

The most important note here, the surcharge (q) is considered only when we calculate the pressure and it is not develop any moment about O because it is not applied force on the wall, and doesn't considered in vertical forces because we considered it in pressure calculation.

If the overturning check is not satisfied, what modification you shall do to make the foundation stable against overturning?? (Important)

In this case, we want to increase the resisting moment (M_R) by increasing the vertical forces and these arms about O.

This can be satisfied by increasing the width of the footing (increase the width of **heel**) to increase the weight of soil above the base and the weight of the base itself as following:



$M_{OT} = 6745.2 \times 7.33 = 49442.3$ Ib. ft.

The table of calculating M_R is:

Section	Area	Weight/unit length of the wall	Moment arm measured from O	Moment about O
1	$20B'$	$20B' \times 115 = 2300B'$	$0.5B' + 6$	$1150B'^2 + 13800B'$
2	20	$20 \times 150 = 3000$	$6 - 0.5 = 5.5$	16500
3	10	$10 \times 150 = 1500$	$5 - \frac{1}{3} = 4.67$	7005
4	12	$12 \times 150 = 1800$	$\frac{6}{2} = 3$	5400
5	$2B'$	$2B' \times 150 = 300B'$	$0.5B' + 6$	$150B'^2 + 1800B'$
$\sum$		$\sum V = 2600B' + 6300$		M_R

$M_R = 1300B'^2 + 15600B' + 28905$

Now, put $FS_{OT} = 2$ (critical case) to calculate B'

$$2 = \frac{M_R}{M_{OT}} = \frac{1300B'^2 + 15600B' + 28905}{49442.3}$$

$1300B'^2 + 15600B' - 69979.6 = 0.0$

Now, in this problem if we calculate B' here, it will be less than 6ft, because calculated the FS is 3.42>2 (as calculated above) and here we put it 2.
So, if the FS is not satisfied (<2) do the above procedures and calculate the new value of B' and then:
The final footing width is: $B = 6 + B'$ (must be larger than original value of B).

d. Check for sliding:

$FS_S = \frac{F_R}{F_d} \geq 2$ (if we consider P_P in F_R)

$F_d = P_{a,h} = 6745.2$ Ib

$F_R = \sum V \times \tan(K_1\phi_2) + K_2c_2B + P_P$

Take $K_2 = K_2 = 2/3$ $\sum V = 21900$ (from table of first check)

$$F_R = 21900 \times \tan\left(\frac{2}{3} \times 35\right) + \frac{2}{3} \times 750 \times 12 + 15363.2 = 30809.94 \text{ Ib.}$$

$$\rightarrow FS_S = \frac{30809.94}{6745.2} = 4.57 > 2 \rightarrow \textbf{OK} \checkmark.$$

If the Sliding stability not satisfied, what modifications you shall do:

Solution (1):

Increase the base width of the footing (width of the heel) to increase vertical forces:

$F_R = \sum V \times \tan(K_1\phi_2) + K_2c_2B + P_P$

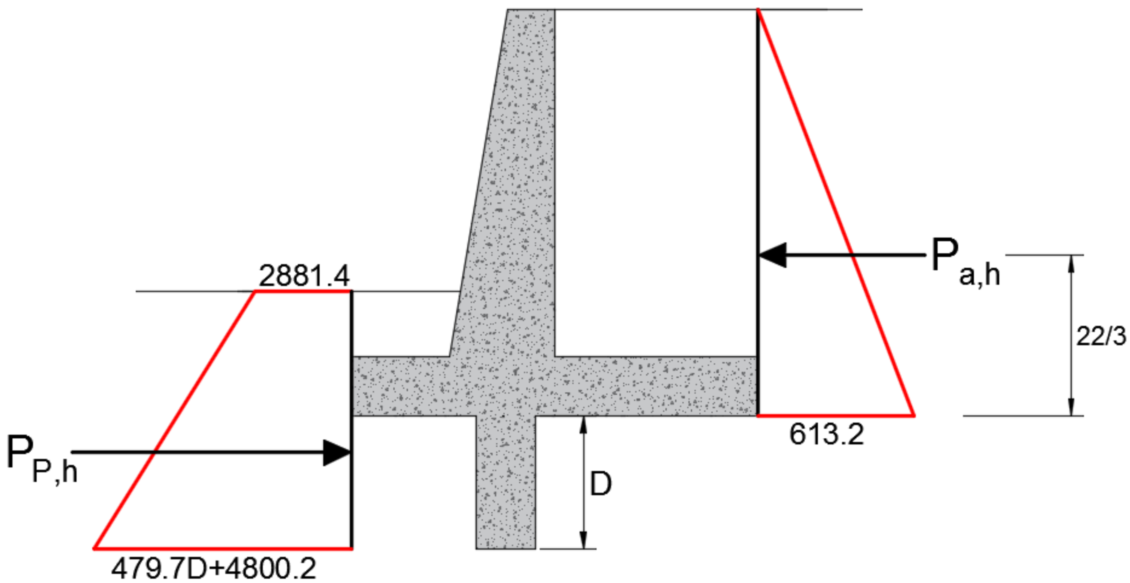
$\sum V = 2600B' + 6300$ (as calculted above in terms of B')

$FS_S = \frac{F_R}{F_d} = 2$ (at critical case)

Now the value of B' can be calculated and then calculate the new width of the footing.

Solution (2):

Use a base key (beam) of depth D under the base of the wall, this base key increase the passive force as following:



As we see, this base key increase the passive force and thereby increase the value of F_R and factor of safety.

$@z = 4 + D$

$$\sigma_{h,P} = (0 + 130 \times (4 + D)) \times 3.69 + 2 \times 750 \times \sqrt{3.69} = 4800.2 \text{ psf}$$

$$P_P = \frac{1}{2} \times (2881.4 + 4800.2 + 479.7\,D) \times (4 + D)$$

You may ask the following tow question (in this case):

1. If a base key of depth 1.5m is constructed under the base of the foundation, calculate the factor of safety against sliding.

$$P_P = \frac{1}{2} \times (2881.4 + 4800.2 + 479.7 \times 1.5) \times (4 + 1.5) = 23103.16 \text{ Ib.}$$

$$F_R = 21900 \times \tan\left(\frac{2}{3} \times 35\right) + \frac{2}{3} \times 750 \times 12 + 23103.16 = 38549.94 \text{ Ib}$$

$$\rightarrow FS_S = \frac{38549.94}{6745.2} = 5.7$$

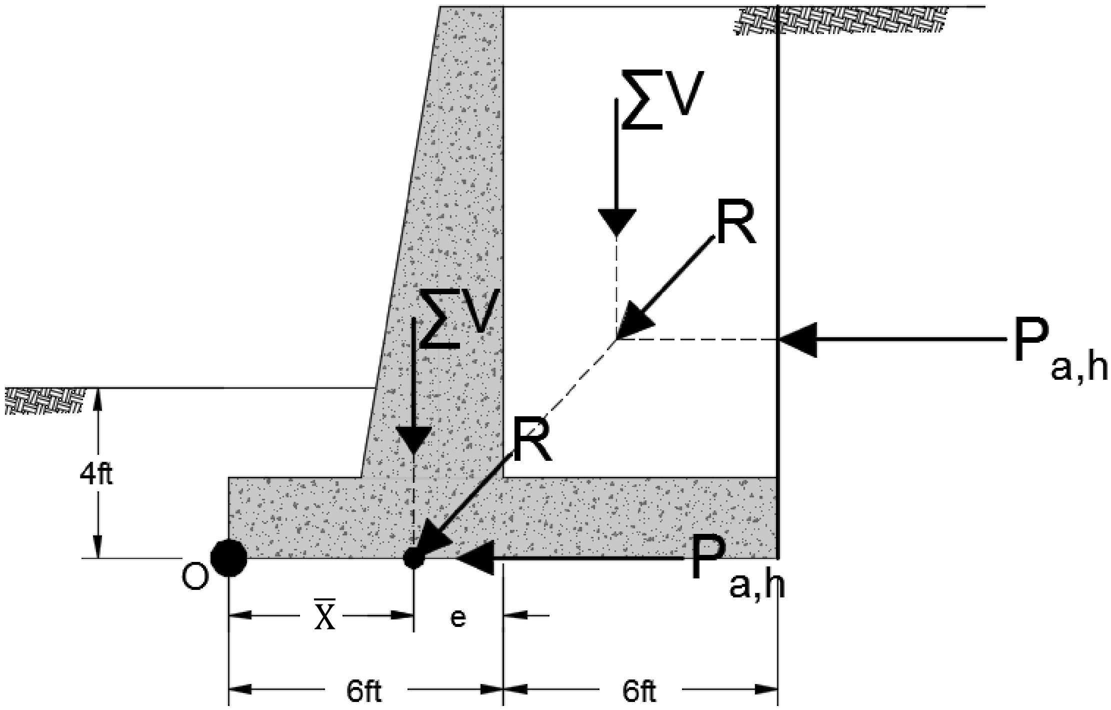
Note that the FS increase when we use base key.

2. If the sliding stability is not satisfied, find the depth of base key located under the base key to make the wall sable against sliding.

Here, the passive force is a function of D (base key depth), so calculate the value of F_R in terms of D, and then:

$$FS_S = \frac{F_R}{6745.2} = 2 \rightarrow F_R = ✓ \rightarrow D = ✓.$$

e.



$$\bar{X} = \frac{M_R - M_{OT}}{\sum V} = \frac{169305 - 49442.3}{21900} = 5.47\text{ft}$$

$$e = \frac{B}{2} - \bar{X} = 6 - 5.47 = 0.53 \text{ ft.}$$

$$\frac{B}{6} = \frac{12}{6} = 2 \rightarrow e = 0.53 < \frac{B}{6} \rightarrow\rightarrow\rightarrow$$

$$q_{max} = \frac{\sum V}{B \times 1}\left(1 + \frac{6e}{B}\right) = \frac{21900}{12 \times 1}\left(1 + \frac{6 \times 0.53}{12}\right) = 2308.6 \text{ psf}$$

$$q_{min} = \frac{\sum V}{B \times 1}\left(1 - \frac{6e}{B}\right) = \frac{21900}{12 \times 1}\left(1 - \frac{6 \times 0.53}{12}\right) = 1341.4 \text{ psf}$$

Calculation of q_u (for the soil below the bas):

$q_u = cN_cF_{cs}F_{cd}F_{ci} + qN_qF_{qs}F_{qd}F_{qi} + 0.5B\gamma N_\gamma F_{\gamma s}F_{\gamma d}F_{\gamma i}$

$c = 750 \quad , \quad q = 4 \times 130 = 520 \quad , \quad \gamma = 130$

$B = B' = B - 2e = 12 - 2(0.53) = 10.94 \text{ ft}$

Shape factors = 1 (RW can be considered strip footing).

For $\phi = 35 \rightarrow N_c = 46.12\,, N_q = 33.3\,, N_\gamma = 48.03$ (from table 3.3)

Depth factors: (We use B not B')

$$\frac{D}{B} = \frac{4}{12} = 0.333 < 1 \text{ and } \phi = 35 > 0.0 \rightarrow\rightarrow$$

$$F_{qd} = 1 + 2\tan\phi\,(1-\sin\phi)^2\left(\frac{D_f}{B}\right)$$

$$= 1 + 2\tan35\,(1-\sin35)^2\,(0.333) = 1.084$$

$$F_{cd} = F_{qd} - \frac{1-F_{qd}}{N_c\tan\phi} = 1.084 - \frac{1-1.084}{46.12\times\tan35} = 1.086$$

$$F_{\gamma d} = 1$$

Inclination Factors:

$$\beta = \Psi = \tan^{-1}\left(\frac{P_{a,h}}{\sum V}\right) = \tan^{-1}\left(\frac{6745.2}{21900}\right) = 17.12 \text{ (with vertical)}$$

$$F_{ci} = F_{qi} = \left(1-\frac{\beta^\circ}{90}\right)^2\left(1-\frac{17.12}{90}\right)^2 = 0.65$$

$$F_{\gamma i} = \left(1-\frac{\beta^\circ}{\phi^\circ}\right) = \left(1-\frac{17.12}{35}\right) = 0.51$$

$$\rightarrow q_u = 750\times46.12\times1.086\times0.65 + 520\times33.3\times1.084\times0.65$$
$$+0.5\times10.94\times130\times48.03\times1\times0.51$$

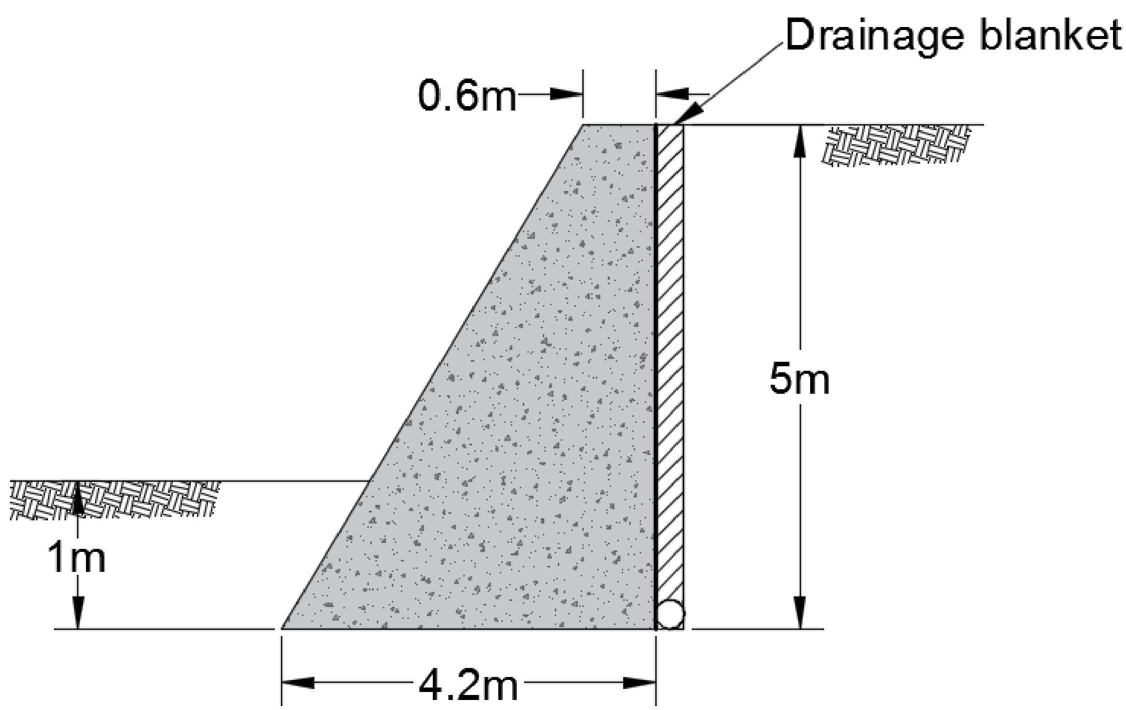
$$\rightarrow q_u = 54036.54$$

$$FS_{B.C} = \frac{q_u}{q_{max}} = \frac{54036.54}{2308.6} = 23.4 > 3 \textbf{ OK} \checkmark.$$

4.

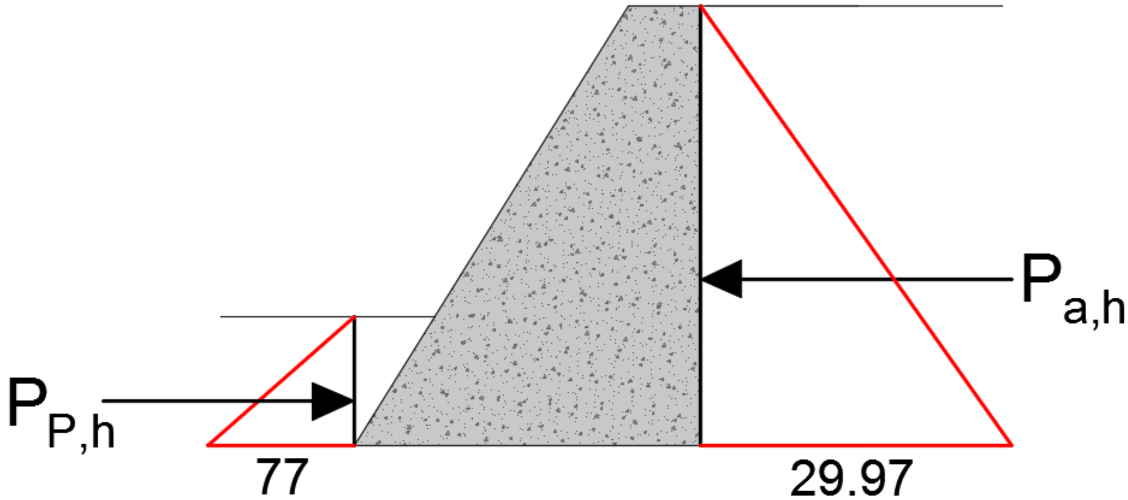
A gravity retaining wall shown in the figure below is required to retain 5 m of soil. The backfill is a coarse grained soil with saturated unit weight =18 kN/m^3, and friction angle of $\phi = 30°$. The existing soil below the base has the following properties; $\gamma_{sat} = 20\ \mathrm{kN/m^3}$, $\phi = 36°$. The wall is embedded 1m into the existing soil, and a drainage system is provided as shown. The ground water table is at 4.5m below the base of the wall. Determine the stability of the wall for the following conditions (assume K_1=K_2 = 2/3):

a- Wall friction angle is zero.
b- Wall friction angle is $20°$.
c- The drainage system becomes clogged during several days of rainstorm and the ground water rises to the surface of backfill (use rankine).
$\gamma_{concrete} = 24\ \mathrm{kN/m^3}$



Solution

a- (wall friction angle = $\delta = 0.0$)
Since $\delta = 0.0$ (we use rankine theory).



(The unit weight of the soil (natural) is not given, so we consider the saturated unit weight is the natural unit weight).

$$K_a = \tan^2\left(45 - \frac{\phi}{2}\right) = \tan^2\left(45 - \frac{30}{2}\right) = 0.333 \text{ (for the retained soil)}$$

$$K_P = \tan^2\left(45 + \frac{\phi}{2}\right) = K_P = \tan^2\left(45 + \frac{36}{2}\right) = 3.85 \text{ (for soil below the base)}$$

Calculation of active lateral earth pressure distribution:

$$\sigma_{h,a} = (q + \gamma H)K_a - 2c\sqrt{K_a}$$

$@z = H = 5\text{m}$ (right side)

$$\sigma_{h,a} = (0 + 18 \times 5) \times 0.333 - 0 = 29.97 \text{ kN/m}^2$$

Calculation of passive lateral earth pressure distribution:

$$\sigma_{h,P} = (q + \gamma H)K_P + 2c\sqrt{K_P}$$

$@z = 1\text{m}$(left side)

$$\sigma_{h,P} = (0 + 20 \times 1) \times 3.85 + 0 = 77 \text{ kN/m}^2$$

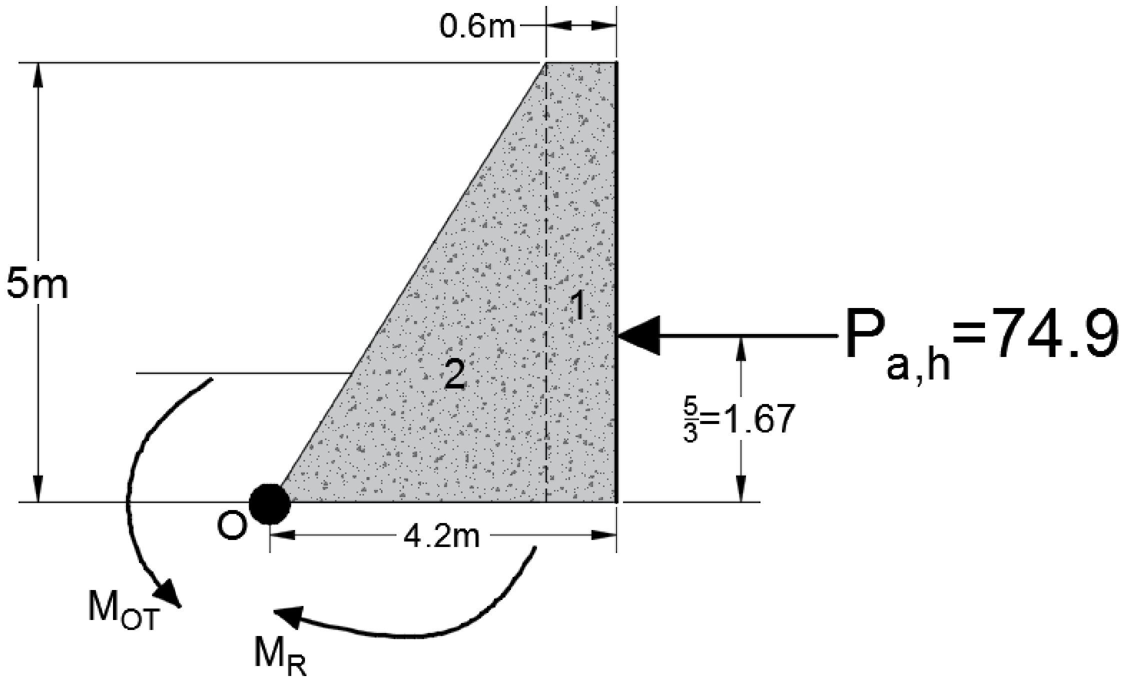
Calculation of active force:

$$P_a = (\text{area of right triangle}) = \frac{1}{2} \times 29.97 \times 5 = 74.9 \text{ kN}$$

Calculation of passive force:

$$P_P = (\text{area of left triangle}) = \frac{1}{2} \times 77 \times 1 = 38.5 \text{ kN}$$

Overturning Stability:



$M_{OT} = 74.9 \times 1.67 = 125.08\ \text{kN.m}$

Now to calculate M_R we divided the soil and the concrete into rectangles and triangles to find the area easily (as shown above) **and to find the arm from the center of each area to point O** as prepared in the following table:
Note that since there is no heel for the wall, the force is applied directly on the wall.

Section	Area	Weight/unit length of the wall	Moment arm measured from O	Moment about O
1	3	$3 \times 24 = 72$	3.9	280.8
2	9	$9 \times 24 = 216$	2.4	518.4
$\sum$		$\sum V = 288$		$M_R = 799.2$

Note that there is no vertical component of active force

$$FS_{OT} = \frac{M_R}{M_{OT}} = \frac{799.2}{125.08} = 6.39 > 2 \rightarrow \textbf{OK} ✓.$$

Sliding Stability:

$$FS_S = \frac{F_R}{F_d} \geq 2 \ \ \text{(if we consider } P_P \text{ in } F_R\text{)}$$

$$F_d = P_{a,h} = 74.9\ kN/m^2$$

$$F_R = \sum V \times \tan(K_1\phi_2) + K_2 c_2 B + P_P$$

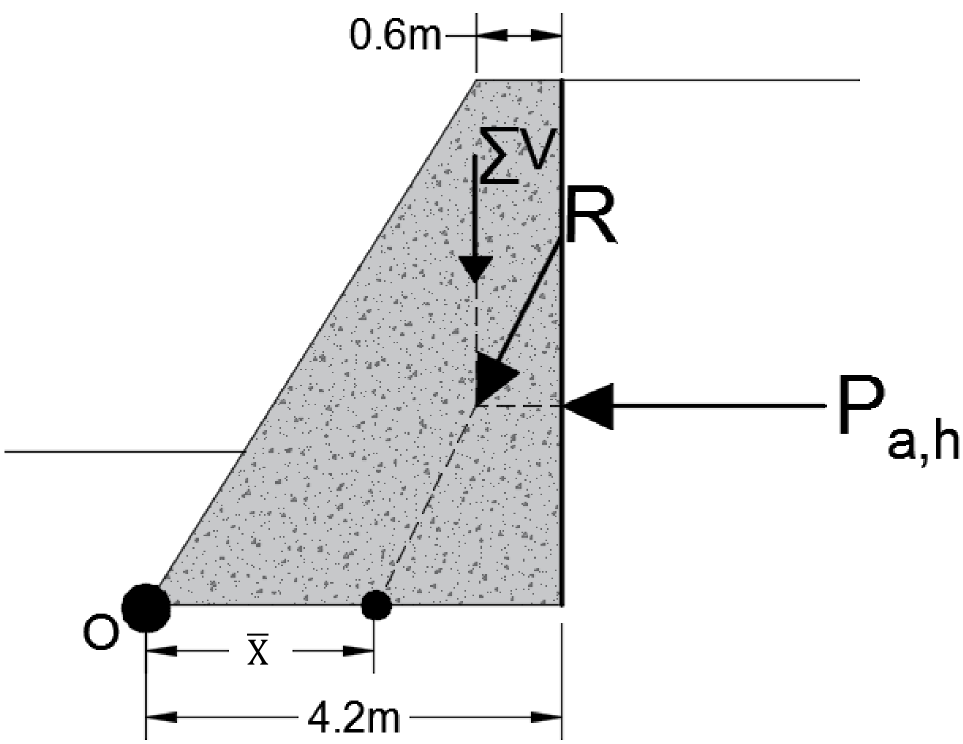
Take $K_2 = K_2 = 2/3$ $\quad \sum V = 288$ (from table of first check)

$P_P = 38.5\ kN/m^2$ (as calculated above)

$$F_R = 288 \times \tan\left(\frac{2}{3} \times 36\right) + \frac{2}{3} \times 0 \times 4.2 + 38.4 = 166.62\ kN.$$

$$\rightarrow FS_S = \frac{166.62}{74.9} = 2.2 > 2 \rightarrow \textbf{OK} \checkmark.$$

Bearing capacity check:



$$\overline{X} = \frac{M_R - M_{OT}}{\sum V} = \frac{799.2 - 125.08}{288} = 2.34\ m$$

$$e = \frac{B}{2} - \overline{X} = \frac{4.2}{2} - 2.34 = -0.24\ m \text{ (R is at right of base center)}$$

$$\frac{B}{6}=\frac{4.2}{6}=0.7 \rightarrow e=0.24<\frac{B}{6} \rightarrow\rightarrow\rightarrow$$

$$q_{max}=\frac{\sum V}{B\times 1}\left(1+\frac{6e}{B}\right)=\frac{288}{4.2\times 1}\left(1+\frac{6\times 0.24}{4.2}\right)=92.08\ \mathrm{kN/m^2}$$

$$q_{min}=\frac{\sum V}{B\times 1}\left(1-\frac{6e}{B}\right)=\frac{288}{4.2\times 1}\left(1-\frac{6\times 0.24}{4.2}\right)=45.06\ \mathrm{kN/m^2}$$

Calculation of q_u (for the soil below the bas):

$q_u = cN_cF_{cs}F_{cd}F_{ci} + qN_qF_{qs}F_{qd}F_{qi} + 0.5B\gamma N_\gamma F_{\gamma s}F_{\gamma d}F_{\gamma i}$

$c = 0.0 \quad , \quad q = 1\times 20 = 20$

Water table is at distance 4.5m > B=4.2m >>> no effect of water table.

$\rightarrow \gamma = 20$

$B = B' = B - 2e = 4.2 - 2(0.24) = 3.72\mathrm{m}$

Shape factors = 1 (RW can be considered strip footing).

For $\phi = 36 \rightarrow N_c = 50.59, N_q = 37.75\ , N_\gamma = 56.31$ (from table 3.3)

Depth factors: (We use B not B')

$$\frac{D}{B}=\frac{1}{4.2}=0.238<1 \text{ and } \phi=36>0.0 \rightarrow\rightarrow$$

$$F_{qd}=1+2\tan\phi\,(1-\sin\phi)^2\left(\frac{D_f}{B}\right)$$

$$=1+2\tan 36\,(1-\sin 36)^2\,(0.238)=1.058$$

$$F_{cd}=F_{qd}-\frac{1-F_{qd}}{N_c\tan\phi}=1.058-\frac{1-1.058}{50.59\times\tan 36}=1.06$$

$$F_{\gamma d}=1$$

Inclination Factors:

$$\beta=\Psi=\tan^{-1}\left(\frac{P_{a,h}}{\sum V}\right)=\tan^{-1}\left(\frac{74.9}{288}\right)=14.6^{\circ}\text{(with vertical)}$$

$$F_{ci}=F_{qi}=\left(1-\frac{\beta^{\circ}}{90}\right)^2\left(1-\frac{14.6}{90}\right)^2=0.7$$

$$F_{\gamma i}=\left(1-\frac{\beta^{\circ}}{\phi^{\circ}}\right)=\left(1-\frac{17.12}{36}\right)=0.59$$

$$\rightarrow q_u=0.0+20\times 37.75\times 1.058\times 0.7$$
$$+0.5\times 3.72\times 20\times 56.31\times 1\times 0.59$$

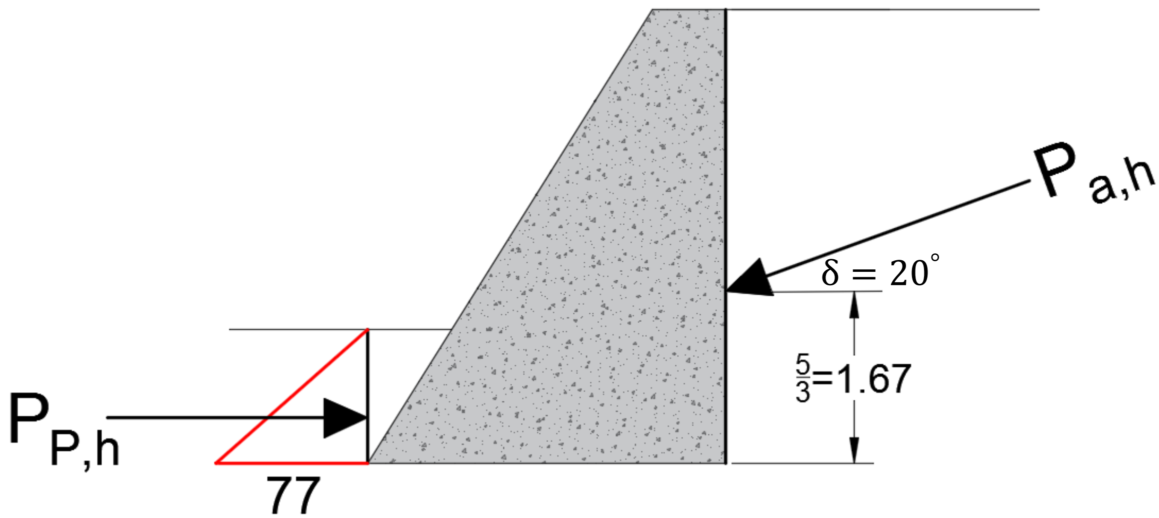
$\rightarrow q_u = 1795 \text{ kN/m}^2$

$FS_{B.C} = \dfrac{q_u}{q_{max}} = \dfrac{1795}{92.08} = 19.5 > 3$ **OK ✓.**

b-(wall friction angle $= \delta = 20$)

Since $\delta = 20$ (we use coulomb's theory for active pressure).

Here, the active force is not horizontal, but it is inclined by angle $\delta = 20$ with horizontal:



$P_a = \frac{1}{2} \times \gamma \times H^2 \times k_a$

$\delta = 20$, $\phi = 30 \rightarrow \delta = \dfrac{2}{3}\phi \rightarrow\rightarrow$

k_a in this case is calculated from (**Table 7.4 P.343**)

$\beta = 90$, $\alpha = 0$, $\phi = 30 \rightarrow\rightarrow k_a = 0.2973$

$P_a = \dfrac{1}{2} \times 18 \times 5^2 \times 0.2973 = 66.9 \text{ kN}$

This force having horizontal and vertical components:

$P_{a,h} = 66.9 \cos(20) = 62.86 \text{ KN}$

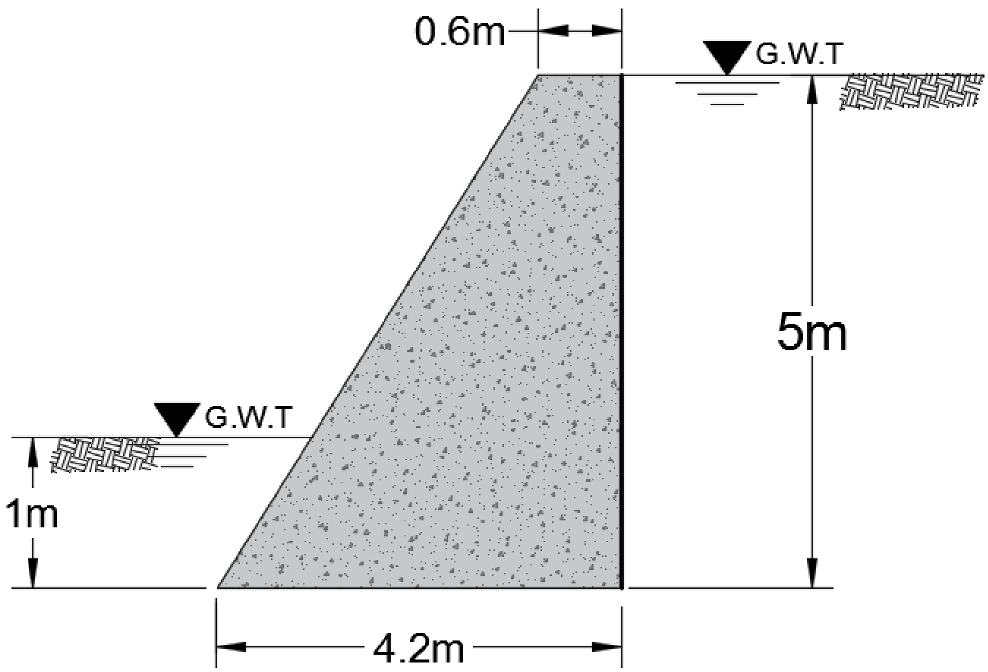
$P_{a,v} = 66.9 \sin(20) = 22.88 \text{ kN}$

Calculation of passive force always done by rankine theory (i.e. passive force doesn't change from first required)

$P_P = 38.5 \text{ kN}$ (as calculate in first required)

Now, you can complete the solution without any problems ☺ .

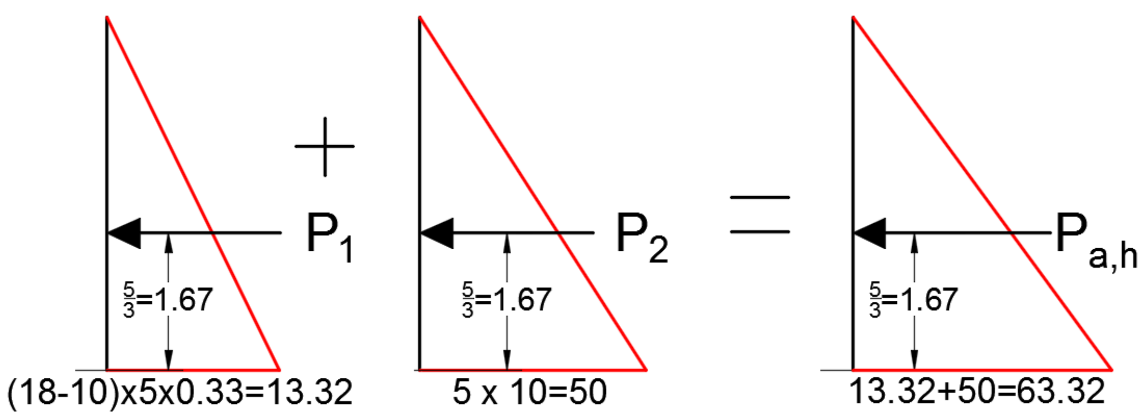
c- When the ground water rises to the surface, the retaining wall is shown below:



What differ???

If we want to use rankine theory (force from soil is gorizontal):

1. Calculation of active force:



Don't forget we calculate effective stress every change, and the we add water alone.

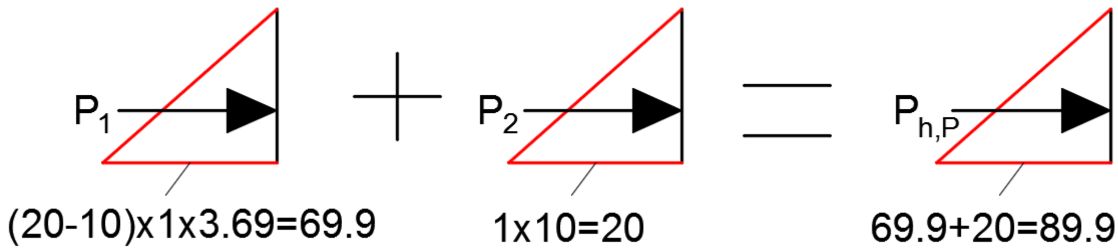
$$P_1 = (\text{force due to effective soil}) = \frac{1}{2} \times 13.32 \times 5 = 33.3 \text{ kN}$$

$$P_2 = (\text{force due to water}) = \frac{1}{2} \times 50 \times 5 = 125 \text{ KN}$$

$P_{a,h} = P_1 + P_2 = 33.3 + 125 = 158.33 \text{ KN}$

Loacation of $P_{a,h}$:
Take the moment at the bottom of the wall to get the location, but here the two forces have the same location, so the resultant of the two forces will have the same location (1.67 from base).

2. Calculation of passive force:



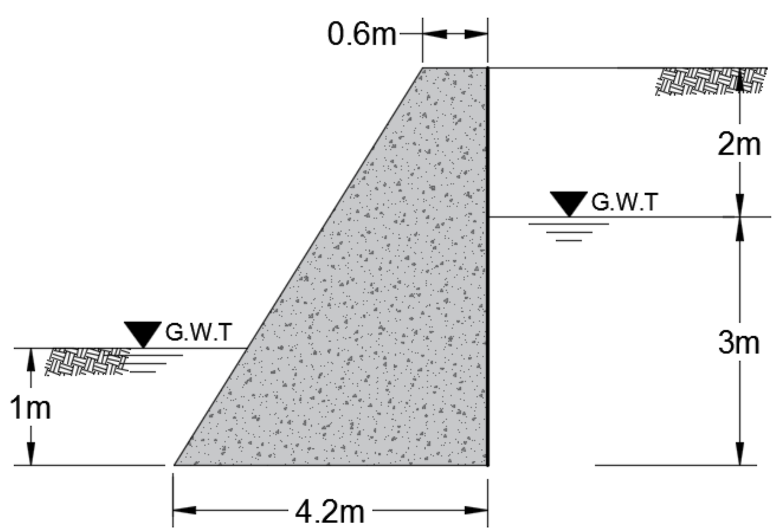
$P_{P,h} = P_1 + P_2$

3. In calculation of vertical forces due to the soil weight always take the effective unit weight and multiply it by the area to get the effective force but this is not required in this problem because the force applied directly on the wall.

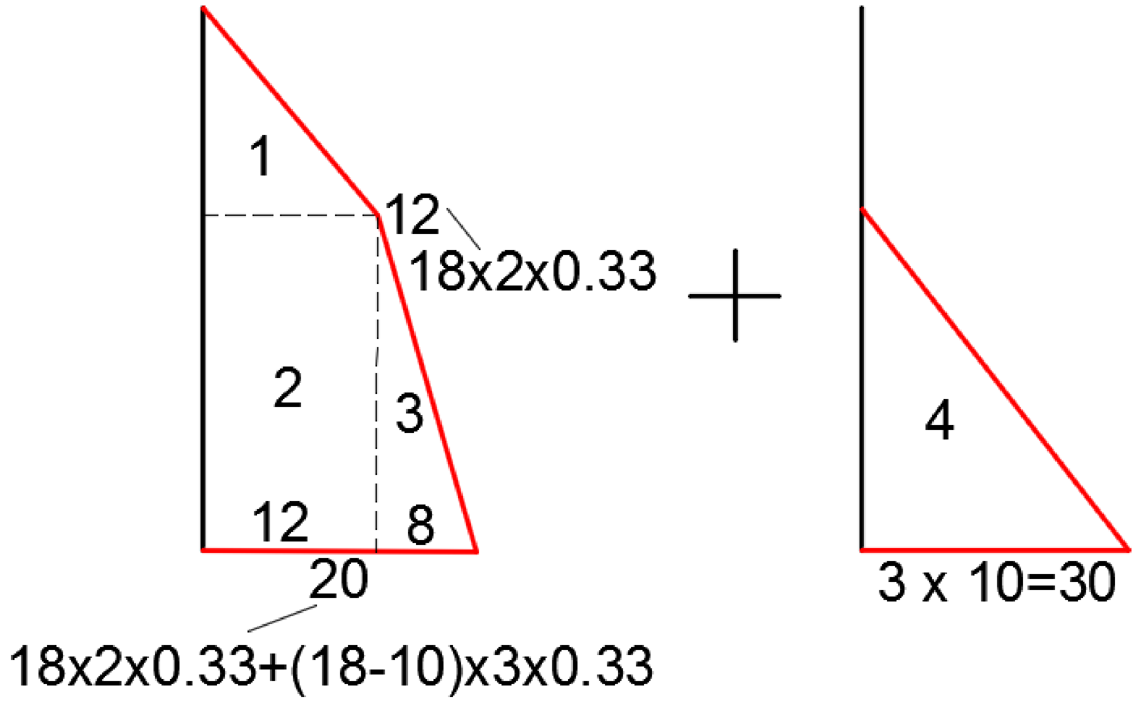
4. In calculating of bearing capacity for the soil below the base, since the water table is above the base (case 3) we take $\gamma = \gamma'$ in Meyerhof Eq.

Now you can complete the solution with the same procedures without any problem ☺.

Now, If the water table is at distance 2m below the surface, what's new???



Calculation of Active force:

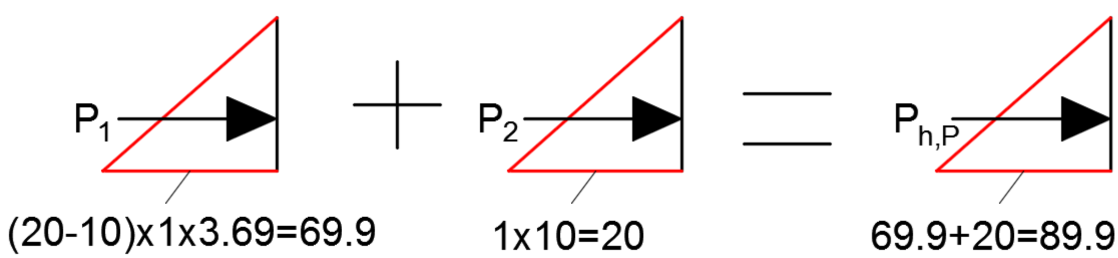


Here we calculate the effective stress every change, and then added the water alone from its beginning:

$P_{a,h} = P_1 + P_2 + P_3 + P_4$

To find the location of $P_{a,h}$ take summation moment at the base of the wall.

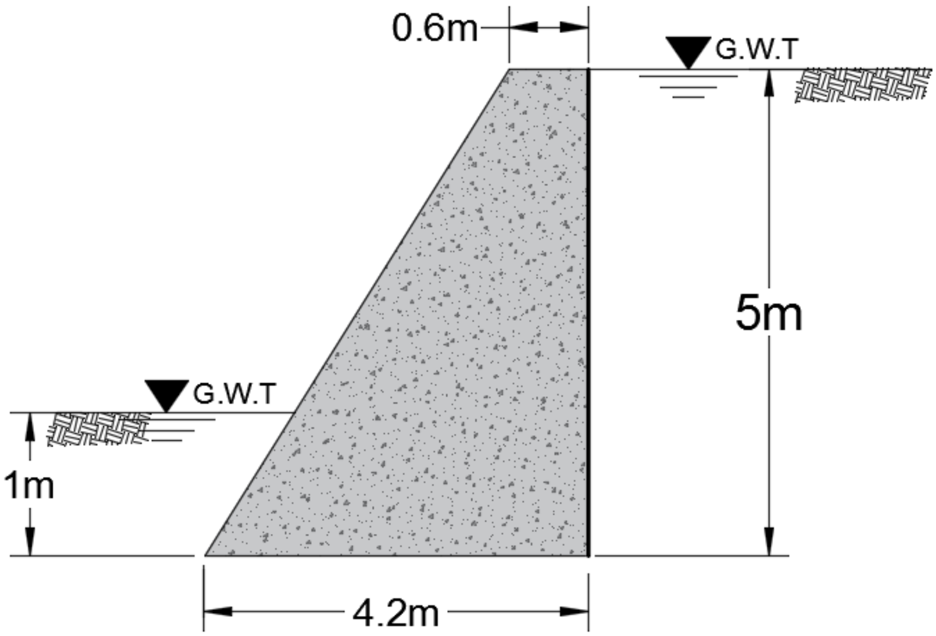
Calculation of passive force will not change



The weight of soil above heel (when heel exist), we divide the soil above the heel for two areas, soil above water table and soil below water table. The area of soil above water table is multiplied by natural unit weight, and the area of soil below water table is multiplied by effective unit weight.

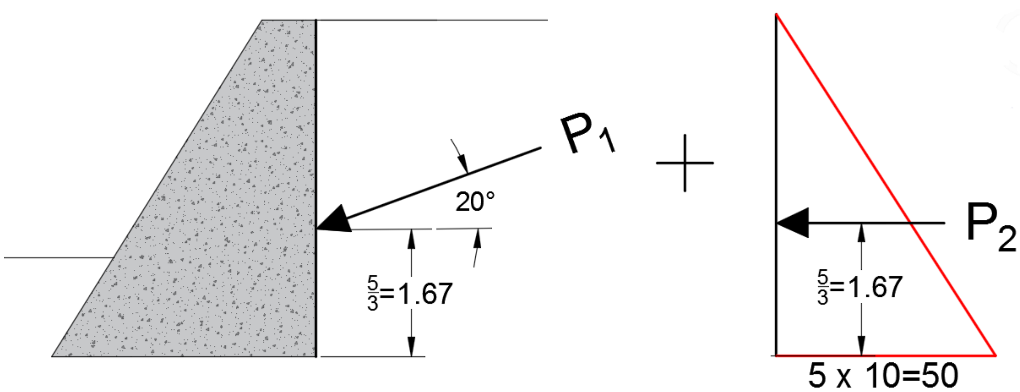
In calculating of bearing capacity, the water is still above the base, so we use effective unit weight in Meyerhof Eq.

The last idea in this chapter:
If you are asked to solve this problem (in case of water table) using Coulomb's theory:



As stated above, the force here will be inclined by angle $\delta = 20$ with horizontal.

Calculation of Active force:



Calculation of P_1:

$$P_1 = \frac{1}{2} \times \gamma \times H^2 \times k_a \qquad \gamma = \gamma' = 18 - 10 = 8$$

$$\delta = 20\ ,\ \ \phi = 30 \rightarrow \delta = \frac{2}{3}\phi \rightarrow\rightarrow$$

k_a in this case is calculated from (**Table 7.4 P.343**)

$\beta = 90\,, \alpha = 0\,, \phi = 30 \rightarrow\rightarrow k_a = 0.2973$

$$P_a = \frac{1}{2} \times 8 \times 5^2 \times 0.2973 = 29.73\ \text{kN}$$

This force having horizontal and vertical components:

$P_{1,h} = 29.73 \cos(20) = 27.93\ \text{KN}$

$P_{1,v} = 29.73 \sin(20) = 10.17\ \text{kN}$

Calculation of P_2:

$$P_2 = (\text{force due to water}) = \frac{1}{2} \times 50 \times 5 = 125\ \text{KN}$$

Calculation of active force:

$P_{a,h} = P_{1,h} + P_2 = 27.93 + 125 = 152.93\ \text{kN}$

$P_{a,v} = P_{1,v} = 10.17\ \text{kN}$

Calculation of passive force:

Will not changes because always we can use rankine theory in calculating of passive force**.**

Other notes remains the same.

Chapter (9)
Sheet Pile Walls

Introduction

Sheet piles are a **temporary** structures used to retain a soil or water for a specific period of time, to build a structure in the other side of this wall.
For example; if we want to build a structure with three basement floors (underground) and this structure surrounded by other structures, when the excavation process starts, if the soil under the surrounding structures doesn't retained by a sheet pile, this soil will fail and will moves to the excavation site, and the structure above this soil may collapse suddenly, so before establishment of excavation process, sheet pile must be constructed to retain this soil and prevent it from fails and after completion of constructed the structure , we can remove this sheet pile because it's function was end.
Another example; if we wanna build a structure in the sea (waterfront structures) we can use sheet piles to retain sea water from flowing to the required area, and then withdraw the water confined between sheet piles and thereby build the required structures, finally remove sheet piles because there functions were end.
The following figures are some explanation of the applications of sheet piles and the shape of sheet pile itself:

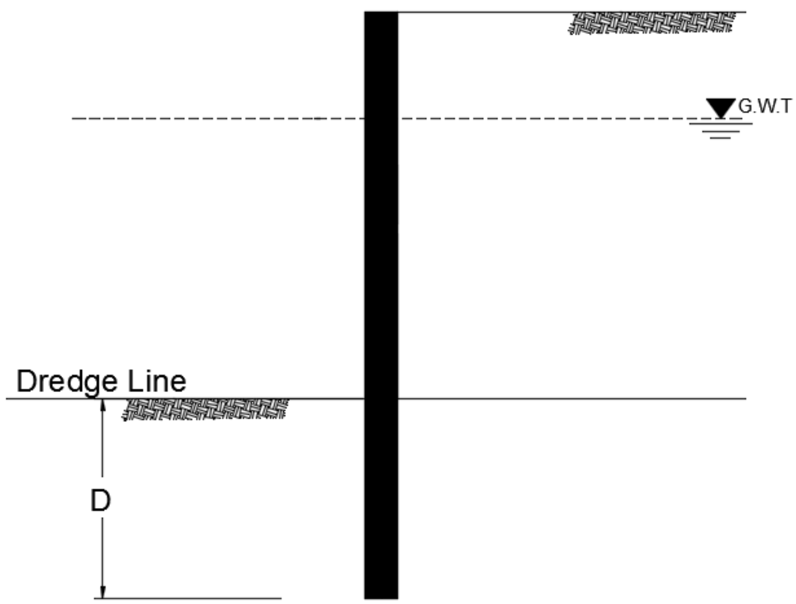
Sheet Pile in basement

Sections of steel Sheet Piles

Sheet Piles in Waterfront structure

Notes:

1. Sheet piles may be made from steel, concrete or wood.
2. As seen in the above pictures, sheet piles must penetrates a specified distance in earth (from both sides) to be stable against applied lateral loads, this depth called depth of penetration, and the following figure explain the main parts of sheet piles:



The line at which the sheet pile starts penetrating in soil from both sides is known by **dredge line**, and the depth of penetration of sheet pile under this line is **D: depth of penetration.**

Designing of sheet piles mainly is to calculate the depth of penetration D and determining the section of sheet pile as will be discussed later.

Types of Sheet Piles

There are two main types of sheet piles:

1. Cantilever Sheet Piles.
2. Anchored Sheet piles.

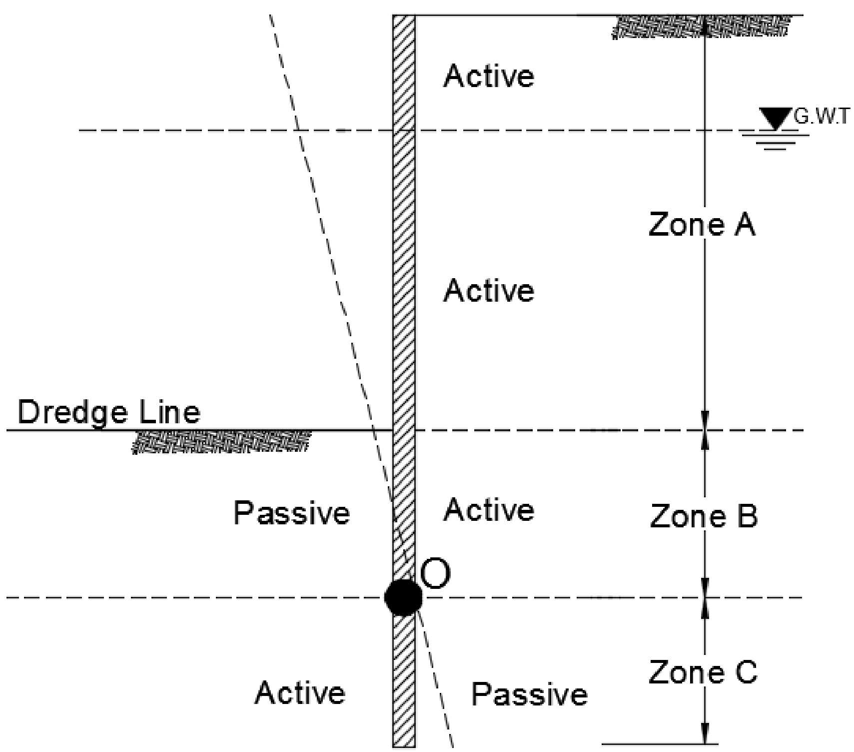
Now, we will learn how to analyze and design each type.

Cantilever Sheet Piles

Cantilever sheet pile walls are usually recommended to use for walls of moderate height ($\leq$ 6m) measured above the dredge line. In such walls, the sheet piles are act as a wide **cantilever beam** above the dredge line.

The main step in analyzing cantilever sheet pile is to knowing the deflection of cantilever sheet pile with depth, and knowing (from deflection shape) the type of LEP (active or passive).

The following figure clarifies the deflection of the cantilever sheet pile due to lateral earth pressure:



As you see, due to the lateral earth pressure the wall will pushed out the soil above the dredge line so the type of LEP above the dredge line is active pressure and no passive pressure because there is no soil exist in the other side (Zone A) in the above figure.
Below the dredge line there exists a soil in both sides of the wall the wall and the wall still moves out (left side) till reaching point O (point of rotation) after point O the wall will moves to right side as shown.
So, soil below dredge line can be divided into two zones; zone B between dredge line and point O, in this zone the wall moves to the left, so the soil on the right exerts active pressure and the soil on the left exerts passive pressure. Zone C between point O to the end of sheet pile, in this zone the wall moves to the right, so the soil on the right exerts passive pressure and the soil on the left exerts active pressure as seen in the figure above.

There are three cases for cantilever sheet piles:
- ✓ **Cantilever Sheet Pile penetrating in Sandy Soil.**
- ✓ **Cantilever Sheet Pile penetrating in Clayey Soil.**
- ✓ **Cantilever Sheet Pile penetrating in** $C-\phi$ **Soil.**

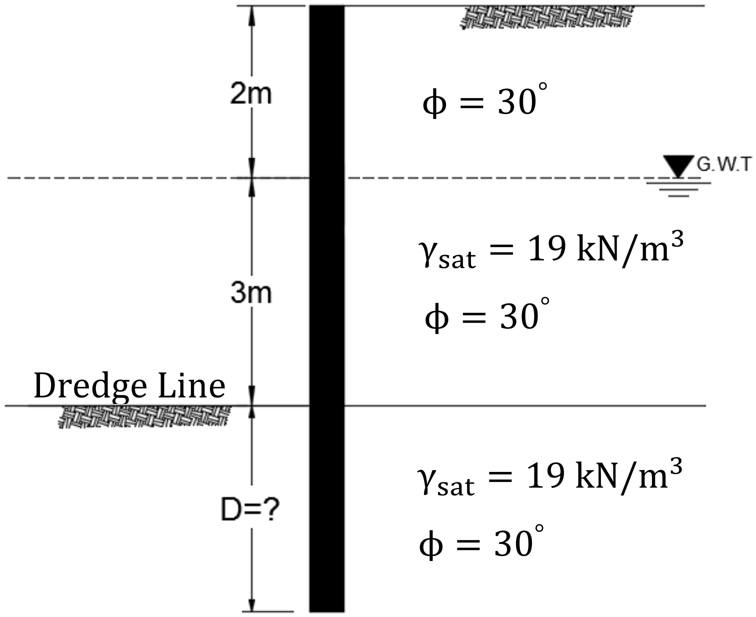
Before discussing each type, the following notes are very important:
- ➢ The first step in designing the sheet pile is to draw the **net LEP** distribution with depth along the sheet pile, the net LEP is the **difference** between passive LEP and active LEP at every change in soil with depth
$\text{i.e. net pressure} = \Delta\sigma = \sigma_{\text{passive}} - \sigma_{\text{active}}$
- ➢ Designing a sheet pile consists of the following two steps:
  1. Calculation the depth of penetration (D).
  2. Determining the section Modulus (S) where: $S = \frac{M_{max}}{\sigma_{all}}$

     $M_{max} = \text{maximum moment along sheet pile}$
     $\sigma_{all} = \text{maximum allwable flexural stress (for sheet pile material).}$
- ➢ Always in this chapter we will use Rankine LEP theory.

The most important one, is drawing the LEP distribution along sheet pile (especially below the dredge line) **correctly (**If you did, completion of designing process will be easy), so now we want to learn how to draw the LEP distribution for all cases of cantilever sheet pile.

Cantilever Sheet Piles Penetrating Sandy Soil

Consider the following example:



The first step always is calculating K_a and K_p for each layer, but here all layers have the same friction angle, so:

$$K_a = \tan^2\left(45 - \frac{\phi}{2}\right) = \tan^2\left(45 - \frac{30}{2}\right) = 0.333$$

$$K_p = \tan^2\left(45 + \frac{\phi}{2}\right) = \tan^2\left(45 + \frac{30}{2}\right) = 3$$

Now we calculate the LEP at each depth:

$$\sigma_{h,a} = (q + \gamma H)K_a - 2c\sqrt{K_a}$$

$$\sigma_{h,p} = (q + \gamma H)K_p + 2c\sqrt{K_p}$$

@ $z = 0.0$ (above dredge line → active pressure only)

$\sigma_{h,a} = (0 + 16 \times 0) \times 0.333 - 0 = 0.0$

$\Delta\sigma_h = \sigma_{h,p} - \sigma_{h,a} = 0.0$

@ $z = 2m$ (above dredge line → active pressure only)

just before = just after because K_a is the same

$\sigma_{h,a} = (0 + 16 \times 2) \times 0.333 - 0 = 10.65 kN/m^2$

$\Delta\sigma_h = \sigma_{h,p} - \sigma_{h,a} = 0 - 10.65 = -10.65\ kN/m^2$

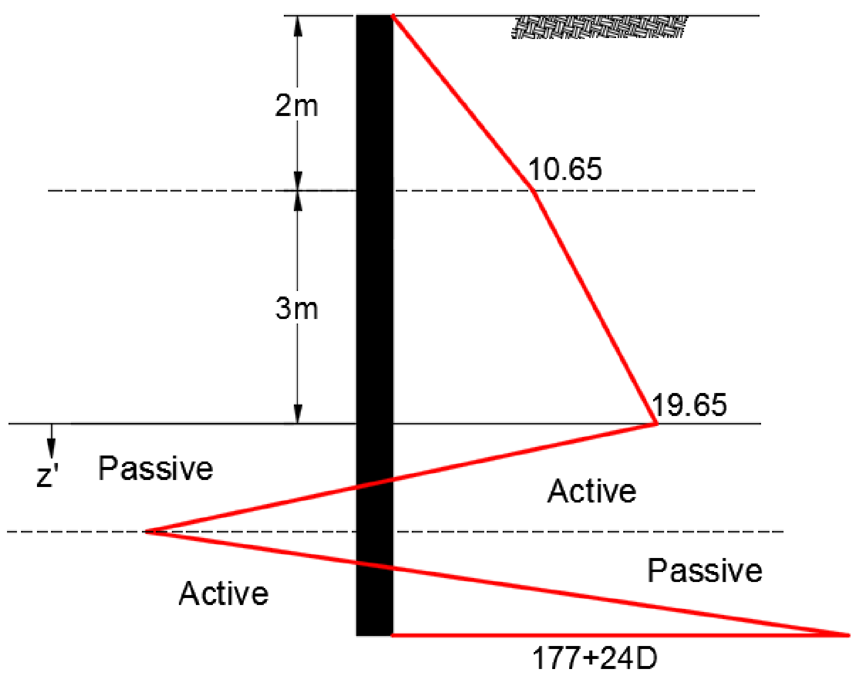
The negative sign means we draw this value at right side (side of active pressure because is the largest pressure at this depth.

$@\ z = 5\text{m}$ (just before → active pressure only)
$\sigma_{h,a} = (0 + 16 \times 2 + (19 - 10) \times 3) \times 0.333 - 0 = 19.65\ \text{kN/m}^2$
$\Delta\sigma_h = \sigma_{h,p} - \sigma_{h,a} = 0 - 19.65 = -19.65\ \text{kN/m}^2$
(Don't forget, effective pressure always)

$@\ z = 5\text{m}$ (just after → active pressure at right, passive pressure at left)
Since K_a is the same before and after, the pressure will be the same
$\sigma_{h,a} = (0 + 16 \times 2 + (19 - 10) \times 3) \times 0.333 - 0 = 19.65\text{kN/m}^2$
$\sigma_{h,p} = (0 + (19 - 10) \times 0)3 + 0 = 0.0$
$\Delta\sigma_h = \sigma_{h,p} - \sigma_{h,a} = 0 - 19.65 = -19.65\ \text{kN/m}^2$

$@\ z = 5 + D$ (passive pressure at right, active pressure at left)
Note that at this depth the types of pressure changes as explained above
Active pressure at left side:
$\sigma_{h,a} = (0 + (19 - 10) \times D) \times 0.333 - 0 = 3D$
Passive pressure at right side:
$\sigma_{h,p} = (0 + 16 \times 2 + (19 - 10) \times 3 + (19 - 10) \times D)3 + 0 = 177 + 27D$
$\Delta\sigma_h = \sigma_{h,p} - \sigma_{h,a} = (177 + 27D) - 3D = 177 + 24D$
Note that the value of $(177 + 24D)$ is positive which means we draw this value as a line in the passive zone at this depth (right side).

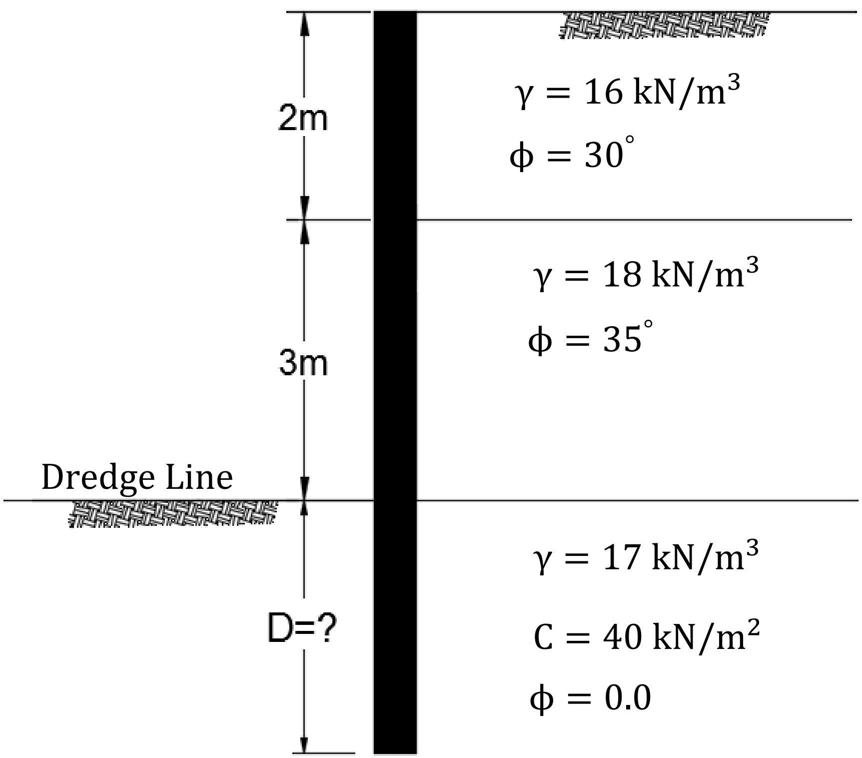
Now, we draw the LEP distribution along the wall:



Now, under dredge line the **net** lateral pressure $\Delta\sigma_h$always will increase by the value of $\gamma z'(K_p - K_a)$ where z': depth below dredge line at any point. And since K_p is always larger than K_a (when $\phi > 0$) the increase in pressure will always in the **direction of passive zones.** So, at the specific point (point O) the pressure will changes from active to passive (right side) and thereby the LEP distribution will tend to moves in the direction of passive zone as shown.

Cantilever Sheet Piles Penetrating Clay

Consider the following example:



The first step always is calculating K_a and K_p for each layer:

$$K_a = \tan^2\left(45 - \frac{\phi}{2}\right)$$

$$K_{a1} = \tan^2\left(45 - \frac{30}{2}\right) = 0.333$$

$$K_{a2} = \tan^2\left(45 - \frac{35}{2}\right) = 0.27$$

$$K_{a3} = \tan^2(45 - 0) = 1$$

$$K_p = \tan^2\left(45 + \frac{\phi}{2}\right)$$
$$K_{p1} = \tan^2\left(45 + \frac{30}{2}\right) = 3$$
$$K_{p2} = \tan^2\left(45 + \frac{35}{2}\right) = 3.69$$
$$K_{p3} = \tan^2(45 + 0) = 1$$

Now we calculate the LEP at each depth:

$$\sigma_{h,a} = (q + \gamma H)K_a - 2c\sqrt{K_a}$$
$$\sigma_{h,p} = (q + \gamma H)K_p + 2c\sqrt{K_p}$$

$@\ z = 0.0$ (above dredge line $\rightarrow$ active pressure only)

$$\sigma_{h,a} = (0 + 16 \times 0) \times 0.333 - 0 = 0.0$$
$$\Delta\sigma_h = \sigma_{h,p} - \sigma_{h,a} = 0.0$$

$@\ z = 2\text{m}$ (above dredge line $\rightarrow$ active pressure only)

just before

$$\sigma_{h,a} = (0 + 16 \times 2) \times 0.333 - 0 = 10.65\ \text{kN/m}^2$$
$$\Delta\sigma_h = \sigma_{h,p} - \sigma_{h,a} = 0 - 10.65 = -10.65\ \text{kN/m}^2$$

just after

$$\sigma_{h,a} = (0 + 16 \times 2) \times 0.27 - 0 = 8.64\ \text{kN/m}^2$$
$$\Delta\sigma_h = \sigma_{h,p} - \sigma_{h,a} = 0 - 8.64 = -8.64\ \text{kN/m}^2$$

$@\ z = 5\text{m}$ (just before $\rightarrow$ active pressure only)

$$\sigma_{h,a} = (0 + 16 \times 2 + 18 \times 3) \times 0.27 - 0 = 23.2\ \text{kN/m}^2$$
$$\Delta\sigma_h = \sigma_{h,p} - \sigma_{h,a} = 0 - 23.2 = -23.2\ \text{kN/m}^2$$

$@\ z = 5\text{m}$ (just after $\rightarrow$ active pressure at right, passive pressure at left)

And there is a value for cohesion($C = 40$) below the drege line

$$\sigma_{h,a} = (0 + 16 \times 2 + 18 \times 3) \times 1 - 2 \times 40 \times \sqrt{1} = 6\text{kN/m}^2$$
$$\sigma_{h,p} = (0 + 0) \times 1 + 2 \times 40 \times \sqrt{1} = 80$$
$$\Delta\sigma_h = \sigma_{h,p} - \sigma_{h,a} = 80 - 6 = 74\ \text{kN/m}^2$$

The positive sign means we draw this value at left side (side of passive pressure because is the largest pressure at this depth.

$@\ z = 5 + D$ (passive pressure at right, active pressure at left)
Active pressure at left side:

$\sigma_{h,a} = (0 + 17 \times D) \times 1 - 2 \times 40 \times \sqrt{1} = 17D - 80$

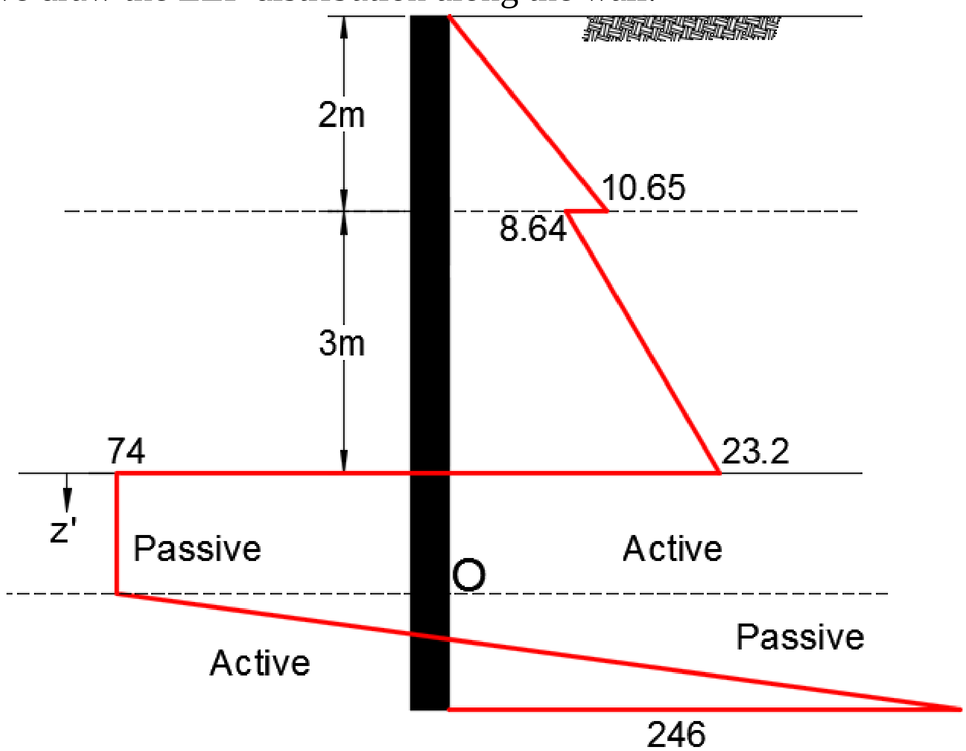
Passive pressure at right side:

$\sigma_{h,p} = (0 + 16 \times 2 + 18 \times 3 + 17 \times D) \times 1 + 2 \times 40 \times \sqrt{1} = 166 + 17D$

$\Delta\sigma_h = \sigma_{h,p} - \sigma_{h,a} = (166 + 17D) - (17D - 80) = 246$

Note that the value of (246) is positive which means we draw this value as a line in the passive zone at this depth (right side).

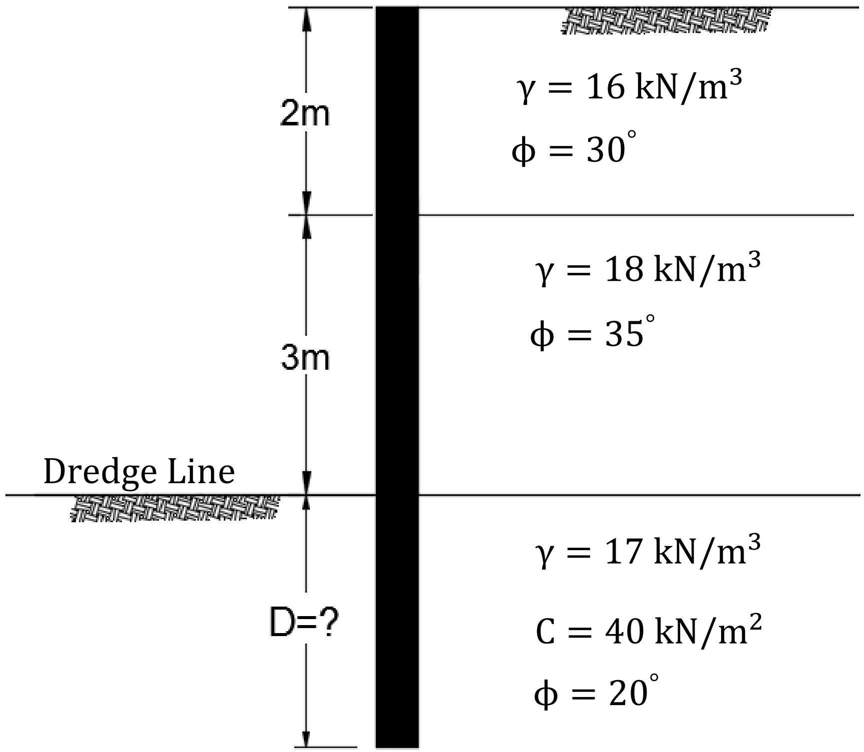
Now, we draw the LEP distribution along the wall:



Now, under dredge line the **net** lateral pressure $\Delta\sigma_h$ always will increase by the value of $\gamma z'(K_p - K_a)$. But here since $(\phi = 0) \rightarrow K_p = K_a = 1$.
So, the increase in pressure will be zero till reaching point O (i.e. the pressure will be constant at this depth)**.** At the specific point (point O) the pressure will changes from active to passive (right side) and thereby the LEP distribution will tend to moves in the direction of passive zone as shown.

Cantilever Sheet Piles Penetrating $C - \phi$ Soil

Consider the following example:



The first step always is calculating K_a and K_p for each layer:

$$K_a = \tan^2\left(45 - \frac{\phi}{2}\right)$$

$$K_{a1} = \tan^2\left(45 - \frac{30}{2}\right) = 0.333$$

$$K_{a2} = \tan^2\left(45 - \frac{35}{2}\right) = 0.27$$

$$K_{a3} = \tan^2\left(45 - \frac{20}{2}\right) = 0.49$$

$$K_p = \tan^2\left(45 + \frac{\phi}{2}\right)$$

$$K_{p1} = \tan^2\left(45 + \frac{30}{2}\right) = 3$$

$$K_{p2} = \tan^2\left(45 + \frac{35}{2}\right) = 3.69$$

$$K_{p3} = \tan^2\left(45 + \frac{20}{2}\right) = 2.04$$

Now we calculate the LEP at each depth:

$$\sigma_{h,a} = (q + \gamma H)K_a - 2c\sqrt{K_a}$$

$$\sigma_{h,p} = (q + \gamma H)K_p + 2c\sqrt{K_p}$$

$@\ z = 0.0$ (above dredge line → active pressure only)

$$\sigma_{h,a} = (0 + 16 \times 0) \times 0.333 - 0 = 0.0$$

$$\Delta\sigma_h = \sigma_{h,p} - \sigma_{h,a} = 0.0$$

$@\ z = 2\text{m}$ (above dredge line → active pressure only)

just before

$$\sigma_{h,a} = (0 + 16 \times 2) \times 0.333 - 0 = 10.65\ \text{kN/m}^2$$

$$\Delta\sigma_h = \sigma_{h,p} - \sigma_{h,a} = 0 - 10.65 = -10.65\ \text{kN/m}^2$$

just after

$$\sigma_{h,a} = (0 + 16 \times 2) \times 0.27 - 0 = 8.64\ \text{kN/m}^2$$

$$\Delta\sigma_h = \sigma_{h,p} - \sigma_{h,a} = 0 - 8.64 = -8.64\ \text{kN/m}^2$$

$@\ z = 5\text{m}$ (just before → active pressure only)

$$\sigma_{h,a} = (0 + 16 \times 2 + 18 \times 3) \times 0.27 - 0 = 23.2\ \text{kN/m}^2$$

$$\Delta\sigma_h = \sigma_{h,p} - \sigma_{h,a} = 0 - 23.2 = -23.2\ \text{kN/m}^2$$

$@\ z = 5\text{m}$ (just after → active pressure at right, passive pressure at left)

And there is a value for cohesion($C = 40$) below the drege line

$$\sigma_{h,a} = (0 + 16 \times 2 + 18 \times 3) \times 0.49 - 2 \times 40 \times \sqrt{0.49} = -13.86\ \text{kN/m}^2$$

$$\sigma_{h,p} = (0 + 0) \times 2.04 + 2 \times 40 \times \sqrt{2.04} = 114.3\ \text{kN/m}^2$$

$$\Delta\sigma_h = \sigma_{h,p} - \sigma_{h,a} = 114.3 - (-13.86) = 128.12\ \text{kN/m}^2$$

$@\ z = 5 + D$ (passive pressure at right, active pressure at left)

Active pressure at left side:

$$\sigma_{h,a} = (0 + 17 \times D) \times 0.49 - 2 \times 40 \times \sqrt{0.49} = 8.33D - 56$$

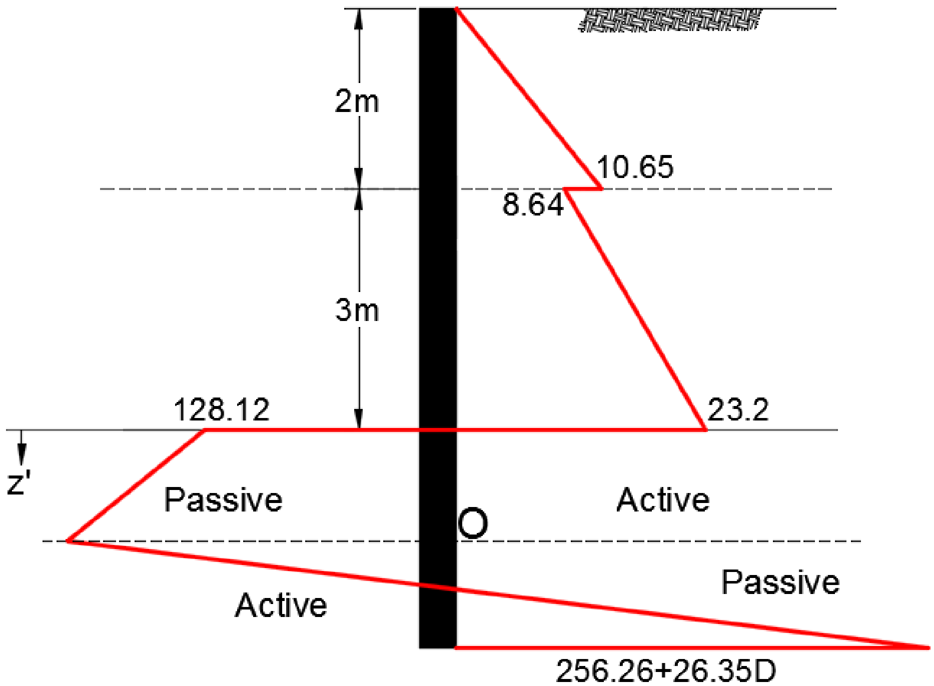
Passive pressure at right side:

$$\sigma_{h,p} = (0 + 16 \times 2 + 18 \times 3 + 17 \times D) \times 2.04 + 2 \times 40 \times \sqrt{2.04}$$
$$= 200.26 + 34.68D$$

$$\Delta\sigma_h = \sigma_{h,p} - \sigma_{h,a} = (200.26 + 34.68D) - (8.33D - 56)$$
$$= 256.26 + 26.35D$$

Note that the value of $(256.26 + 26.35D)$ is positive which means we draw this value as a line in the passive zone at this depth (right side).

Now, we draw the LEP distribution along the wall:



Now, under dredge line the **net** lateral pressure $\Delta\sigma_h$ always will increase by the value of $\gamma z'(K_p - K_a)$. And since $\phi = 20 \rightarrow K_p > K_a$ →the increase in pressure will be in the **direction of passive zones.** So, after the value of 128.12 the stress will be increased gradually in the direction of passive zone till reaching (point O) the pressure will changes from active to passive (right side) and thereby the LEP distribution will tend to moves in the direction of passive zone as shown.

Now If we draw the net LEP distribution correctly, we can calculate the depth of penetration D by applying equilibrium equations:

$$\sum F_x = 0.0 \text{ (Along the sheet pile)}$$

$$\sum M = 0.0 \text{ (At the bottom of sheet pile)}$$

Note:

If M_{max} is required always take a section with distance (x) above point O Because in cantilever sheet piles the maximum moment always above O.

See examples 9.1 and 9.3

Anchored Sheet Piles

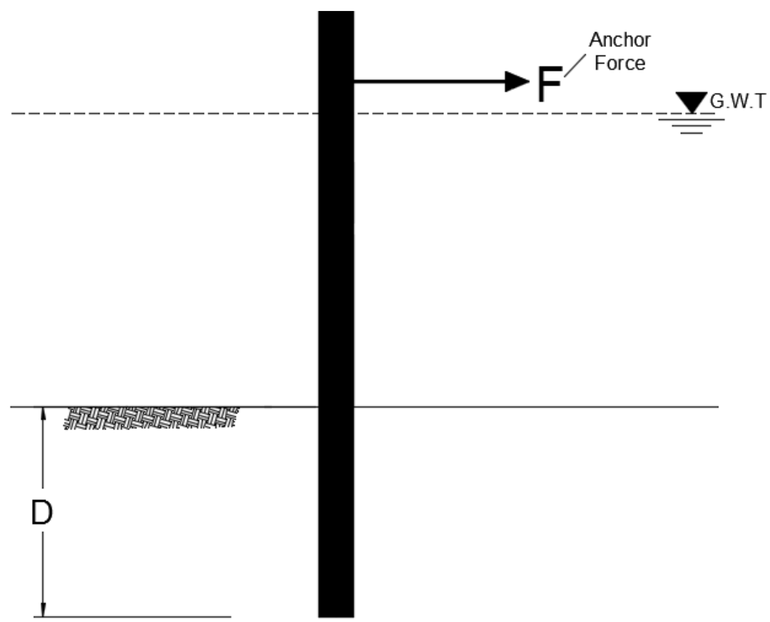
When the height of the backfill soil behind a sheet pile exceeds 6m, the deflection on the sheet pile will be great and thereby the depth of penetration and the section of sheet pile will be large to meet this large deflection. To reduce this deflection, sheet pile should be supported from its upper edge (usually at distance 1m-2m from the top), this support is called anchor and the sheet pile with anchor called **Anchored Sheet Pile.**

There are two ways for analysis of anchored sheet piles:

- **Free Earth Support Method.**
- **Fixed Earth Support Method.**

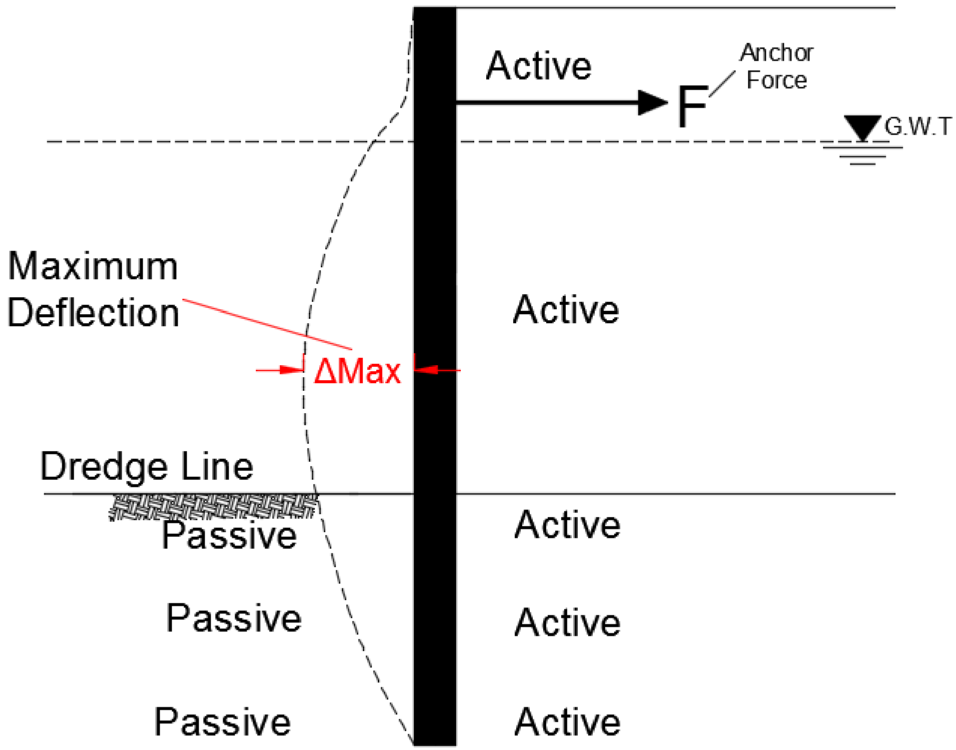
In our discussion we will mainly discuss free earth support method.

The following figure shows anchored sheet pile:



Anchored Sheet pile (Free Earth Support Method)

In this method, the soil is assumed as a simply support (pin support) at the end of sheet pile, and also the wall is simply supported from its upper edge by anchor. So the deflection of sheet pile will be similar to the deflection of simply supported beam as shown in the following figure:



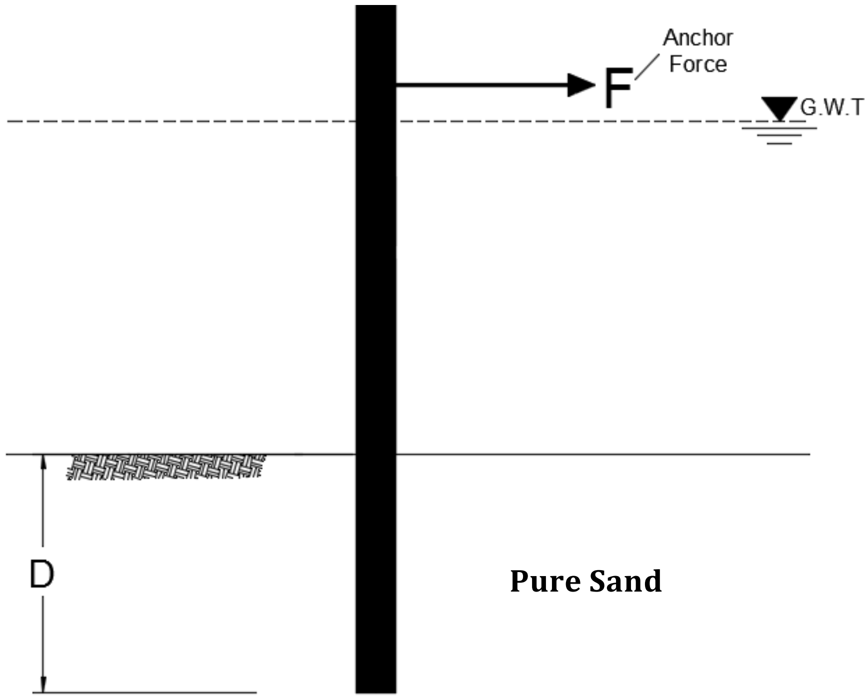
Important notes on the above figure:

1. As you see, the deflection of the sheet pile is similar to the deflection of simply supported beam, so if we need M_{max} we take a section **above the dredge line** (at point of maximum deflection "zero shear").
2. Note that the soil in right side at all depths pushes the wall to the left side, so the soil in the right side will exerts active LEP at all depths of sheet pile and no inflection point (as in cantilever sheet pile), also, under the dredge line, the soil on the wall will pushed into the left side soil, thus the LEP of the left soil is **passive pressure** to the end without any inflection (إنقلاب).
3. Depending on note(2), when we drawing the net pressure distribution under the dredge line, the increase in pressure will be always in the left side (passive side) to the end without any inflection.

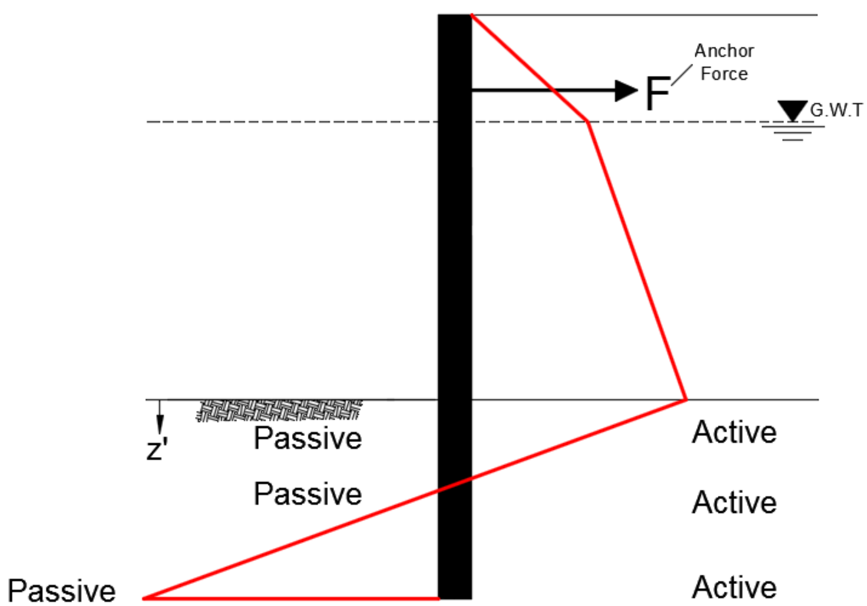
Now, we will sketch the pressure distribution for anchored sheet pile using free earth support method when the sheet pile penetrating in different types of soil.

Anchored Sheet Piles Penetrating Sandy Soil

Consider the following example:



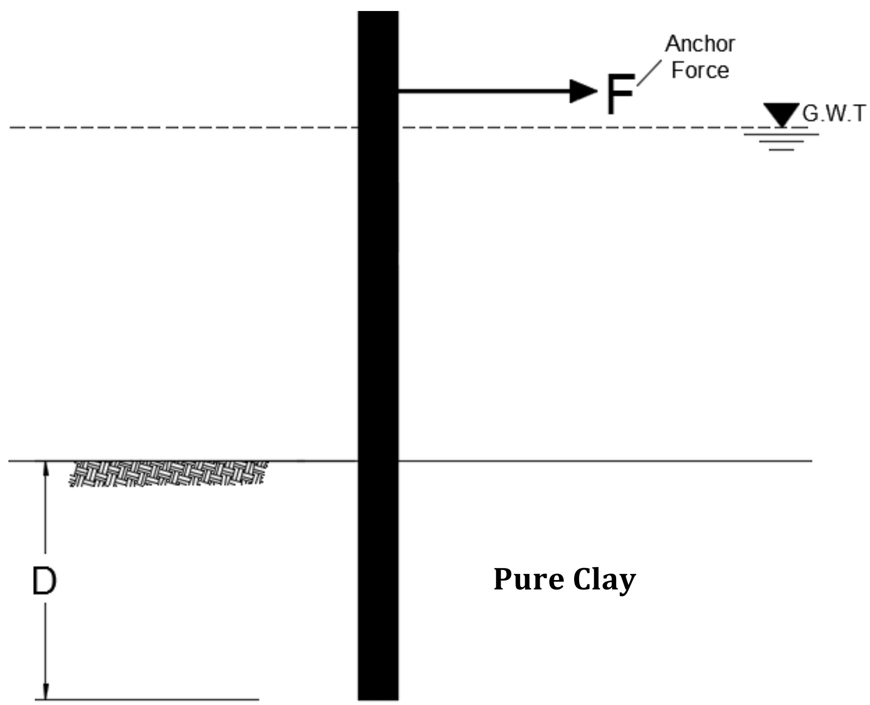
The net pressure distribution along the sheet pile will be as following:



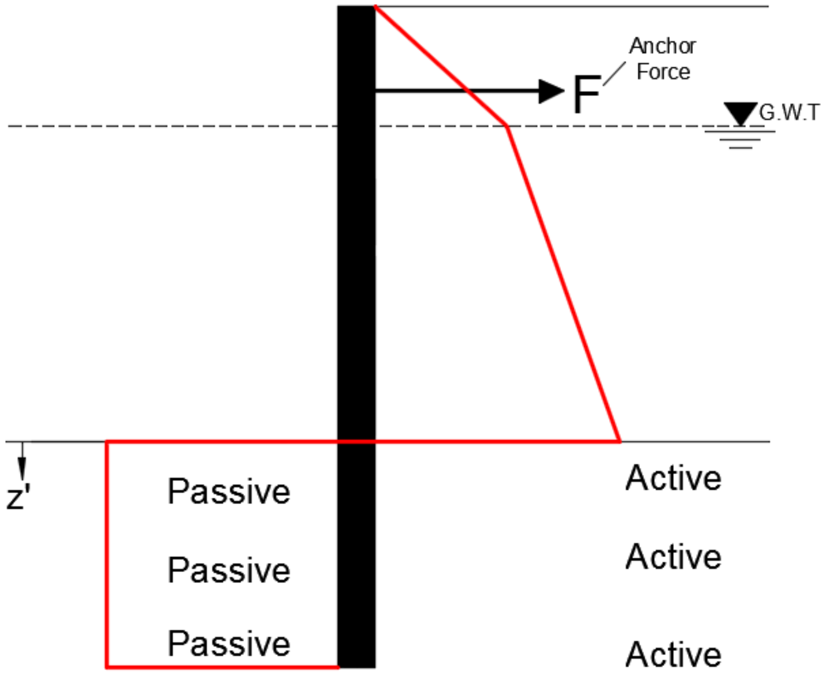
Below the dredge line:
The **net** lateral pressure $\Delta\sigma_h$ always will increased by the value of $\gamma z'(K_p - K_a)$. And since the soil is **pure sand** $(\phi > 0) \rightarrow K_p > K_a$ →the increase in pressure will be in the **direction of passive zones.** So, below dredge line, the stress will be increased gradually in the direction of passive zone **till reaching the end** of sheet pile because there is no inflection point (no change in LEP types) below the dredge line.

Anchored Sheet Piles Penetrating Clay

Consider the following example:



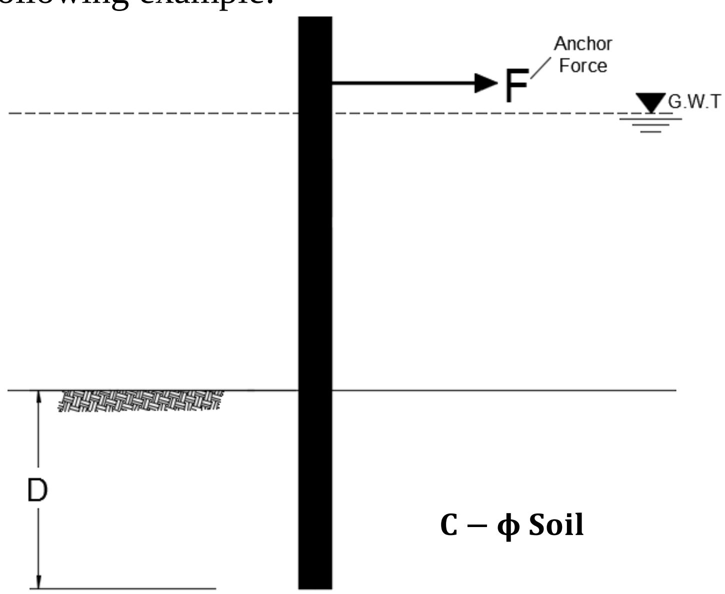
The net pressure distribution along the sheet pile will be as following:



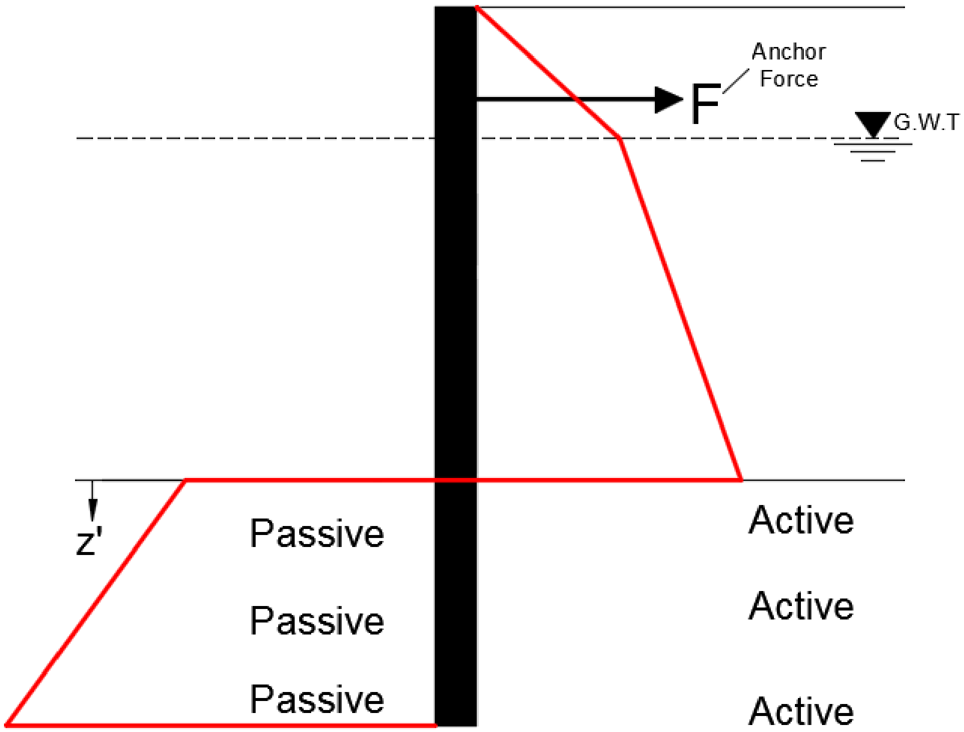
Below the dredge line:
The **net** lateral pressure $\Delta\sigma_h$ always will increased by the value of $\gamma z'(K_p - K_a)$. But since the soil is **pure clay** $(\phi = 0) \rightarrow K_p = K_a \rightarrow$ the increase in pressure will **zero till reaching the end of sheet pile because there is no changes in the type of LEP.**

Anchored Sheet Piles Penetrating $C - \phi$ Soil

Consider the following example:



The net pressure distribution along the sheet pile will be as following:



Below the dredge line:
The **net** lateral pressure $\Delta\sigma_h$always will increase by the value of $\gamma z'(K_p - K_a)$. And since the soil is $\mathbf{C - \phi}$ **soil** $(\phi > 0) \rightarrow K_p > K_a$ →the increase in pressure will be in the **direction of passive zones.** So, below dredge line, the stress will be increased gradually in the direction of passive zone **till reaching the end** of sheet pile because there is no inflection point (no change in LEP types) below the dredge line.

Important Note:
Read the fixed earth support method from textbook page 476 and read example 9.9 pages (477 – 479).

Holding Capacity of Anchor Plates in Sand (Page 488)

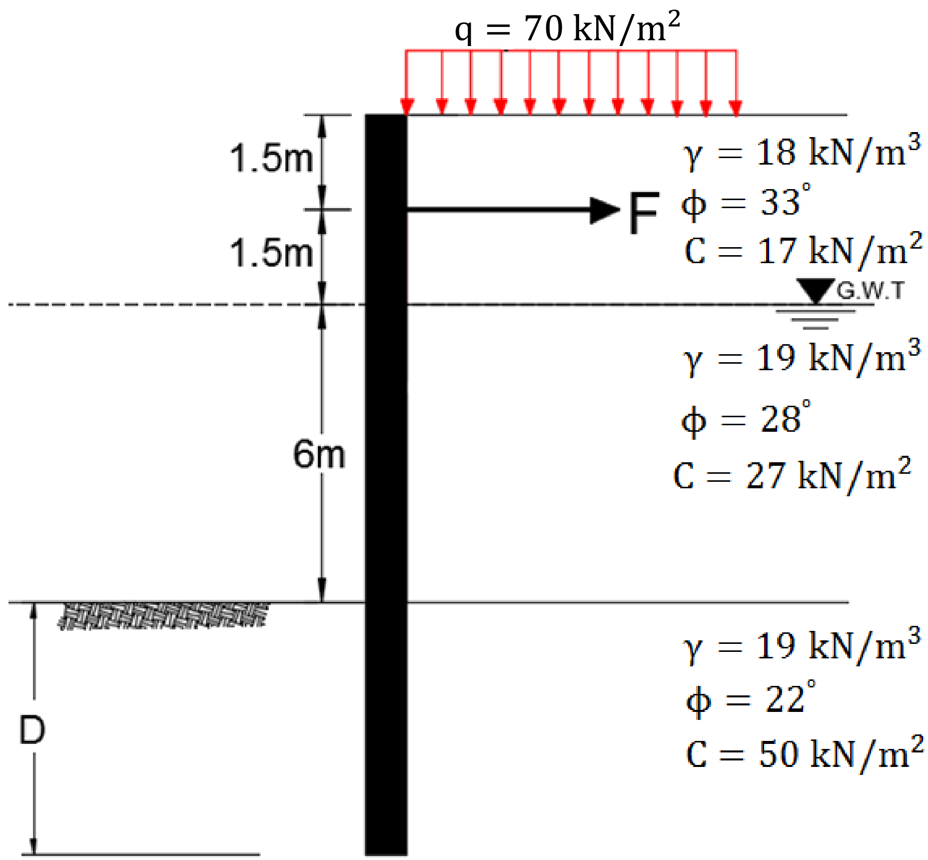
Solve Example 9.11page (493-494) **(Important)**

Problems

1.

For the anchored sheet pile shown below, do the following:

1. Draw the lateral earth pressure distribution with depth.
2. Calculate the depth of penetration (D).
3. Calculate the anchor force per unit length of the sheet pile.
4. Calculate section modulus if σ_{all}=175 MPa.



Solution

The first is calculating K_a and K_p for each layer:

$$K_a = \tan^2\left(45 - \frac{\phi}{2}\right)$$

$$K_{a1} = \tan^2\left(45 - \frac{33}{2}\right) = 0.29$$

$$K_{a2} = \tan^2\left(45 - \frac{28}{2}\right) = 0.36$$

$$K_{a3} = \tan^2\left(45 - \frac{22}{2}\right) = 0.45$$

$$K_p = \tan^2\left(45 + \frac{\phi}{2}\right)$$

The required value of K_p is the value of the third layer (layer below the dredge line)

$$K_{p3} = \tan^2\left(45 + \frac{22}{2}\right) = 2.2$$

Now we calculate the net LEP at each depth:

$$\sigma_{h,a} = (q + \gamma H)K_a - 2c\sqrt{K_a}$$

$$\sigma_{h,p} = (q + \gamma H)K_p + 2c\sqrt{K_p}$$

@ z = 0.0 (above dredge line → active pressure only)

$$\sigma_{h,a} = (70 + 18 \times 0) \times 0.29 - 2 \times 17 \times \sqrt{0.29} = 2\ \text{kN/m}^2$$

$$\Delta\sigma_h = \sigma_{h,p} - \sigma_{h,a} = 0 - 2 = -2\ (2 \text{ in active direction})$$

@ z = 3m (above dredge line → active pressure only)

just before ($K_a = 0.29$, $C = 17$)

$$\sigma_{h,a} = (70 + 18 \times 3) \times 0.29 - 2 \times 17 \times \sqrt{0.29} = 17.65\ \text{kN/m}^2$$

$$\Delta\sigma_h = \sigma_{h,p} - \sigma_{h,a} = 0 - 17.65 = -17.65\ \text{kN/m}^2$$

just after ($K_a = 0.36$, $C = 27$)

$$\sigma_{h,a} = (70 + 18 \times 3) \times 0.36 - 2 \times 27 \times \sqrt{0.36} = 12.24\ \text{kN/m}^2$$

$$\Delta\sigma_h = \sigma_{h,p} - \sigma_{h,a} = 0 - 12.24 = -12.24\ \text{kN/m}^2$$

@ z = 9m (just before → active pressure only)($K_a = 0.36$, $C = 27$)

$$\sigma_{h,a} = (70 + 18 \times 3 + (19 - 10) \times 6) \times 0.36 - 2 \times 27 \times \sqrt{0.36} = 31.68\ \text{kN/m}^2$$

$$\Delta\sigma_h = \sigma_{h,p} - \sigma_{h,a} = 0 - 31.68 = -31.68\ \text{kN/m}^2$$

@ z = 9m (just after → active pressure at right, passive pressure at left)

And($K_a = 0.45$, $K_p = 2.2$, $C = 50$) below the drege line

Active pressure (at right)

$$\sigma_{h,a} = (70 + 18 \times 3 + (19 - 10) \times 6) \times 0.45 - 2 \times 50 \times \sqrt{0.45} = 13\ \text{kN/m}^2$$

Passive pressure (at left)

$$\sigma_{h,p} = (0 + (19 - 10) \times 0) \times 2.2 + 2 \times 50 \times \sqrt{2.2}$$
$$= 148.3\ \text{kN/m}^2$$
$$\Delta\sigma_h = \sigma_{h,p} - \sigma_{h,a} = 148.3 - 13 = 135.3\ \text{kN/m}^2$$

$@\ z = 9 + D$ (Active pressure at right, Passive pressure at left "no inflection")

Active pressure at (at right):

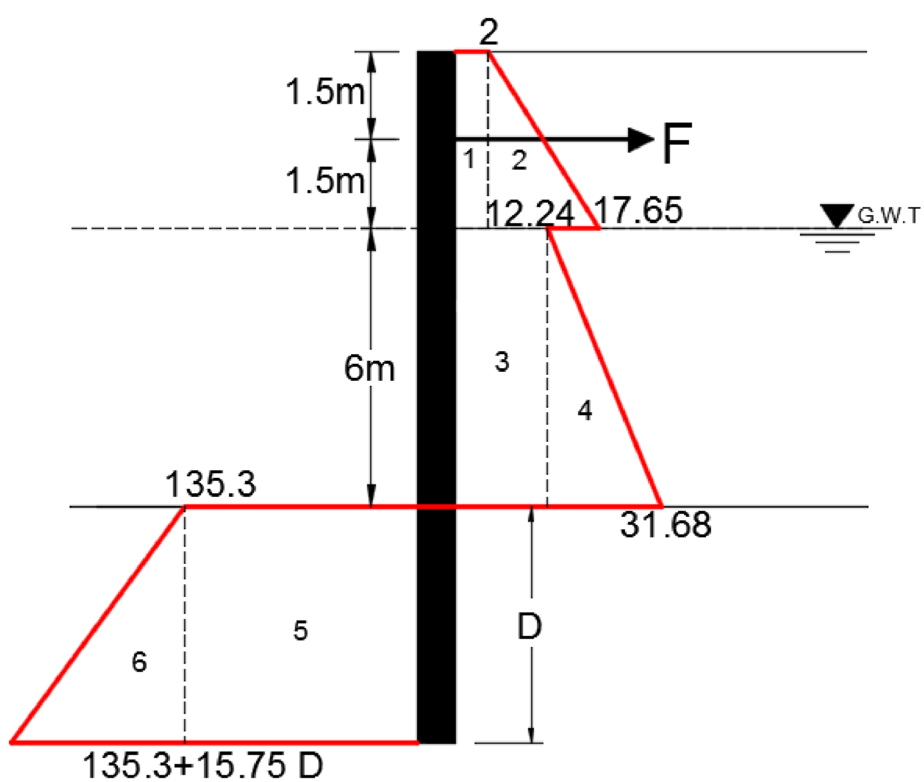
$$\sigma_{h,a} = (70 + 18 \times 3 + (19 - 10) \times 6 + (19 - 10) \times D) \times 0.45$$
$$-2 \times 50 \times \sqrt{0.45} = 13 + 4.05\ D$$

Passive pressure (at left):

$$\sigma_{h,p} = (0 + (19 - 10) \times D) \times 2.2 + 2 \times 50 \times \sqrt{2.2}$$
$$= 148.3 + 19.8\ D$$
$$\Delta\sigma_h = \sigma_{h,p} - \sigma_{h,a} = (148.3 + 19.8\ D) - (13 + 4.05\ D)$$
$$= 135.3 + 15.75\ D$$

Note that the value of $(135.3 + 15.75\ D)$ is positive which means we draw this value as a line in the passive zone at this depth (left side).

Now we draw the net stress distribution along the sheet pile:



As you see, there are two unknowns (D and F).
The most suitable method to find D and F is:

To find D → take $\sum M_{@F} = 0.0$

To find F → take $\sum F_x = 0.0$

Assume forces 1 ,2 ,3 and 4 are positive and forces F , 5 and 6 are negative.
We prepare the following table (to simplified the solution):

Force	Magnitude	Arm from F	Moment about F
P_1	$2 \times 3 = 6\ (+)$	0.0	0.0
P_2	$0.5 \times (17.65 - 2) \times 3 = 23.48\ (+)$	$1.5 - \frac{3}{3} = 0.5$	$11.74\ (+)$
P_3	$12.24 \times 6 = 73.44\ (+)$	$3 + 1.5 = 4.5$	$330.48\ (+)$
P_4	$0.5 \times (31.68 - 12.24) \times 6 = 58.32\ (+)$	$6 - \frac{6}{3} + 1.5 = 5.5$	$320.76\ (+)$
P_5	$135.3\ D\ (-)$	$7.5 + 0.5\ D$	$1014.75D + 67.65D^2\ (-)$
P_6	$0.5 \times (15.75D) \times D = 7.88\ D^2 (-)$	$7.5 + \frac{2}{3}D$	$59.1D^2 + 5.25D^3 (-)$
F	$F\ (-)$	0.0	0.0

$$\sum M_{@F} = 0.0 \rightarrow 11.74 + 330.48 + 320.76 = 1014.75D + 67.65D^2$$
$$+59.1D^2 + 5.25D^3$$
$$\rightarrow 5.25\ D^3 + 126.75\ D^2 + 1014.75\ D - 663 = 0.0$$

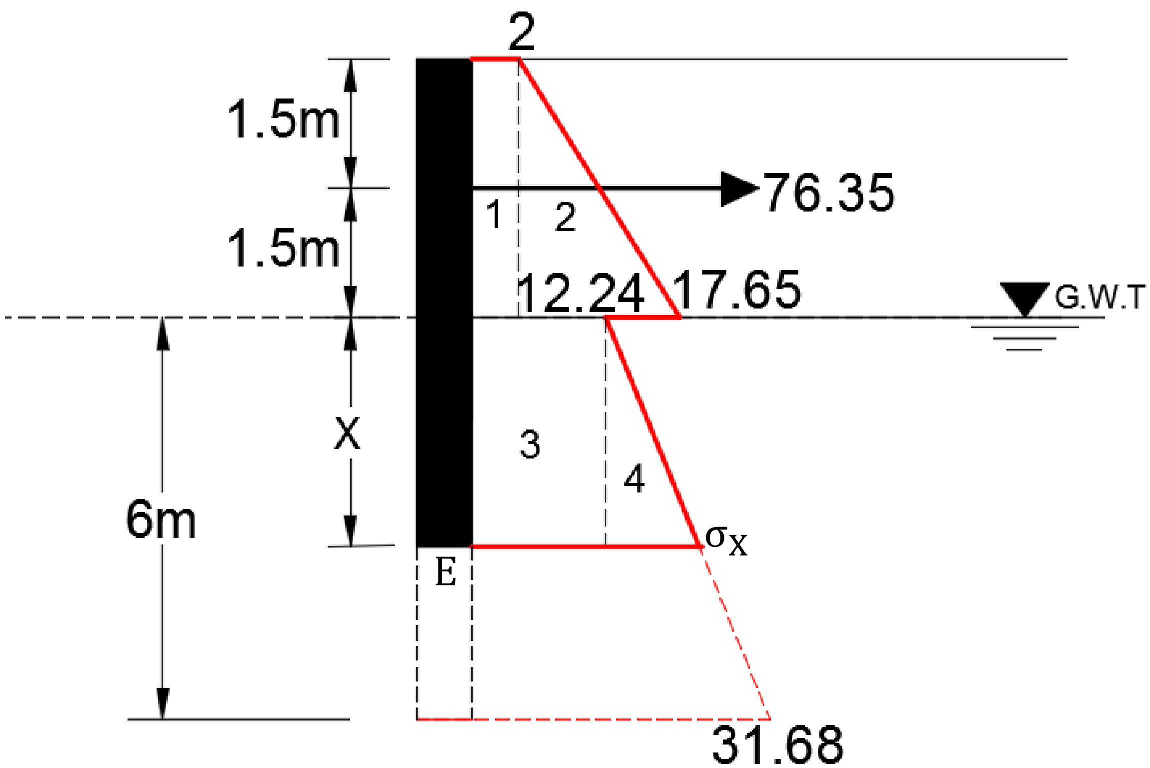
By trial and error or by calculati $\rightarrow D = 0.606\ \text{m}$ ✓.

$$\sum M_{Fx} = 0.0 \rightarrow 6 + 23.48 + 73.44 + 58.32 = 135.3 \times 0.606$$
$$+7.88 \times 0.606^2 + F \rightarrow\rightarrow F = 76.35\ \text{kN}$$ ✓.

Now to find section modulus:

$$S = \frac{M_{max}}{\sigma_{all}}$$

So we need to calculate M_{max}, and we mentioned previously, the maximum moment in anchored sheet pile will be above dredge line (at point of zero shear), so we can make a section above the dredge line and calculate M_{max} as following:



Now, to calculate the areas 3 and 4 in terms of X we must calculate the pressure at the distance X (σ_X) by interpolation.

$@X = 0.0 \rightarrow \sigma = 12.24$, $@X = 6 \rightarrow \sigma = 31.68$ $@X = X \rightarrow \sigma = \sigma_X$

$$\frac{31.68 - 12.24}{6 - 0} = \frac{\sigma_X - 12.24}{X - 0} \rightarrow \sigma_X = 3.24\,X + 12.24$$

Now we can calculate the force 3 and 4 in terms of X:

$P_3 = 12.24\,X$

$P_4 = 0.5 \times (\sigma_X - 12.24)X$

$\quad = 0.5 \times (3.24\,X + 12.24 - 12.24)X = 1.62X^2$

Now at distance X, summation forces must be zero (point of zero shear) to get maximum moment:

$F = P_1 + P_2 + P_3 + P_4$ (F, P_1 and P_2 are taken from table above)

$76.35 = 6 + 23.48 + 12.24\,X + 1.62\,X^2$

$\rightarrow 1.62\,X^2 + 12.24\,X - 46.87 = 0.0$

$\rightarrow X = 2.8\text{ m}$

$\rightarrow P_3 = 12.24 \times 2.8 = 34.27$ and $P_4 = 3.24 \times 2.8^2 = 25.4$

Now to get maximum moment, take summation moment at point E (point of zero shear):

$$M_{max} = \sum M_{@E}$$

$$M_{max} = 76.35 \times (1.5 + 2.8) - 6 \times (1.5 + 2.8) - 23.48 \times \left(\frac{3}{3} + 2.8\right)$$

$$-34.27 \times \left(\frac{2.8}{2}\right) - 25.4 \times \left(\frac{2.8}{3}\right)$$

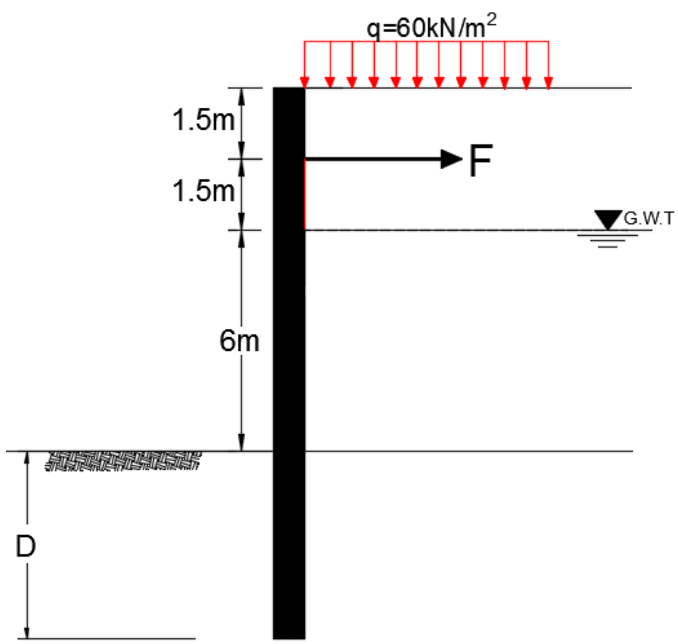
$$\rightarrow\rightarrow M_{max} = 141.6 \text{ kN.m}$$

$$\sigma_{all} = 175 \text{ MPa.} = 175{,}000 \text{ kPa.} \left(\text{kN/m}^2\right)$$

$$S = \frac{M_{max}}{\sigma_{all}} = \frac{141.6}{175000} = 0.81 \times 10^{-3} \ \text{m}^3/\text{m ofwall} \checkmark.$$

Important Note:

In the above problem, the water table is at both sides of sheet pile, so pore water pressure will canceled from both sides, however if the water table is at one side of the sheet pile (right side) as shown in the following figure:



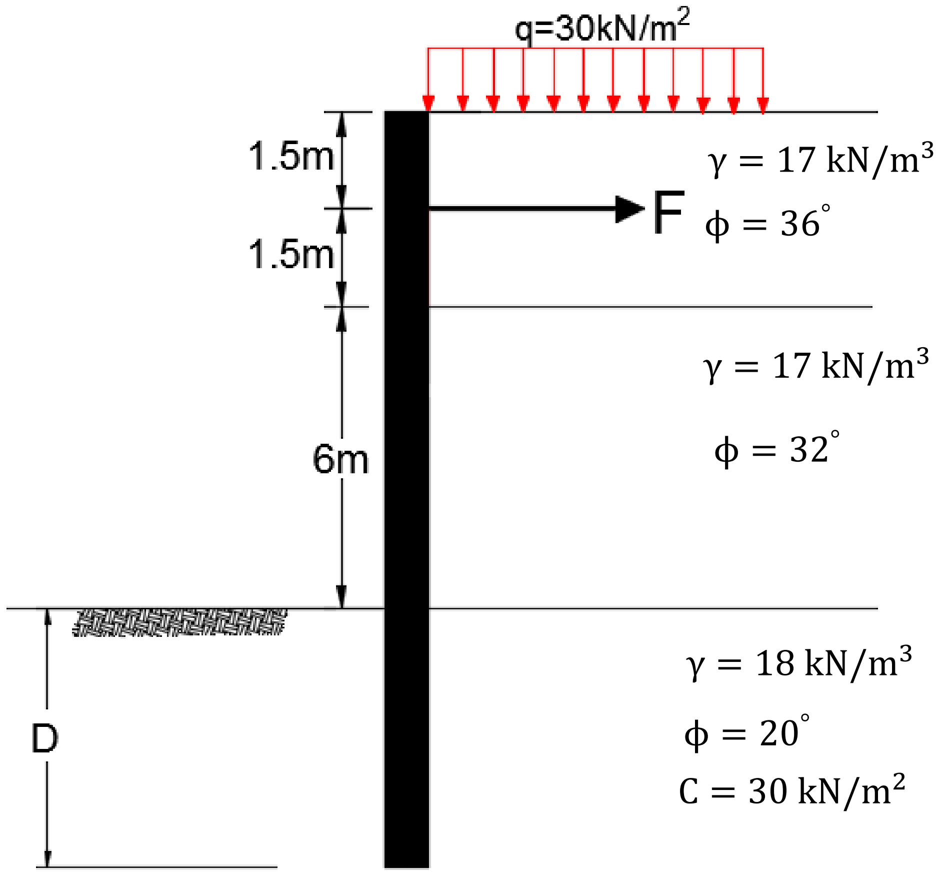
When calculating the pressure at right side (active pressure), pore water pressure must be added as following:

$$\sigma_{h,a} = (q + \gamma H)K_a - 2c\sqrt{K_a} + \gamma_w \times h_w \quad \text{(at each depth below the GWT)}$$

2.

For the anchored sheet pile shown below, do the following:

1. Draw the lateral earth pressure distribution with depth.
2. Calculate the depth of penetration (D).
3. Calculate the anchor force per unit length of the sheet pile.
4. Calculate Maximum moment.



Solution

Note that there is no water table in this problem.

Solve the problem by yourself ☺ with the same procedures in the problem above.

Final Answers:

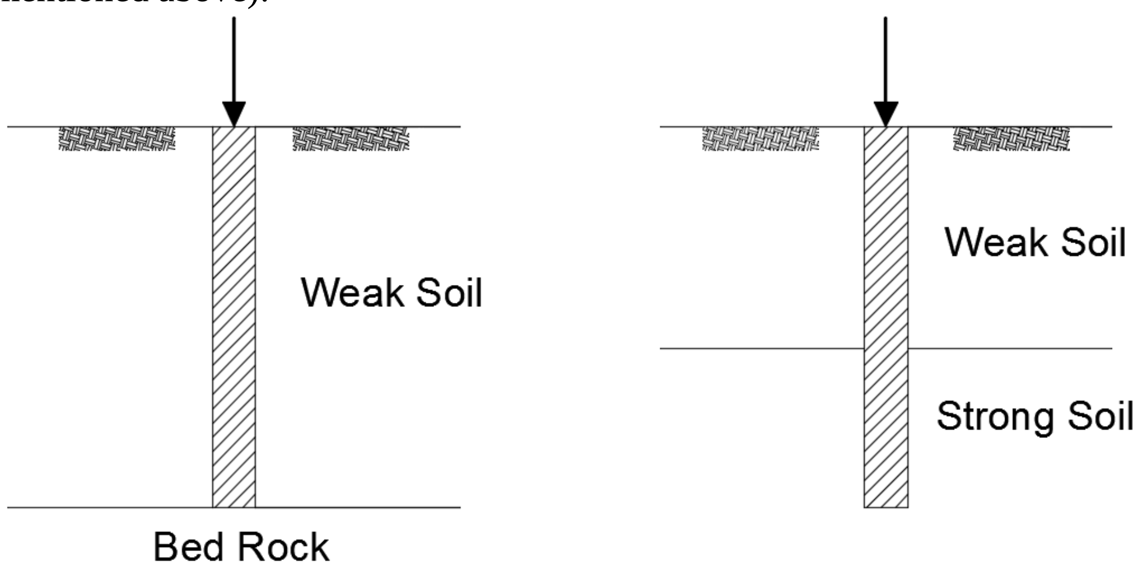
$D = 2.06\ \text{m} \qquad F = 149\ \text{kN} \qquad M_{max} = 351\ \text{kN.m}$

See Examples 9.5 and 9.10 in your textbook

Chapter (11)
Pile Foundations

Introduction

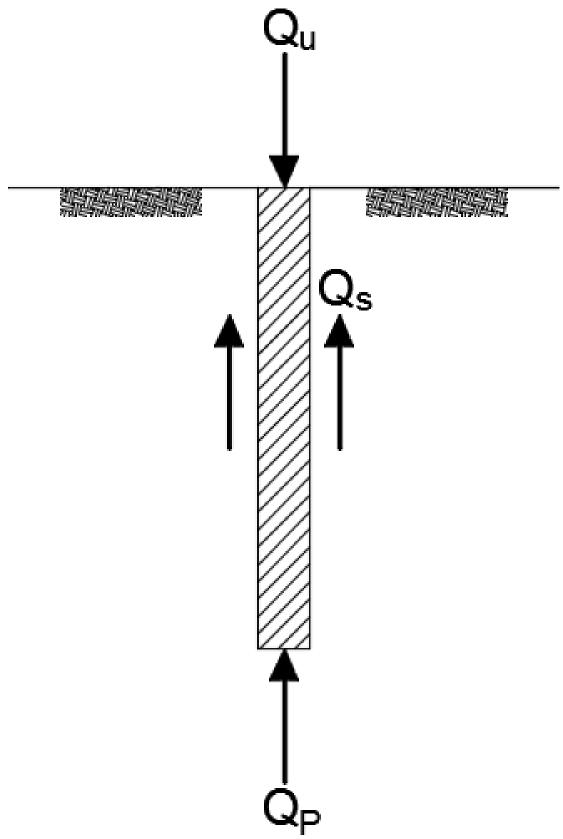
Piles are structural members that are made of steel, concrete, or timber. They are used to build pile foundations (classified as deep foundations) which cost more than shallow foundations (discussed in Chapters 3, 4, 5, and 6).
Despite the cost, the use of piles often is necessary to ensure structural safety. The most case in which pile foundations are required, is when the soil supporting the structure is **weak soil** (expansive soil, or collapsible soil, etc...) we use piles to transmit the foundation load to the nearest bed rock layer, and if bed rock is not encountered, we use piles to transmit the load to the nearest stronger soil layer to ensure the safety for the structure.
The following figure clarifies the function of pile foundation (which mentioned above):



Capacity of Piles

The ultimate load capacity of the pile may be expressed as:

$Q_u = Q_P + Q_s$



When the pile penetrates weak soil to rest on strong soil or bed rock, the pile will supported by the bed rock or the strong soil from at the pile end (end point of pile), So:

$Q_P =$ Load carried at the pile end point

In addition, when the pile penetrates the soil, the shearing resistance between the soil and the pile should be considered in Q_s where:

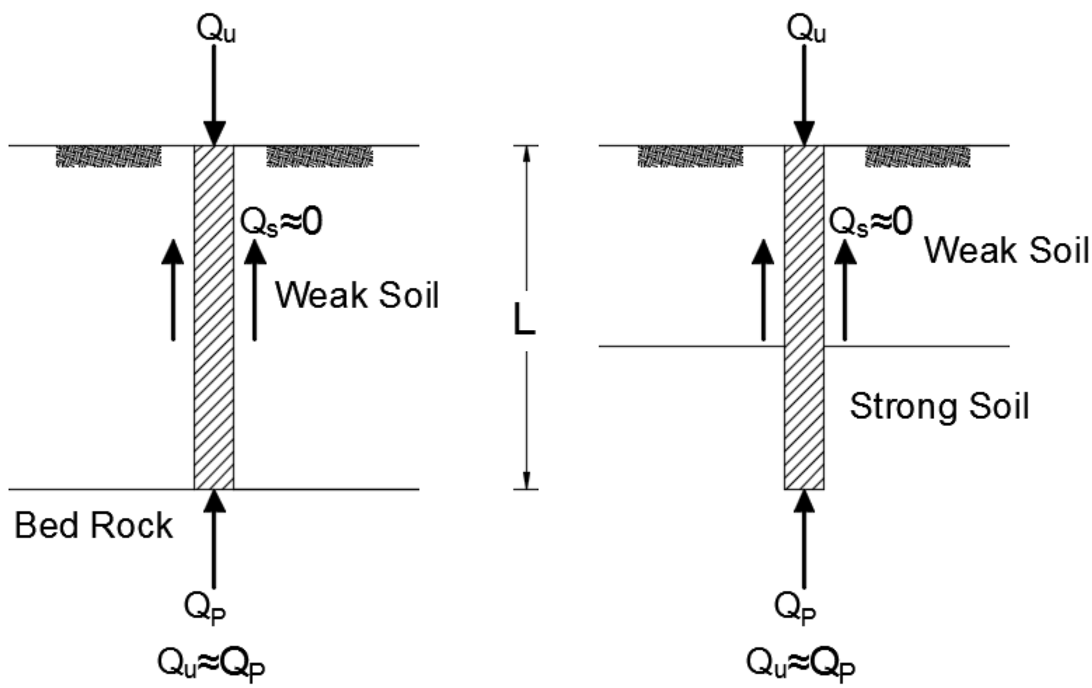
$Q_s =$ Load carried by the skin friction developed at the sides of the pile (caused by shearing resistance between the soil and the pile)

Types of Pile

1. Point Bearing Piles:

If the soil supporting the structure is weak soil, pile foundation will used to transmit the load to the strong soil layer or to the bed rock (if encountered), here the pile will resist the entire load depending on its end point load Q_P and the value of Q_s (frictional resistance) is very small in this case, so:

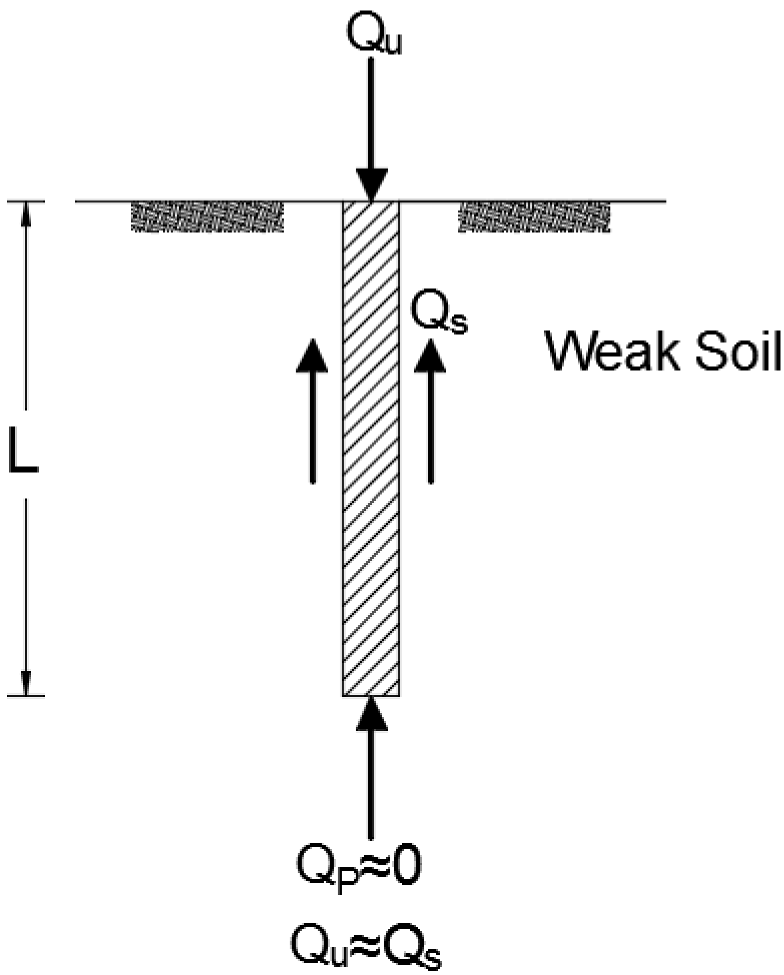
$Q_u = Q_P + Q_s \quad Q_s \approx 0.0 \rightarrow \; Q_u = Q_P$ (Point Bearing Piles)



2. Friction Piles:

When no strong layer or rock is present at reasonable depth at a site, point bearing piles becomes very long (to reach strong layer) and uneconomical. In these type of soil profiles, piles are driven through the softer (weaker) soil to specified depth, and here the point bearing load (Q_P) is very small and can be considered zero, however the load on the pile will resisted mainly by the frictional resistance between soil and pile (Q_s) so:

$Q_u = Q_P + Q_s \quad Q_P \approx 0.0 \rightarrow \; Q_u = Q_s$ (Friction Piles)



In practice, we assume the pile resist the applied loads by its point bearing load and its frictional resistance to estimate the ultimate load the pile can carry.

$$Q_u = Q_P + Q_s$$

In the following sections, we will learn how to calculate the value of Q_P and Q_s and thereby Q_u for sand and clay and $C - \phi$ soil

Calculation of Point Bearing Load(Q_P)

We will use **Meyerhof method** to calculate the value of Q_P for sand and clay.

Calculation of Q_P for sand:

$Q_P = A_P \times q_P \leq Q_L$

A_P =Cross-sectional area of the **end point** of the pile (**bearing area** between pile and soil).

$q_P = q' \times N_q^*$

$q' =$ **Effective vertical stress** at the level of the end of the pile.

$N_q^* =$ Load capacity Factor (depends only on $\phi -$ value)

N_q^* is calculated from (**Figure 11.13** P. 557) or (**Table 11.15** P. 558)

$Q_L =$ Limiting value for point resistance

$Q_L = 0.5 \times A_P \times p_a \times N_q^* \times \tan\phi$

$p_a =$ atmospheric pressure $= (100\ kN/m^2$ or $2000\ Ib/ft^2)$

So, for sandy soil the value of Q_P is:

$Q_P = A_P \times q' \times N_q^* \leq 0.5 \times A_P \times p_a \times N_q^* \times \tan\phi$

Important Note:

The soil profile may consists of several sand layers, the value of friction angle (ϕ) which used to calculate Q_P as shown in the above equation is the friction angle for the soil that **supporting the pile end** (for last soil layer).

Calculation of Q_P for Clay:

$Q_P = A_P \times c_u \times N_c^*$

$c_u =$ Cohesion for the soil supported the pile at its end.

$N_c^* =$ Bearing capacity factor for clay $= 9$ (when $\phi = 0.0$)

$Q_P = 9 \times A_P \times c_u$

Calculation of Q_P for $C - \phi$ Soile:

If the supporting the pile from its end is $C - \phi$ soil:

$Q_P = (A_P \times q' \times N_q^* \leq 0.5 \times A_P \times p_a \times N_q^* \times \tan\phi) + A_P \times c_u \times N_c^*$

But here, the value of $\phi \neq 0.0 \rightarrow N_c^* \neq 9$ (you will given it according ϕ)

But, if you are not given the value of N_c^* at the existing value of $\phi \rightarrow$ Assume $N_c^* = 9$ and complete the solution.

Calculation of Frictional Resistance (Q_s)

Calculation of Q_s for sand:

The general formula for calculating Q_s is:

$$Q_s = P \times \sum f_i \times L_i$$

$P = \text{pile perimeter} = \pi \times D\ (\text{if the pile is circular}, D = \text{Pile diameter})$

$= 4 \times D\ (\text{if the pile is square}, D = \text{square dimension})$

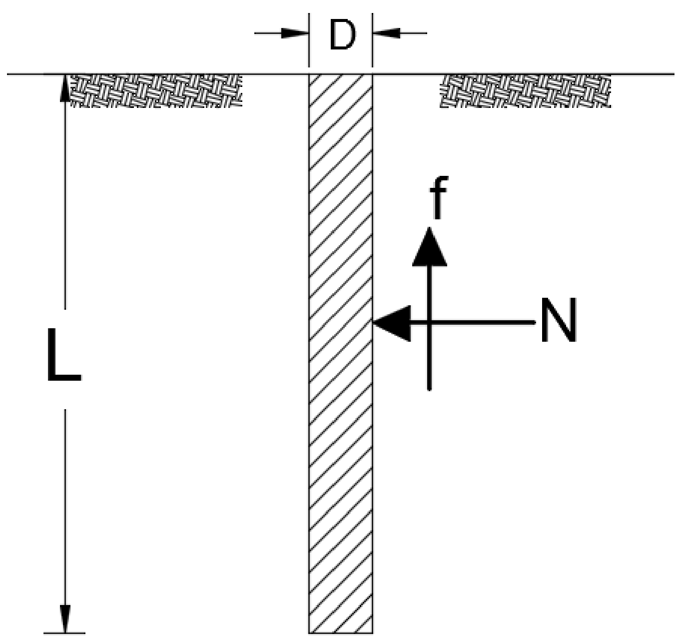
$f_i = \text{unit friction resistance at any depth}$

$L_i = \text{depth of each soil layer}$

Now, how we calculate the value of f_i (for each soil layer):

$f = \mu_s \times N$

Here the value of (f) is vertical, so N must be perpendicular to f (i.e. N must be horizontal) as shown in the following figure:



$\mu_s = \text{friction coefficient between soil and pile} = \tan\delta$

$\delta = \text{soil} - \text{pile friction angle} = 0.8\phi \rightarrow \mu_s = \tan(0.8\phi)\ (\text{for each layer})$

N = **Horizontal stress** from the soil to the pile

$\rightarrow N = \sigma'_v \times K\ (\text{for each soil layer})$

$\sigma'_v = \text{vertical effective stress for each layer}$

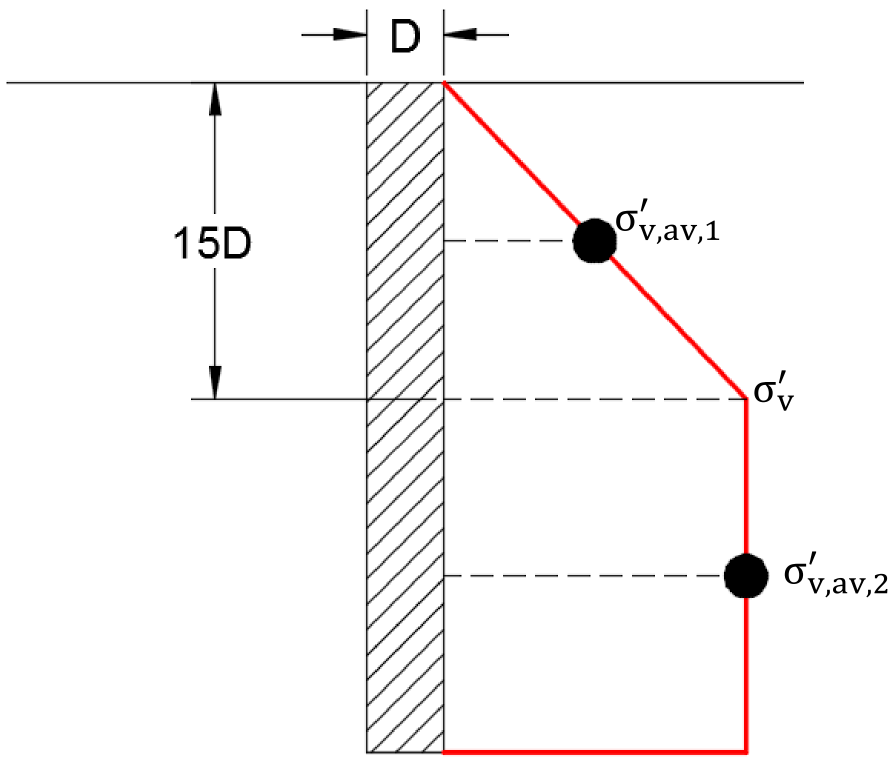
But, to calculate σ'_v for each soil layer, to be representative, we take the average value for σ'_v for each layer.

$K = \text{Effective earth pressure coefficient}$

$K = 1 - \sin\phi$ or $K = 0.5 + 0.008\,D_r$ $\quad D_r =$ relative density (%)
If you are not given the relative density for each layer, use $K = 1 - \sin\phi$ or you may given another formula to calculate K.

Now, $N = \sigma'_{v,av} \times K$ (for each soil layer)
$\rightarrow\rightarrow f = \tan(0.8\phi) \times \sigma'_{v,av} \times K$ (for each soil layer)
Now, how we calculate the value of $\sigma'_{v,av}$ for each soil layer:
We draw the vertical effective stress along the pile, **but** the stress will linearly increase to a depth of (15D), after this depth the stress will be **constant** and will not increase. (this is true only if we deal with sandy soil).

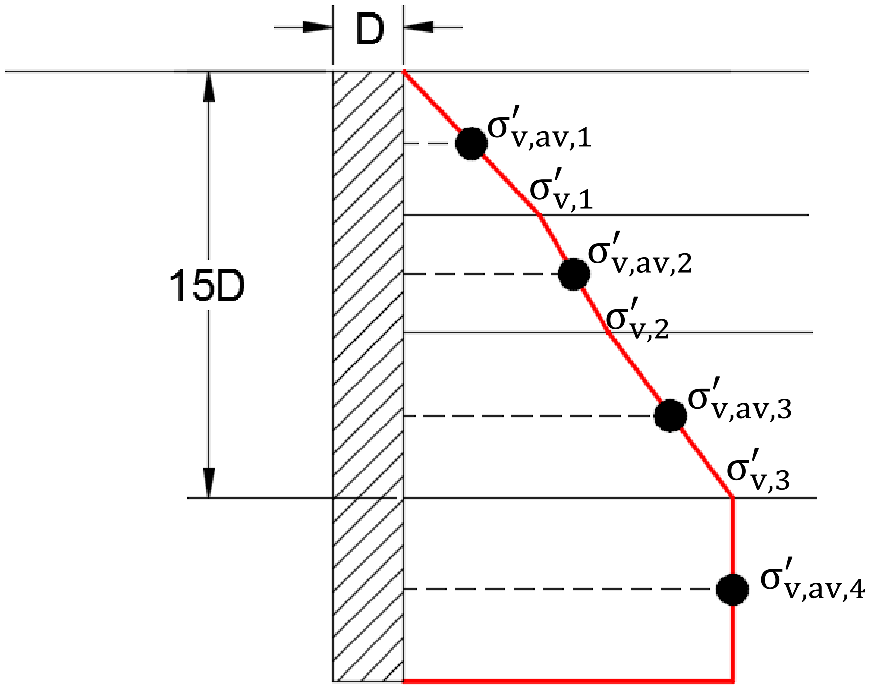
If there is one soil layer before reaching 15D:



$$\sigma'_{v,av,1} = \frac{0 + \sigma'_v}{2} = 0.5\sigma'_v$$

$$\sigma'_{v,av,2} = \frac{\sigma'_v + \sigma'_v}{2} = \sigma'_v$$

If there are more than one soil layer before reaching 15D:



$$\sigma'_{v,av,1} = \frac{0 + \sigma'_{v,1}}{2} = 0.5\sigma'_{v,1}$$

$$\sigma'_{v,av,2} = \frac{\sigma'_{v,1} + \sigma'_{v,2}}{2}$$

$$\sigma'_{v,av,3} = \frac{\sigma'_{v,2} + \sigma'_{v,3}}{2}$$

$$\sigma'_{v,av,4} = \frac{\sigma'_{v,3} + \sigma'_{v,3}}{2} = \sigma'_{v,3}$$

Finally we can calculate the value of Q_s as following:

$$Q_s = P \times \sum \tan(0.8\phi_i) \times \sigma'_{v,av,i} \times K_i \times L_i$$

$i =$ each soil layer

Note:

We take soil layer every change in soil properties or every change in slope of vertical stress.

Calculation of Q_s for clay:

There are three methods used to calculate Q_s in clay:

1. $\boldsymbol{\lambda}$ Method

$$Q_s = P \times \sum f_i \times L_i$$

But here we take the entire length of the pile:

$$Q_s = P \times L \times \sum f_i$$

$$\sum f_i = f_{av} = \lambda \times \left(\sigma'_{v,av} + 2\, c_{u,av}\right)$$

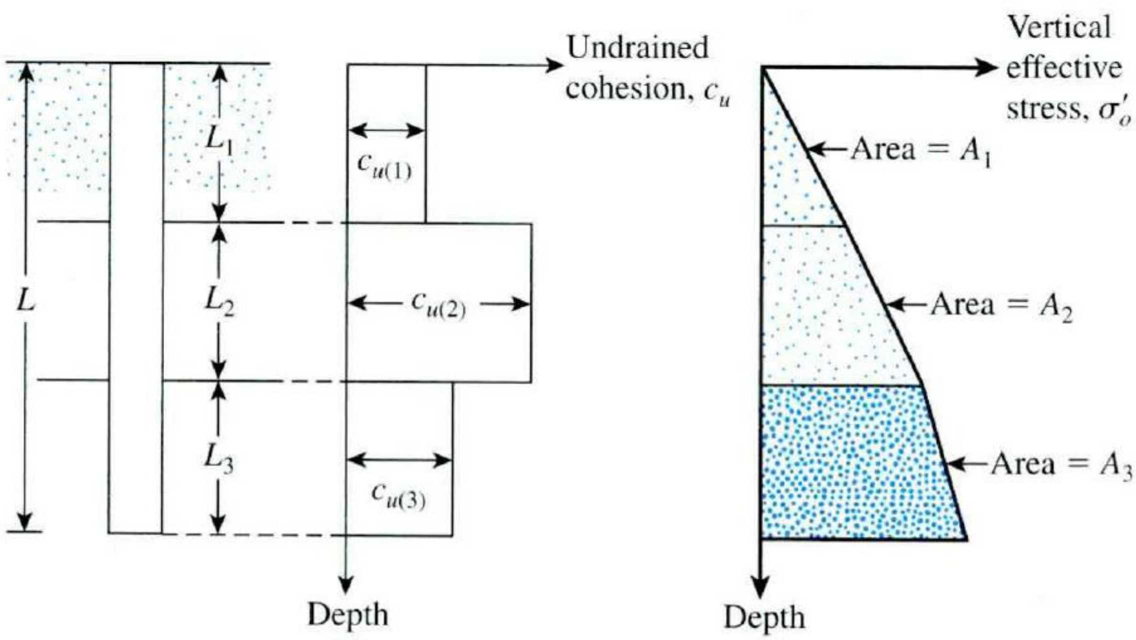
$\sigma'_{v,av}$ = mean effective vertical stress for the entire embedment length

$c_{u,av}$ = mean undrained shear strength for the entire embedment length

λ = function of pile length (L) (calculated from **Table 11.9** P. 576)

Calculation of $\boldsymbol{\sigma'_{v,av}}$ and $\boldsymbol{c_{u,av}}$:

We prepare the following graph (assuming three soil layers):



Note that the soil is clay, and the stress is not constant after 15D, the stress is constant (after 15D) in sand only.

$$c_{u,av} = \frac{L_1 \times c_{u,1} + L_2 \times c_{u,2} + L_3 \times c_{u,3}}{L}$$

$$\sigma'_{v,av} = \frac{A_1 + A_2 + A_3}{L}$$

2. α Method

$$Q_s = P \times \sum f_i \times L_i$$

$$f_i = \alpha_i \times c_{u,i}$$

$\alpha_i = \text{function of} \left(\frac{c_{u,i}}{p_{atm}}\right)$ (calculated from **Table 11.10** P. 577)

$$Q_s = P \times \sum \alpha_i \times c_{u,i} \times L_i$$

3. β Method

$$Q_s = P \times \sum f_i \times L_i$$

$$f_i = \beta_i \times \sigma'_{v,av,i}$$

$\sigma'_{v,av,i}$ = average vertical effective stress for each clay layer

$$\beta_i = K_i \times \tan\phi_{R,i}$$

ϕ_R = drained friction angle of remolded clay (given for each layer)

K_i = earth pressure coefficient for each clay layer

$K = 1 - \sin\phi_R$ (for normally consolidated clay)

$K = (1 - \sin\phi_R) \times \sqrt{OCR}$ (for overconsolidated clay)

Important Note:

If the soil is $(C - \phi)$soil, we calculate Q_s for sand alone and for clay alone and then sum the two values to get the total Q_s

Now, from all above methods, we can calculate the ultimate load that the pile could carry:

$$Q_u = Q_p + Q_s$$

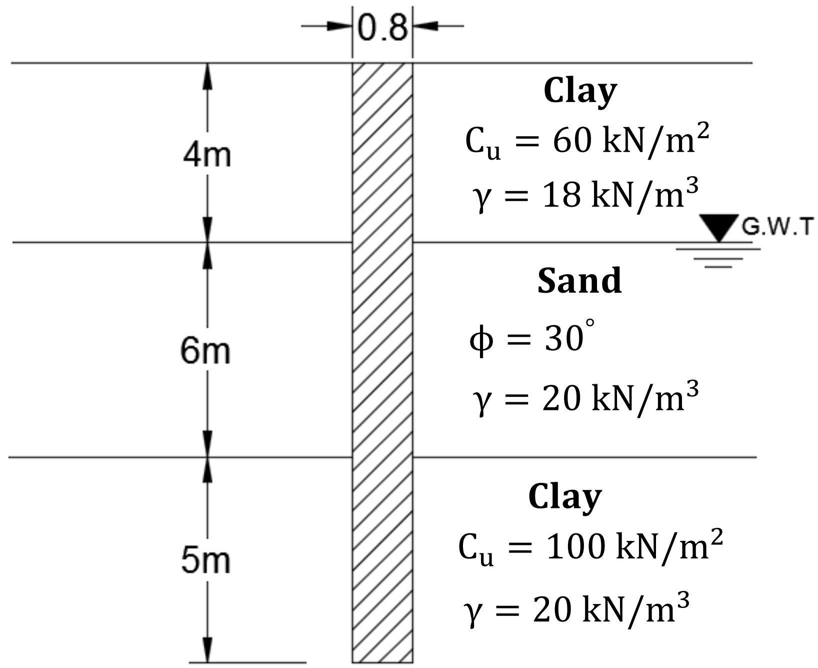
If we want to calculate the allowable load:

$$Q_{all} = \frac{Q_u}{FS} \quad (FS \geq 3)$$

Problems

1.

Determine the ultimate load capacity of the 800 mm diameter concrete bored pile given in the figure below.



Solution

Calculation of Q_P:

Note that the soil supporting the pile at its end is clay, so:

$Q_P = A_P \times c_u \times N_c^* \qquad N_c^* = 9$ (pure clay $\phi = 0.0$)

$$A_P = \frac{\pi}{4} \times 0.8^2 = 0.502\ m^2$$

$c_u = 100\ kN/m^2$ (for the soil supporting the pile at its end)

$Q_P = 0.502 \times 100 \times 9 = 452.4\ KN$

$Q_L = 0.0$ since $\tan\phi = 0.0 \rightarrow Q_P = 452.4\ KN$

Calculation of Q_s:

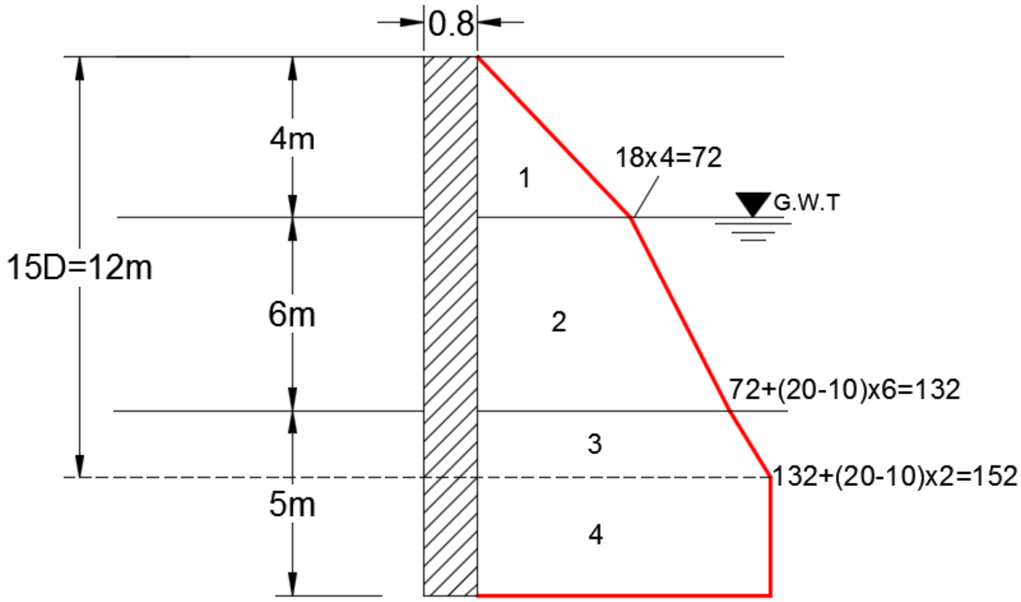
Since there are one sand layer and two clay layer, we solve firstly for sand and then for clay:

For sand:

The stress will increase till reaching 15D

$15D = 15 \times 0.8 = 12m$

Now we draw the vertical effective stress with depth:



$$Q_s = P \times \sum (\tan(0.8\phi_i) \times \sigma'_{v,av,i} \times K_i) \times L_i$$

Note that the value of ϕ for layers 1, 3, and 4 is zero (clay), so we calculate Q_s only for the layer 2 (sand layer).

$P = \pi \times D = \pi \times 0.8 = 2.51\text{ m}$

$$\sigma'_{v,av,2} = \frac{72 + 132}{2} = 102\ \text{kN/m}^2$$

$L_2 = 6\text{m}$

$\phi_2 = 30^{\circ}$

$K_2 = 1 - \sin\phi_2 = 1 - \sin 30 = 0.5$

$\rightarrow Q_{s,sand} = 2.51 \times (\tan(0.8 \times 30) \times 102 \times 0.5) \times 6 = 342\text{ kN}$

For Clay:

If we want to use λ Method:

$Q_s = P \times L \times f_{av}$

$f_{av} = \lambda \times \left(\sigma'_{v,av} + 2\, c_{u,av}\right)$

$P = 2.51\text{m} \quad L = 4 + 6 + 5 = 15\text{m}$

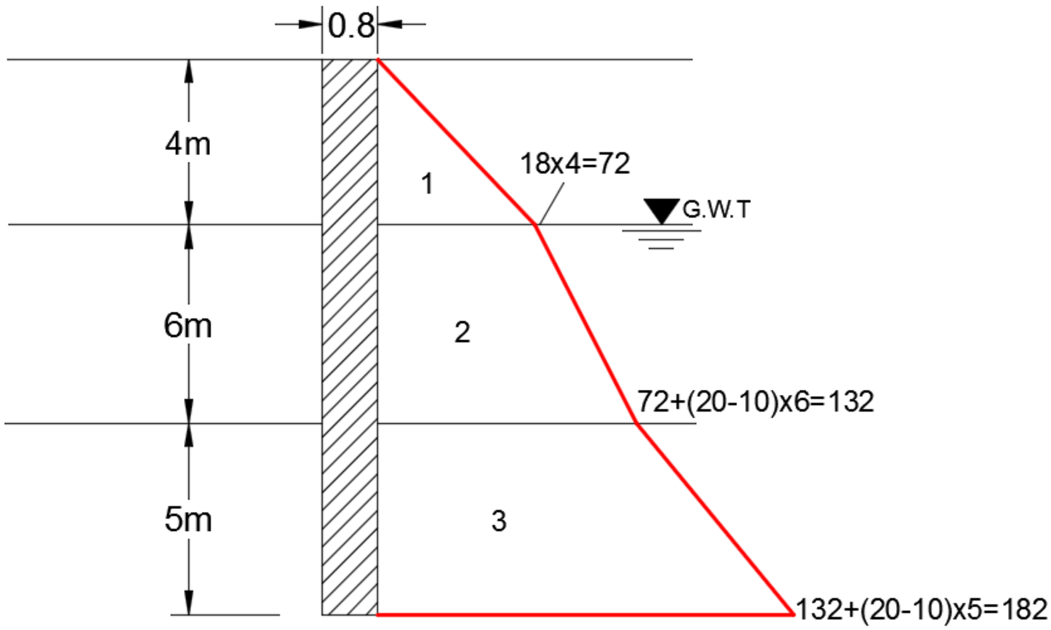
$\lambda = 0.2$ (at $L = 15\text{m}$ from Table 11.9)

$$c_{u,av} = \frac{L_1 \times c_{u,1} + L_2 \times c_{u,2} + L_3 \times c_{u,3}}{L}$$

$$c_{u,av} = \frac{4 \times 60 + 6 \times 0 + 5 \times 100}{15} = 49.33\ \text{kN/m}^2$$

$$\sigma'_{v,av} = \frac{A_1 + A_2 + A_3 + \cdots A_n}{L}$$

We draw the vertical effective pressure with depth:



$$A_1 = \frac{1}{2} \times 72 \times 4 = 144$$

$$A_2 = \frac{1}{2} \times (72 + 132) \times 6 = 612$$

$$A_3 = \frac{1}{2} \times (132 + 182) \times 5 = 785$$

$$\sigma'_{v,av} = \frac{144 + 612 + 785}{15} = 102.73\ \mathrm{kN/m^2}$$

$$f_{av} = 0.2 \times (102.73 + 2\ \times 49.33) = 40.28$$

$$\rightarrow Q_{s,clay} = 2.51 \times 15 \times 40.28 = 1516.54\ \mathrm{kN}$$

If we want to use α Method:

$$Q_s = P \times \sum \alpha_i \times c_{u,i} \times L_i$$

For layer (1)

$$c_{u,1} = 60 \rightarrow \frac{c_{u,1}}{p_{atm}} = \frac{60}{100} = 0.6 \rightarrow \alpha_1 = 0.62$$

For layer (2)

$$c_{u,2} = 0.0 \rightarrow \frac{c_{u,2}}{p_{atm}} = \frac{0}{100} = 0 \rightarrow \alpha_2 = 0$$

For layer (3)

$$c_{u,3} = 100 \rightarrow \frac{c_{u,3}}{p_{atm}} = \frac{100}{100} = 1 \rightarrow \alpha_3 = 0.48$$

$$Q_{s,clay} = 2.51 \times [(0.62 \times 60 \times 4) + 0 + (0.48 \times 100 \times 5)] = 975.88 \text{ kN}$$

$$Q_{s,total} = Q_{s,sand} + Q_{s,clay}$$

$$Q_{s,total} = 342 + 1516.54 = 1858.54 \text{ kN (when using } \lambda - \text{method)}$$

$$Q_{s,total} = 342 + 975.88 = 1317.88 \text{ kN (when using } \alpha - \text{method)}$$

$$Q_u = Q_p + Q_{s,total}$$

$$Q_u = 452.4 + 1858.54 = 2310.94 \text{ kN (when using } \lambda - \text{method)} \checkmark.$$

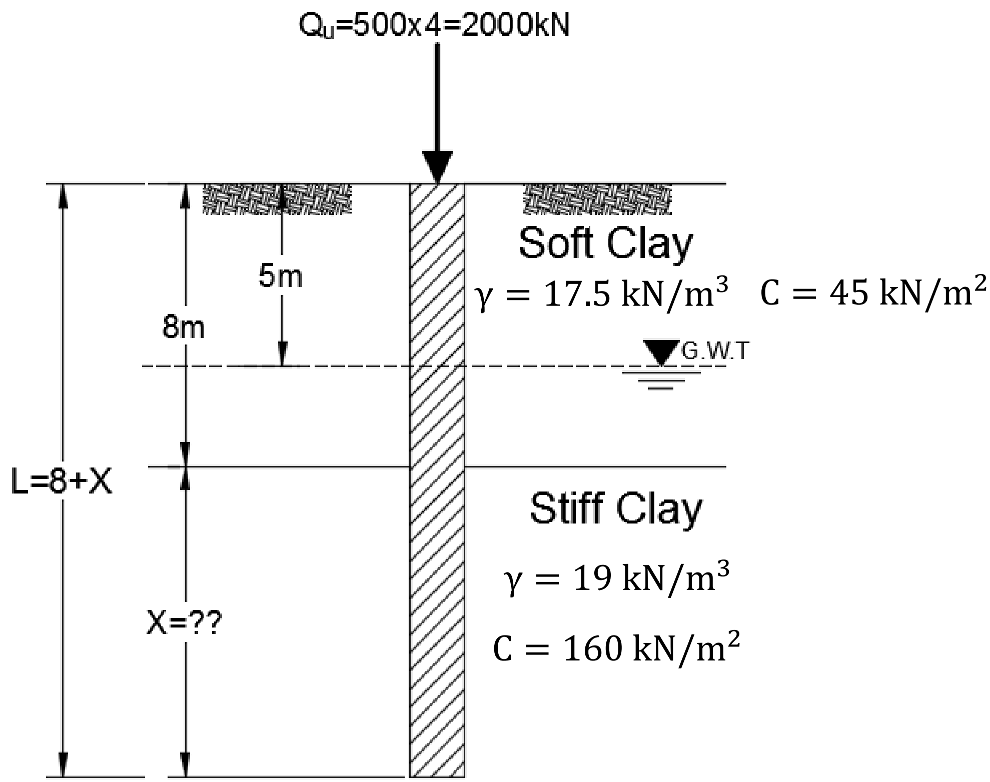
$$Q_u = 452.4 + 1317.88 = 1770.28 \text{ kN (when using } \alpha - \text{method)} \checkmark.$$

2.

A pile is driven through a soft cohesive deposit overlying a stiff clay, the average un-drained shear strength in the soft clay is 45 kPa. and in the lower deposit the average un-drained shear strength is 160 kPa. The water table is 5 m below the ground and the stiff clay is at 8 m depth. The unit weights are 17.5 kN/m^3 and 19 kN/m^3 for the soft and the stiff clay respectively. **Estimate the length** of 500 mm diameter pile to carry a load of 500 kN with a safety factor of 4. Using **(a).** $\alpha -$ method **(b).** $\lambda -$ method

Solution

There is no given graph in this problem, so you should understand the problem and then draw the following graph by yourself:



$Q_{all} = 500\ \mathrm{kN}$, $FS = 4 \rightarrow Q_u = 500 \times 4 = 2000\ \mathrm{KN}$

$Q_u = Q_P + Q_s$

Calculation of Q_P:

Note that the soil supporting the pile from its end is clay, so:

$Q_P = A_P \times c_u \times N_c^*$ $\quad N_c^* = 9$ (pure clay $\phi = 0.0$)

$A_P = \frac{\pi}{4} \times 0.5^2 = 0.196\ \mathrm{m^2}$

$c_u = 160\ \mathrm{kN/m^2}$ (for the soil supporting the pile at its end)

$Q_P = 0.196 \times 160 \times 9 = 282.24\ \mathrm{KN}$

$Q_L = 0.0$ since $\tan\phi = 0.0 \rightarrow Q_P = 282.24\ \mathrm{KN}$

Calculation of Q_s:

Note that all layers are clay.

(a). $\alpha -$ method

$$Q_s = P \times \sum \alpha_i \times c_{u,i} \times L_i$$

$P = \pi \times D = \pi \times 0.5 = 1.57\mathrm{m}$

For layer (1)

$$c_{u,1} = 45 \rightarrow \frac{c_{u,1}}{p_{atm}} = \frac{45}{100} = 0.45 \rightarrow \alpha_1 = 0.71 \text{ (by interpolation from table)}$$

For layer (2)

$$c_{u,2} = 160 \rightarrow \frac{c_{u,2}}{p_{atm}} = \frac{160}{100} = 1.6 \rightarrow \alpha_2 = 0.38$$

$$Q_s = 1.57 \times [(0.71 \times 45 \times 8) + (0.38 \times 160 \times X)] = 401.3 + 95.45X \text{ Kn}$$

$$Q_u = Q_P + Q_s = 282.24 + 401.3 + 95.45X = 683.54 + 95.45X$$

But $Q_u = 2000 \rightarrow 2000 = 683.54 + 95.45X \rightarrow X = 13.8\text{m}$

$\rightarrow\rightarrow L = 8 + 13.8 = 21.8 \cong 22\text{m}$✓.

(b). $\lambda -$ method

$Q_s = P \times L \times f_{av}$

$f_{av} = \lambda \times \left(\sigma'_{v,av} + 2\, c_{u,av}\right)$

$P = 1.57\text{m} \quad L = 8 + X$

We want to calculate λ from the table, but λ is a function of pile length which is required, so in this types of problems when the solution is required according $\lambda -$ method you are strongly recommended to assume a reasonable value of L.

Assume $X = 7 \text{ m} \rightarrow L = 8 + 7 = 15$

$\lambda = 0.2$ (at $L = 15\text{m}$ from Table 11.9)

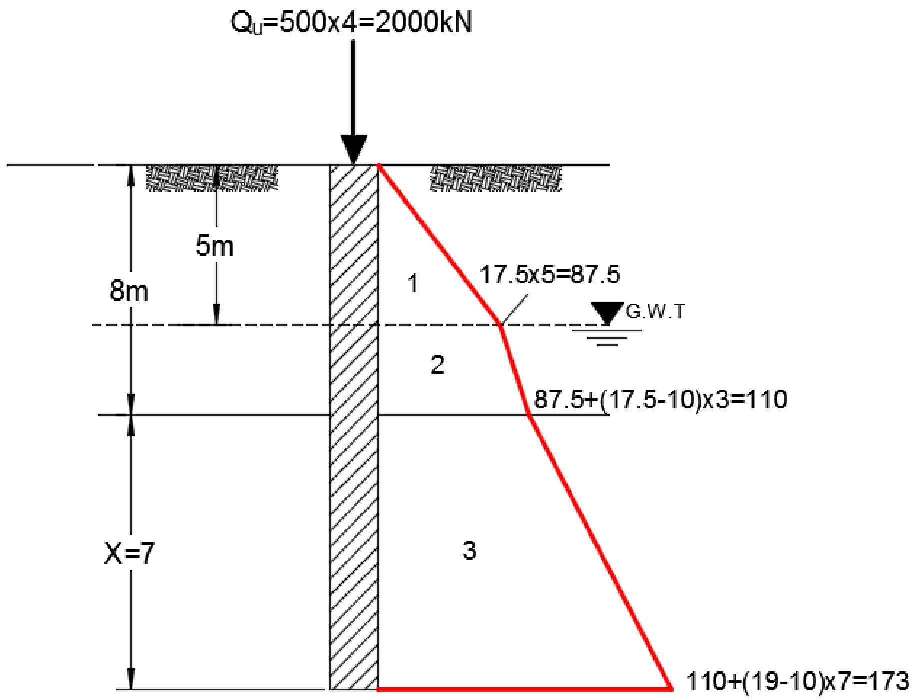
$$c_{u,av} = \frac{L_1 \times c_{u,1} + L_2 \times c_{u,2} + L_3 \times c_{u,3}}{L}$$

$$c_{u,av} = \frac{8 \times 45 + 7 \times 160}{15} = 98.67 \text{ kN/m}^2$$

$$\sigma'_{v,av} = \frac{A_1 + A_2 + A_3 + \cdots A_n}{L}$$

We draw the vertical effective pressure with depth:

For the upper layer (Soft clay) assume the saturated unit weight is the same as the natural unit weight (17.5) because no enough information about them.



$$A_1 = \frac{1}{2} \times 87.5 \times 5 = 218.75$$

$$A_2 = \frac{1}{2} \times (87.5 + 110) \times 3 = 296.25$$

$$A_3 = \frac{1}{2} \times (110 + 173) \times 7 = 990.5$$

$$\sigma'_{v,av} = \frac{218.75 + 296.25 + 990.5}{15} = 100.36 \text{ kN/m}^2$$

$$f_{av} = 0.2 \times (100.36 + 2 \ \times 98.67\ ) = 59.54$$

$$\rightarrow Q_s = 1.57 \times 15 \times 59.54 = 1402.167 \text{ kN}$$

$Q_u = Q_p + Q_s = 282.24 + 1402.167 = 1684.4 < 2000 \rightarrow\rightarrow\rightarrow$ We need to increase L to be closed from 2000

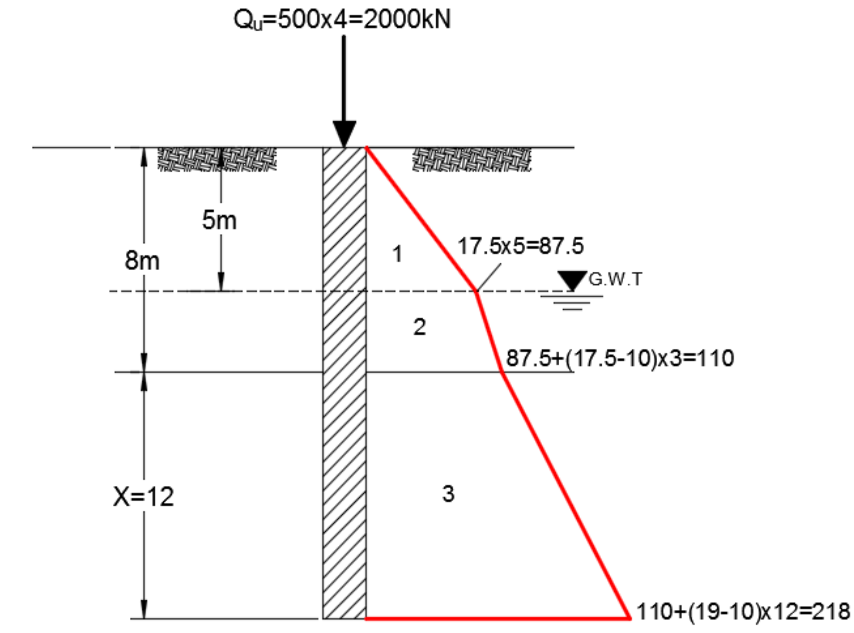
Try $X = 12$ m $\rightarrow L = 8 + 12 = 20$
$\lambda = 0.173$ (at $L = 20$m from Table 11.9)

$$c_{u,av} = \frac{L_1 \times c_{u,1} + L_2 \times c_{u,2} + L_3 \times c_{u,3}}{L}$$

$$c_{u,av} = \frac{8 \times 45 + 12 \times 160}{20} = 114 \text{ kN/m}^2$$

$$\sigma'_{v,av} = \frac{A_1 + A_2 + A_3 + \cdots A_n}{L}$$

We draw the vertical effective pressure with depth:



$$A_1 = \frac{1}{2} \times 87.5 \times 5 = 218.75$$

$$A_2 = \frac{1}{2} \times (87.5 + 110) \times 3 = 296.25$$

$$A_3 = \frac{1}{2} \times (110 + 218) \times 12 = 1968$$

$$\sigma'_{v,av} = \frac{218.75 + 296.25 + 1968}{20} = 124.15\ \text{kN/m}^2$$

$f_{av} = 0.173 \times (124.15 + 2\ \times 114\ ) = 60.92$

$\rightarrow Q_s = 1.57 \times 20 \times 60.92 = 1912.94\ \text{kN}$

$Q_u = Q_p + Q_s = 282.24 + 1912.94 = 2195.12 > 2000$

Note that at X= 12 m the value of Q_u is closed to 2000 but its need to slightly decrease, so we can say $X \cong 10\text{m} \rightarrow L \cong 18\text{m}$✓.

As you see, the solution is by trial and error, so when you assume the value for L, be logic and be realistic to save time. ☺☺☺

3.

A concrete pile is 20 m length and 360 mm x 360 mm in cross section. The pile is fully embedded in **sand** which unit weight is 16.8 kN/m^3 and $\phi = 30^\circ$ You are given also $N_q^* = 56.7$. Calculate:

a) The ultimate load (Q_p), by using Meyerhof's method.

b) Determine the frictional resistance (Qs), if k = 1.3 and $\delta = 0.8\phi$.

c) Estimate the allowable load carrying capacity of the pile (Use FS = 4).

Solution

a) $\mathbf{Q_P}$ =??

$Q_P = A_P \times q' \times N_q^* \leq Q_L$

$Q_P = A_P \times q' \times N_q^*$

$A_P = 0.36 \times 0.36 = 0.1296\ \text{m}^2$

$q' = 16.8 \times 20 = 336\ \text{kN/m}^2$

$N_q^* = 56.7$ (given)

$\rightarrow Q_P = 0.1296 \times 336 \times 56.7 = 2469\ \text{KN}$

Now we check for Upper limit:

$Q_L = 0.5 \times A_P \times p_a \times N_q^* \times \tan\phi$

$p_a = \text{atmospheric pressure} = 100\ \text{kPa}\ ,\ \ \phi = 30^\circ(\text{given})$

$Q_L = 0.5 \times 0.1296 \times 100 \times 56.7 \times \tan 30 = 212.13\ \text{kN}$

$Q_L < Q_P \rightarrow Q_P = Q_L = 212.13\ \text{kN}$ ✓.

b) $\mathbf{Q_s}$ =??

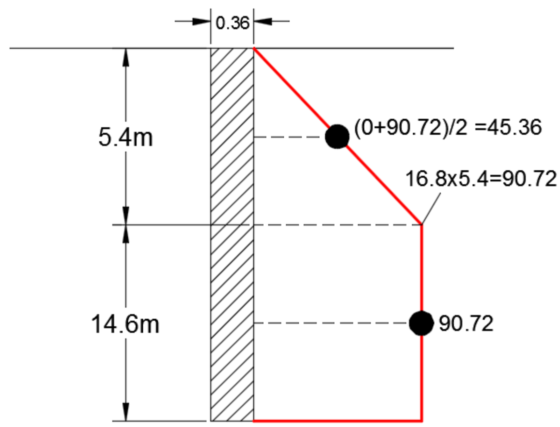
The soil is pure sand, so:

$$Q_s = P \times \sum \tan(0.8\phi_i) \times \sigma'_{v,av,i} \times K_i \times L_i$$

But, since the soil is sand the stress will varies at depth of 15D then will be constant on the remained pile length.

$15D = 15 \times 0.36 = 5.4m$

The shown figure is the pressure distribution with depth:



$P = 4 \times 0.36 = 1.44$ (Square cross section)

$K_1 = K_2 = 1.3$ (given)

$0.8\phi_i = 0.8 \times 30 = 24^{\circ}$

$Q_s = 1.44 \times 1.3 \times \tan(24)\left[\sigma'_{v,av,i} \times L_i\right]$

$Q_s = 0.8334[45.36 \times 5.4 + 90.72 \times 14.6]$

$\rightarrow Q_s = 1308 \text{ kN}$ ✓.

c) $\mathbf{Q_{all}}$ =??

$$Q_{all} = \frac{Q_u}{FS}$$

$Q_u = Q_P + Q_s = 212.13 + 1308 = 1520.13 \text{ kN}$, $FS = 4$ (given)

$$\rightarrow Q_{all} = \frac{1520.13}{4} = 380 \text{ kN}$$ ✓.