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"Much Palaver About Greater Than Zero and Such Stuff" – First Year Engineering Students' Recognition of University Mathematics

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Abstract Research under the key word 'secondary-tertiary transition problem' in mathematics education points to a range of difficulties students face when passing from learning mathematics at school to attending undergraduate mathematics courses. One of these problems concerns the change in criteria for what counts as a legitimate mathematical activity. Based on this observation, the aim of this study is to investigate the extent to which students enrolled in undergraduate mathematics courses are aware of such changes in criteria and how their reflective recognition relates to their academic success. The main body of empirical data comprises interviews with 60 undergraduate students, who were enrolled in different engineering programmes at two Swedish universities and attended compulsory mathematics courses. These interview data are complemented by the grades achieved by these students on all mathematics courses during their first year of enrolment. A group of their lecturers were also interviewed. In order to explore what counts as a legitimate mathematical activity, participants were presented with excerpts from different mathematics textbooks and asked which of these they would describe as more or less mathematical and why. As theoretical resources we selectively employ notions by means of which Bernstein conceptualised pedagogic discourse, elements of Halliday and Hasan's social semiotics, and Eco's idea of the model reader. The investigation shows that students focus on a considerably wide range of aspects of mathematics texts by which they (mis)recognise the specificity of the discourse, and how this relates to their academic success. The study not only provides a differentiated picture of students' reflective recognition of levels of rigour, abstraction and formalisation in mathematics, but also offers a methodological contribution.

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Introduction

The outcomes of national surveys and international research studies point to a range of problems students face as a consequence of their passage from school to university mathematics studies (see e.g. the review by Gueudet 2008). In their review, De Guzman et al. (1998) pointed to epistemological and cognitive, sociological and cultural, as well as didactical discontinuities between school and undergraduate mathematics teaching and learning. One of these discontinuities concerns the change in criteria for what counts as a legitimate mathematical activity. These changes have commonly been described as a tendency towards more rigour, a higher level of abstraction, and formalisation.

Based on these observations, the aim of this study is to investigate the extent to which students enrolled in undergraduate mathematics courses are aware of such differences, how they recognise and articulate them, and how their reflective recognition relates to their academic success. The empirical data for the study comprise interviews that we conducted with 60 students who were enrolled in civil engineering programmes at two Swedish higher education institutions and attended the compulsory mathematics courses.[1] In order to access the legitimate modes of mathematical discourse in this context, we also interviewed a group of their lecturers. In all these interviews, we presented participants with excerpts from four mathematics textbooks (with proofs or applications and different foci on technicality) and asked which of these they would describe as more or less mathematical and why. For determining the students' academic success we used their grades achieved on all mathematics courses during the first year of study at university. This is because – at least theoretically – the course examinations test whether new modes of producing and communicating mathematical 'truths' have become part of the students' consciousness.

Our analysis shows the extent to which students have become aware of the criteria for mathematical rigour and how this awareness relates to their academic success. More importantly, it provides a nuanced picture of how the students articulate what they recognize as mathematics in the course of their apprenticeship into disciplinary discourse. As we treat this process as fundamentally social, this study avoids relying on descriptions of the 'transition-problem' that assume an essence of academic mathematics that is fundamentally different from school mathematics. The analytical tools that were developed in interaction with the empirical material instead allow a fine-tuned analysis that operates at the level of text.

[1] These interviews were part of a larger project in which we followed these students during their first year at the university. This project, *The transition from school mathematics to university mathematics*: *an integrated study of a cultural gap*, was funded by the Swedish Research Council (Vetenskapsrådet). While short preliminary reports from this project have been presented at conferences (e.g. Jablonka et al. 2012), this article draws on a more extensive literature review and a much larger set of interviews. It also includes an elaboration of our analytical framework and so offers a methodological contribution.

The article consists of five main sections. In the first section we review literature that deals with a range of aspects of the secondary-tertiary transition in mathematics education; we look more closely at studies that point to discrepancies in the criteria for what counts as mathematics. Then we present our analytical framing, which selectively employs some notions of Bernstein's conceptualisation of pedagogic discourse, elements of Halliday and Hasan's social semiotics, as well as Eco's idea of the model reader. This section also includes an outline of how we operationalised the analytical framework for organising the empirical data. In the next section, we describe the methods for data generation and provide a rationale for the selection and use of the four texts in the interviews. This rationale is based on an analysis of these texts that reflects our analytical framework. The next section presents the findings, including a summary of the lecturers' discussion of the four texts; a simple quantitative analysis that points to trends in student responses in relation to their academic success; and a detailed presentation of the ways in which students recognise (or not) the specificity of the mathematical discourse. Finally, we further discuss our findings and offer some conclusions.

Dimensions of the Secondary–Tertiary Transition in Mathematics Education

A concern about discrepancies between school and university mathematics education was raised already in the 1960s (Unesco 1966), and since the 1990s (e.g. ICMI 1997) the notion of a *transition problem* became established. Two of the main motives for many studies of the transition problem concern pass rates and participation. In Sweden pass rates for mathematics courses were at the level of 70 % for engineering students during the beginning of the 21st century (HSV 2005, p. 45), with a strong variation between different study programmes. Local reports show that an increasing number of students pass the exams only after several attempts. It is also reported that participation in the various teaching formats (such as lectures, study groups, tutorials, mathematics skills help centres) offered at some universities has decreased in recent years (local information). Comparatively low pass rates for first year undergraduate mathematics courses have also been observed in other countries (Dieter and Törner 2010; Engineering Council 2000; De Guzman et al. 1998; Vollstedt et al. 2014). The issue has gained more importance with policies that aim at widening participation in higher education for under-represented groups, mostly framed in terms of gender, ethnicity and social class (Pampaka et al. 2012). Student participation and pass rates were also a concern at the two institutions where our study was carried out; in particular as the enrolment in the different engineering programmes showed links between programme choice and social class origin of the students, which are reflected in our sample.

The following review of surveys and research studies carried out in Sweden and in other contexts serves to provide some local background and a synthesis of a range of dimensions that have been investigated in connection with the transition problem in mathematics education. We look more closely at studies that point to discrepancies in the criteria for what counts as mathematics as these are central to the focus of this article.

Change in Expected Learning Habits and Study Organisation

Studying (mathematics) at university is often described as requiring a higher degree of autonomy than at secondary school on the side of the students in organising their learning (e.g. De Guzman et al. 1998; Wingate 2007). This was also acknowledged by many of the students as a major difference to school in a Swedish survey with a sample that covered 59 % of the target population of second semester students in undergraduate engineering and science programmes (HSV 2005, p. 32). Hence their academic success might rely on skills in organising their time, on particular mathematics specific learning strategies (Rach and Heinze 2013) as well as on skills in note taking during lectures, reading mathematics textbooks, and judgment of suitability of other resources. Unexpected changes of the “didactical contract” (Gueudet 2008; Pepin 2014) have been found to cause stress and depression (Jackson et al. 2000). De Vleeschouwer and Gueudet (2011) theorise their findings by introducing three different “levels of contract”: a general contract (independent of the subject studied), a didactical contract for mathematics in the institution, and a didactical contract for a specific mathematical content. The differences in authority relationships between teacher and student as compared to school appear to operate across these levels. While there needs to be acknowledgement of differences in pastoral care and tutorship offered at different institutions, a common experience expressed by the students in the Swedish report (HSV 2005, p. 32) concerns the university’s setting with large group lectures that increases the social distance between teachers and students and amongst students. On similar observations in other contexts, De Guzmán et al. (1998, p. 755) comment that the anonymity at a large university can be “quite a frightful experience”.

Different Teaching Formats and Modes of Assessment

The most common teaching format for undergraduate mathematics at Swedish universities is a combination of lectures and study groups. Swedish school classrooms, in contrast, show a high proportion of lessons devoted to individual activity where students work at their own pace, occasionally scaffolded by the teacher (Skolinspektionen 2010). Hence students have little or no experience with the lecture format. The Swedish survey (HSV 2005, p. 32) depicted many students pointing at the increased tempo and amount of material to be covered in university lectures as a characteristic difference to their school experience. De Guzmán et al. (1998, p. 750) summarise similar observations from a range of countries. In a study of students’ experiences of the transition in various subjects in the UK, including mathematics and engineering, Pampaka et al. (2012) found that perceptions vary considerably between different programmes. They note, however, that common across all programmes, a negative experience of the transition was linked to a move towards more teacher-centred transmissionist pedagogies.

With the teaching format, also the forms and function of assessment change. While in school classrooms students might receive some formative feedback, in undergraduate mathematics courses there is commonly only summative assessment through an obligatory closed book examination. However, the content and form of these examinations might be less standardised and offer more flexibility for the lecturers than school examinations, as for instance reported by Thomas and Klymchuk (2012) from a New Zealand context. The authors contrast this with teaching to the test and a higher amount of bureaucratic work for school teachers with local assessments. The students in their study found examinations at university "much better and more efficient than at school" (p. 289). In the context of our study, the lecturers responsible for the courses had control over the content of the examinations as well as the marking criteria.

Differences in Pedagogical Awareness of Teachers

The image of university mathematics teachers reported from Swedish engineering students is rather positive (HSV 2005, p. 32). While some critique was raised in this survey, a large majority appreciated the engagement as well as the knowledge of their teachers, while complaints referred to the lack of steering and control in the study groups, to problems with the lectures and old-fashioned teaching methods (HSV 2005, p. 46). Similar perceptions were revealed in the study by De Guzman et al. (1998) and Alsina (2001). Nardi's (2008) study portrays the mathematicians who participated in her project as having the capacity for developing pedagogical awareness when addressing didactical issues that arise in undergraduate courses. More specifically, in their study of undergraduate mathematics teachers Nardi et al. (2005) identified a "spectrum" of pedagogical awareness, including four levels labelled "naive and dismissive", "intuitive and questioning", "reflective and analytic", and "confident and articulate" (p. 293). The students involved in our study were generally positive about the pedagogic approaches of their lecturers.

Curriculum Misalignment

In the Swedish survey (HSV 2005, pp. 32–33), only 40 % of the engineering students in the second semester thought they had used their mathematical knowledge in other subjects to a considerable extent. Similarly, De Guzmán et al. (1998, p. 756) noted a general underappreciation of the role of mathematics in a diversity of subjects in the French, Spanish and Canadian contexts. Rather than perceiving mathematics as directly useful, the engineering students involved in our study described more general gains from their mathematics studies, such as acquiring generic problem solving skills (Bergsten and Jablonka 2013). A "utilitarian trend", as noticed by some in UK school education (Hoyles et al. 2001, p. 835), can cause conflicting messages as to the purpose of studying mathematics at school and at university.

A couple of studies reveal more specific discrepancies between mathematical faculties' expectations and the school curriculum. For example, topics not included in the Swedish school syllabuses but expected to be mastered, range from formal definitions of limits and functions, analytic geometry to inequalities (Brandell et al. 2008; HSV 1999). The picture of the universities' expectations is not uniform though. Mathematical proof, for example, is not necessarily a focus in basic university courses (Hemmi 2006). Students' own perceptions of their general pre-knowledge is inhomogeneous, and often more positive than that of their teachers (e.g. HSV 2005, p. 34).

Mismatches in mathematical content of secondary and tertiary level mathematics are also reported from other countries (Engineering Council 2000; Heck and van Gestel 2006; Hourigan and O'Donoghue 2007; Hoyles et al. 2001; Kajander and Lovric 2005; Robert 1998; Thomas and Klymchuk 2012). In summarising these studies, discrepancies in the focus on distinct mathematical activities relate to oppositions between factual knowledge and use of formularies and tables, routine skills and mathematical problem solving, computational fluency and use of technology, as well as informal argumentation and mathematical proof. At a more general level, in their textbook study, Bosch et al. (2004) point to the incompleteness of the "praxeologies" in high school mathematics as compared to university mathematics. This mismatch between the curricula points to differences in coherence of the mathematical discourse, which at university is achieved by elaboration of the principles.

Changes in Level of Formalisation and Abstraction

As indicated above, several studies suggest that curriculum discrepancies are not only a problem of providing breadth, privileging different skills and using different tools, but are often conceptualised as change in the *type* of mathematical practice and discourse (e.g. Hoyles et al. 2001; Robert 1998, cited in Gueudet 2008 p. 240; Vollstedt et al. 2014). For instance, according to Jooganah and Williams (2010), mathematics in school is mainly procedural requiring from the students only little conceptual depth, while university mathematics places "emphasis on proof and rigour requiring them to engage with and critically think about concept definitions" (p. 116). Similarly, in their comparative study of teaching and assessment at school and university, Thomas and Klymchuk (2012) observed that at school emphasis is on practice of worked examples, mathematical modelling and recalling factual knowledge, while at university conceptual reasoning, rigour and problem solving are more valued. The students in their study also found the university courses "deeper and more challenging" (p. 298).

Some studies point to a lack of explicitness of criteria for what counts as a legitimate accomplishment both in university mathematics textbooks as well as in lectures (cf. Dreyfus 1999). For example, the criteria for what exactly has to be proven and what can be taken as obvious in a proof, are often not obvious; switches between inductive and deductive arguments occur unnoticed; and informal and formal reasoning have a different status in oral and written mathematical discourse. For instance, Raman (2002) inferred from an analysis of textbooks that in the transition from pre-calculus to calculus, students lack opportunities for co-ordinating informal and formal aspects of mathematical meaning. She suggests that different epistemological messages in textbooks for different levels may contribute to students' difficulties (Raman 2004). Hemmi (2006) observed that mathematics at university is presented in a comparatively advanced technical language, which students perceive as more cumbersome. This reflects what Österholm found in his study (2008) which revealed difficulties faced by students confronted with a mathematical text containing symbols in comparison with a less technical version of the same text.

In the Swedish survey (HSV 2005), 75 % of the students found the mathematics courses difficult, and 85 % of the students said the university sets up new demands (p. 32). On an open question regarding these differences in requirements, the most common answers were "higher demands of understanding"and "higher level/higher demands/more content". Similarly, in a study involving three universities in France, Spain and Canada, the tasks to be dealt with were perceived by the students as more "abstract" (De Guzman et al. 1998 p. 749). The authors pointed out (pp. 752–753) that tertiary mathematics includes "unifying and generalising concepts" (such as the notion of a vector space). More specifically, Robert (1998, cited in Gueudet 2008) emphasised that mathematics at university level involves processes of "formalizing, unifying, generalizing and simplifying" (Gueudet 2008, p. 240). In mathematics lectures this is often related to a Euclidean style of presentation in a sequence of definition–theorem–proof (Weber 2004). Jooganah and Williams (2010) focused on the issue of rigour and proof and concluded that "the different activity systems of school and university involve contradictory mathematical practices" (p. 113) and that the nature of proof poses an epistemological shift regarding the character of mathematical knowledge. From a cognitive perspective, the difference has been described as a switch from intuitive to formal mathematical thinking (Tall 2008). Also, the notion of "a fundamental conceptual divide between school and university mathematics" (Hoyles et al. 2001, p. 832) has been used in this context. Leviatan (2008) goes as far as to describe the shift in criteria as a "cultural gap":

> … tertiary mathematics is more abstract and emphasizes the inquisitive as well as the rigorous nature of mathematics. Many first-year college students find it difficult to adapt to a *culture* where concepts are *abstract*, yet require *rigorous* definitions; theorems have to be *proved*, and their assumptions meticulously *verified* before their results can be applied, etc. (p. 105; emphasis in original)

In the literature about tertiary mathematics education, similar ways of pointing at some general features claimed to characterise mathematics at university level without further detailed elaboration or empirical footing, appears to be common (e.g. Luk 2005). A Canadian study, however, empirically investigated what university mathematicians think constitutes mathematical practice and discourse at university level. In an on-line survey Mura (1993) asked the open question "How would you define mathematics?". From a content analysis of 106 received responses (from an original sample of 444 mathematicians), 12 themes were identified. Some mathematicians' answers included more than one of these themes. The most common themes were (Mura 1993, pp. 379–381):

- Design and analysis of models abstracted from reality; their application ($n = 30$)
- Logic, rigour, accuracy, reasoning, especially deductive reasoning; the application of laws and rules ($n = 26$)
- The study of formal axiomatic systems, of abstract structures and objects, of their properties, and relationships ($n = 25$)
- An art, a creative activity, a product of the imagination; harmony and beauty ($n = 15$)
- A science; the mother, the queen, the core, a tool of other sciences ($n = 13$)
- A language, a set of notations and symbols ($n = 10$)
- Reference to specific mathematical topics (number, quantity, shape, space, algebra, etc.) ($n = 10$)

Though there is a dominance of aspects alluding to what might be subsumed under "rigour" (level of formalisation, technicality and abstraction), the relation to "reality" (modelling and applications) was by many seen as a core aspect of what university mathematicians do. According to Mura's (1993) interpretation the diversity of the answers indicates that many mathematicians "seem to have little interest in reflecting on the nature of mathematics" (p. 384). Mura discusses the findings in terms of different philosophies of mathematics (Ernest 1989) but does not address the issue about how these views may translate to the institutionalised mathematics taught to undergraduate students.

Conclusion

While school curricula and pedagogic approaches across the contexts of the studies summarised above are quite diverse, the characterisations of university practices and discourses appear more uniform. From the studies it is evident that the secondary-tertiary transition problem in mathematics education concerns differences in forms of pedagogy, curriculum and assessment, as well as in types of role involvement and institutional culture, which have been analysed from a diverse range of vantage points, including epistemological and sociological perspectives. While in most of the studies the transition is framed as a problem, Hernandez-Martinez et al. (2011) see it "as a question of identity in which persons see themselves developing due to the distinct social and academic demands that the new institution poses" (p. 119), that is, as something potentially positive for their development and future opportunities (see also Hernandez-Martinez and Williams 2013). In her review of the research on the secondary-tertiary transition in mathematics education, Gueudet (2008) differentiates between immediate *ruptures* met by new types of semiotic objects and tasks, and a *long enculturation process* as students progressively become familiar to a new language and new ways of thinking. In line with these perspectives, in our study we do not assume from the outset that there is a clearly delineable transition problem. We see the university courses in mathematics, as in any other subject, as aiming at an apprenticeship into a disciplinary discourse, which might be realised by a range of different pedagogic strategies. We treat this process as fundamentally social.

Analytical Framework

As outlined in the literature review above, differences between school mathematics and traditional undergraduate mathematics have been described in terms of qualitative changes in mathematical activities privileged at university. These include a focus on deductive reasoning (rather than inductive); an orientation towards making the principles behind mathematical procedures discursively explicit through proofs; a focus on pure rather than applied mathematics; and a reduction of informal (often called 'intuitive') arguments. We draw on Basil Bernstein's ideas of knowledge *classification* and *recognition rules* as a rationale for investigating students' awareness of these differences. Further, the differences investigated in studies of the secondary-tertiary transition also point to changes in pedagogic relationships between teachers and students. In order to capture these differences in the excerpts from mathematics textbooks presented to the participants, we look at the forms of guidance provided for their intended readers, in particular at the visibility of a *didactic layer*. In order to enable a nuanced analysis of the different texts presented to the participants as well as for the organisation of the lecturers' and students' responses, we employ analytical tools from social semiotics.

Theoretical Departure

We start with the assumption that students, throughout their mathematics education, participate in a range of pedagogic practices, which emulate different mathematical discourses[2]. In selectively drawing on Basil Bernstein's complex theorising of pedagogic discourse, we assume that in order to be successful, students need to understand the principles for distinguishing between contexts and recognise the speciality of the discourse, in which they engage (cf. Bernstein 1981); they have to acquire the *recognition rules* (Bernstein 1996). In Bernstein's theory, recognition rules are outcomes of principles of knowledge classification that reflect dominant power relations, which are described in terms of the nature of "relations between categories, whether these categories are between agencies, between agents, between discourses, between practices" (Bernstein 1996, p. 20). Classification is *strong* or *weak*, depending on the level of interaction between these categories. In characterising forms of university curricula for engineering students, the specialised academic canon located in single disciplines or their sub-branches reflects strong classification of practices and discourses, while a move towards interdisciplinary or project-based approaches weakens classification (e.g. the CDIO approach[3]; see Crawley 2001). At the higher education institutions of our study, the former is the case, at least in the first 2 years of the engineering programmes. In addition to characterising relations between categories, Bernstein (e.g. 1996) used the concept of *framing* for capturing the specificity of the communicative realisation of pedagogic discourse in pedagogic practice. Weak framing means apparent control of the student over this realisation. However, as Bernstein repeatedly stressed, when there is pedagogic transmission, the relation between transmitter and acquirer is essentially asymmetrical. Theoretically, framing and classification can vary independently, but as Bernstein earlier (1971) noted, strong knowledge classification (if one takes "knowledge" as a given) limits teachers' and in consequence students' choices about what can be realised in the communication. Hence we do not conceive of framing varying independently of the classificatory principles.

Consequently, weak framing does not imply lack of control by the teacher, but only deferment of revealing the principles for what counts as a legitimate contribution. This points to different pedagogic strategies that may be based on different authority relationships between teachers and students. In order to capture these differences in the excerpts from mathematics textbooks presented to the participants, we look at the forms of guidance provided for their intended readers, in particular at the visibility of a *didactic layer*. In drawing on Eco's notion of *model reader*

[2] See Bergsten et al. (2010) for a discussion of university calculus.
[3] CDIO is an abbreviation of Conceive – Design – Implement – Operate.

assumed and shaped by textual strategies (e.g. Eco 1979), Sierpinska (1997) categorised texts that are concerned with formatting the model reader as their main goal as *didactic texts*. Sierpinska analytically separates two types of discourses, a discipline specific discourse and a didactic discourse, which are realised as different *layers* in textbooks. While the *mathematical layer* is concerned with developing what obviously belongs to mathematical discourse (definitions, theorems, proofs, worked examples, exercises and solutions), the *didactic layer* is concerned with shaping the learner's use of the textbook. Sierpinska acknowledges that didactic action is also implicit in the mathematical layer (e.g. by choice of examples addressing common misinterpretations). Sierpinska (1997, p. 7) classified the empirical texts in terms of a quantitative relationship between the two layers as having a comparatively *thin* (or *thick*) didactic layer. However, as all mathematics (text-)books are concerned with shaping the readers' interpretation of the mathematical layer, the didactic layer can only be recognised if the author directly addresses the reader with some guidance on how to use the book or offers some commentary to the main body of the text. We refer to these features as the *disembedded didactic layer*, and to the shaping of the model reader's interpretation of the mathematical layer (based on assumptions about their competence and knowledge) as the *embedded didactic layer*.

In our study, we were interested in the students' recognition of what counts as a legitimate mathematical activity, which is a necessary condition for their own capacity for realising a legitimate communication. The knowledge classification of undergraduate university mathematics, as the literature review suggests, creates specific subject-related recognition rules that differ from school mathematics and from more applied mathematics. Hence for selecting the excerpts from undergraduate textbooks that we presented to the participants, we took differences in the strength of classification as one criterion. Further, we were interested in the extent to which these university level textbooks exhibit a didactic layer that is reminiscent of a school textbook or pedagogy that could be recognised by the students. The differences in the book pages that we presented to the lecturers (mathematicians) and students point to differences in classification as well as in the visibility of a didactic layer (see the section 'classification and didactic layers in the texts' in the description of the design of the study below, in particular Table 2).

Analytical Tools

For the description of the different texts as well as for the organisation of the lecturers' and students' responses, we employ analytical tools developed by Halliday and Hasan (1989) in their work on social semiotics. Language is then modelled as interacting with the social context of its use, with different modes of meanings ("metafunctions") embedded in a text allowing predictions of corresponding features of the situation. We are not concerned, however, with identifying some intrinsic functional organisation of language in mathematics-related discourses (the "mathematical register"). Neither do we assume that the different metafunctions are necessarily realised by elements from different parts of a language system (particular semantic and lexico-grammatical resources), as proposed by Halliday and Hasan (1989). We rather use the notions of *experiential*, *logical*, *interpersonal* and *textual meanings* realised in a text for operationalising aspects of our analytical framework. In recognising that elaborations in language play an essential role in academic education, Halliday and Hasan (1989, pp. 44–45), state that:

> […] anyone who is learning by listening to a teacher, or reading a textbook, has to:
>
> 1a. understand the processes being referred to, the participants in these processes, and the circumstances-time, cause, etc-associated with them [EXPERIENTIAL].
>
> 1b. understand the relationship between one process and another or one participant and another, that share the same position in the text [LOGICAL].
>
> 2. recognise the speech function, type of offer, command, statement, or question, attitudes and judgements embodied in it, and the rhetorical features that constitute it as a symbolic act [INTERPERSONAL] and
>
> 3. grasp the news value and topicality of the message, and the coherence between one part of the text and every other part [TEXTUAL]

The capitalised terms in brackets refer to different modes of meaning, which in turn correspond to different aspects of the social context, that is to the *field* (1a, 1b), the *tenor* (2) and the *mode* (3) of a discourse. The field refers to the activity and topic with which the participants are engaged in which the language figures as an essential component, the tenor to the relationships, status and roles of the participants, and the mode to what the language is expected to achieve in the context.

At the theoretical level we assume that classificatory principles are realised in the experiential and logical meaning. As realisations of strong (or weak) classification in the textbook pages we take the use of technical terms (or not), the employment of more (or less) mathematics-specific forms of argument (i.e. deduction), as well as the exclusion (or inclusion) of themes from other disciplines or fields of practice.

The embedded didactic layer we see realised by resources that achieve coherence and structure of the text (textual meaning), but also by elements that could possibly be analysed as realising the logical meaning. This is because mathematics texts often use conjuncts to establish relations between series of propositions and specialise or expand main themes into sub-themes. Further, we note that a separation between interpersonal and textual meanings appeared difficult to uphold when dealing with the empirical material. The structuring of the text was indeed seen by many students as an interference of the author to make the material more or less accessible.

We use the description of the text functions to provide some systematic description of the textbook pages as well as to structure the presentation of the lecturers' and students' arguments at the level of detail appropriate for this purpose. For organising the interview data we operationalized the framework as described in Table 1.

Table 1 Operationalization of the framework for organising the empirical data

Experiential/Logical Meaning	Textual/Interpersonal Meaning
Reference to classes of mathematical signifiers (e.g. numbers, variables, formulas, notation, terms, graphs of functions); Reference to other subjects by means of examples or general names (e.g. bacteria, biology, force, physics); Reference to mathematical relations (e.g. equals, greater than) or mentioning single metamathematical notions (e.g. 'proof') without talking about their function in terms of coherence of the exposition.	Reference to the structure and coherence of the text; Reference to specific acts (e.g. explaining, showing steps, making assumptions, defining and proving) in relation to the function of the text; Reference to the coherence of the discipline of mathematics; Reference to the intended audience or impact of the text on themselves as readers (e.g. 'understanding', 'not knowing').

Design of the Study

Methods of Data Generation and Compilation

The study is based on a set of data comprising individual interviews with 60 students enrolled in different masters level engineering programs at two Swedish universities. These were 5-year major programs in computer technology, energy and environment, industrial economy, mechanical engineering, as well as technical physics and electric engineering. All mathematics courses in the programs were taught by mathematicians employed at a mathematics department. At each of the two universities, the first year calculus and linear algebra courses were common to the different study programmes, with common exams. We collected the students' results from all mathematics exams during the first year of study as well as from a diagnostic mathematics test administered by the universities shortly after enrolment.

The interviews on which we draw in our analysis were conducted after about half a year of the students' enrolment at university after the students had finished their examinations in the introductory linear algebra and/or calculus courses[4]. Here we only focus on one part of these individual interviews that dealt with the students' recognition rules. In the interviews, the students were shown four excerpts of texts (1–2 pages each), all from Swedish language mathematics textbooks at undergraduate level. They were invited to look at the texts and in an open question asked whether and why they perceived any of them as comparatively more or less 'mathematical' (according to their own interpretation of the term mathematical) and to suggest a ranking. The interviews were audio-recorded and transcribed and the relevant parts coded following the operationalization of the analytical framework of the study, as outlined above. A detailed description and analysis of the four texts is provided below in the next subsection.

The participating students were selected by a theoretically informed sampling (Dowling and Brown 2010) that aimed at representing students enrolled in a range of different study programmes as well as different achievement levels on the diagnostic

[4] In the larger project about the transition from secondary to tertiary mathematics education of which this study is a part, this was the second of a total of three interviews conducted during their first year of study.

mathematics test. This was important as we assume a relationship between recognition of classificatory principles, identity and academic achievement in line with our theoretical departure.

Almost all students who had been asked via email or visits to classrooms to participate in the study volunteered to do so. They were informed about the study with an opportunity to discuss it and signed written consent forms. All interviews took place at the mathematics departments in a classroom or in a meeting room and were audio-taped. The interviewers were the authors of this article.

The overall success levels on the mathematics examinations were for each student summarised as low (L), middle (M) or high (H). This categorisation was based on the average final mark over all the mathematics exams a student took during the first year of study; the marks used by the institution were 0 for Fail, 3 for Pass, 4 for Pass with distinction, and 5 for Pass with excellence. Category L included all students with an average mark below 2.5, category H students with average mark above 3.5, and M all other students. There were four or five exams during the year, with two optional resits for each course in the case of a Fail. Generally, students at levels M and H passed all the exams (in a few cases all except one) with H students receiving higher marks than M students, while students at level L did not pass all exams, with some students passing a few exams after several attempts.

Lecturers of the mathematics courses at one of the universities participating in the project were invited to a focus group interview to discuss the transition problem. The eight university lecturers/professors of mathematics who volunteered to participate all worked at the same mathematics department. As they knew each other as colleagues and constituted a homogenous group in terms of their extensive teaching experience and involvement with undergraduate students, we hoped the interaction between them in a group interview could develop freely into a shared opinion of the group but also expose issues of disagreements (Morgan 1997). The interview, which took place after the student interviews had been completed, was audio-recorded and transcribed. Only one part of the interview, where the lecturers were asked to rank the same four texts that were given to the students in terms of which were more or less mathematical (according to their own interpretation of the term mathematical), is used for this study.[5]

Texts Used in the Interviews

We took each textbook as an instance of the realisation of a particular pedagogic discourse, which the students' might recognise. In the interview, for example, one student said about a textbook excerpt, "This is how they do it here". In the interviews with the lecturers we intended to explore whether the students' recognition reflected the discourse institutionalised at the departments.

Below we present a systematic description of the textbook pages based on an analysis by means of our analytical tools. Typical excerpts from each of the four texts (and English translations) are included in Appendix. We do not attempt to analyse the function of particular linguistic resources chosen by the authors in comparison to possible alternatives, but only present the modes of meaning in a structured way. This

[5] See Bergsten and Jablonka (2015) for a discussion of the methodology and other outcomes of this focus group interview.

description serves as an introduction of the texts. It also enables us to relate the dimensions of mathematical discourse recognized by the students to some textual features in a systematic way. In addition, we draw on this description in ranking the texts in terms of strength of classification and for describing the guidance of the model reader in terms of embedded and disembedded didactic layers.

Text A

Experiential/Logical Meaning In a short introductory paragraph a general statement establishes a relationship between "many processes in nature" and "exponential functions", the latter being of help in "describing" the "growth" of these processes. Some examples are given. The next (longest) section of the text deals with one example of "organic growth". The participants, processes and conditions in this example include individual bacteria and a colony of bacteria that grows; conditions are not specified (see Excerpt A1 in Appendix). The growth is described as a recursive function in two steps, and then generalized to $t = n$. By the definition $1 + p = a$, a is introduced as "growth factor". Substitutions into the formula are then made for non-integer time periods $\left(t = \frac{1}{2}, t = \frac{1}{3}\right)$. At the end of this section, exponential growth for the bacteria colony is declared after deriving $N(t) = N_0 a^t$ (see Excerpt A2). In the last section one example is calculated (deriving $N(t)$ from two values for given amounts), without mentioning bacteria anymore but "equations" (this term was not mentioned before and hence 'equations' is a new theme in the text). The solution is given in symbolic notation.

Textual/Interpersonal Meaning An anonymous knowledgeable author speaks to unknowing readers (employing a general "we", the community of mathematicians, and a "reader-we", for example in "as we have seen earlier" in the same book). There is one direct question posed that is then answered by an exposition. The worked example is introduced with an imperative, which is reminiscent of exercise sections in school mathematics textbooks. The reader is guided through solving the equations. Overall, the text is expository, and the themes are logically connected (then, so, now, as, etc.). Some semantic choices reflect a narrative structure (bacteria growing in time), even though the tense is present tense. The coherence of the topic is achieved through repetitive use of technical terms or respective mathematical symbols, and through reference to statements earlier in the text. There is also an introductory paragraph indicating what is to come. All equations are printed aligned to the centre.

Text B

Experiential/Logical Meaning The main theme of the text is "power functions", with one section about positive and another about negative exponents. The text for positive exponents introduces symbolic notations and properties of "all power functions" for positive exponents and basis, and then for a root function as inverse power function. The existence of inverse functions is justified by reference to a diagram in the text displaying three graphs for the exponents 2, 2.4 and 3 (see Excerpt B1). This sub-section ends with declaring a notation for rational exponents, justified by its coherence with "the rule for powers". The sub-section on negative exponents then culminates in two numbered "theorems" about growth properties of monotonous functions, and of power functions in particular (see Excerpt B2 for the first of these theorems; the second states that the power function x^a is increasing for positive exponents and decreasing for negative exponents, and has the inverse $x^{1/a}$). These theorems appear as a consequence of the exposition through one exemplar (shown in Excerpt B2) and a second diagram showing six graphs of power functions with unspecified negative exponents. Altogether, the text contains four diagrams with graphs of functions. The description of the functions in the graphs employs non-technical terms. The text contains a footnote about the mathematical meanings of "root", which is presented as a dialogue between two students (see Excerpt B3). For this parallel text, the corresponding experiential meaning is the verbalized learning activity of these students.

Textual/Interpersonal Meaning The text employs a "we" that includes both reader and author of the text ("we summarise", "we have seen"), as well as the more general "we" for the community of mathematicians. The reader-author "we" in the introductory paragraph refers to the previous chapter and to what will be "defined" and "discussed" in the section. Then the author-reader "we" states a possible expectation [referring to exponents], "We'll possibly see it coming that the graph for $x^{2,4}$ lies between the graphs for x^2 and x^3. This is totally correct, …". This recruits the reader into having an expectation, which then is evaluated by the general "we". Hence there is a trace of the teacher's function of assessing student contributions, by a friendly teacher and students who do have the right feel for functions and understand what they have previously encountered ("discussed", a choice suggesting an interaction between reader and author). Together with the footnotes containing student-student dialogues, this offers identification with a group of reader-students who are guided through, and discuss, the same topics. The rest of the text is expository; except the description of the functions in the graphs suggests a narrative structure (approaching, raising, falling, coming closer, etc.). Overall structuring is achieved by numbered headings ("5.3 Power Functions"), and two subheadings that specialise these ("5.3.1 Positive exponents"; "5.3.2 Negative exponents") and numbered theorems. Two equations appear centred, but also within the running text there are equations and inequalities in consistent symbolic notation.

Text C

Experiential/Logical Meaning The text states "Theorem 10" followed by a "proof" (one form of the intermediate value theorem) which appears beside a diagram with a graph of a function with several local maxima and minima (see Excerpt C). The diagram includes labels with the same notation as in the theorem and in the proof. In the proof, reference is made to another theorem in the same book. The proof in one part employs the technique of indirect proof and explicitly states assumptions. All sentences contain mathematical notations. The text continues with a short section entitled "Remark" that invites to conduct a "thinking experiment" in relation to whether the theorem would also be true for a function defined only for rational numbers.

Textual/Interpersonal Meaning Throughout the text an anonymous knowledgeable general "we" speaks to an unknowing reader in the form of an exposition, with the aim to "prove" something. It frequently uses a general inclusive imperative ("consider", "assume", "let"). The "remark" at the end includes an author-reader "we" participating in the thinking experiment, which suggests some negotiation between author and reader. Coherence is achieved through logical relations and substitution of symbols. There is a running head visible at the top of the page with the numbered chapter heading "Continuous Functions", as well as the end of the paragraph from the preceding page "reminding" that a function that is continuous on a closed interval is continuous at each point of the interval.

Text D

Experiential/Logical Meaning The text sets out its theme with the heading "work with varying force". It starts with a general description of a process (a body being displaced) and poses the question that is going to be answered, "How does one calculate the work if the force $K(x)$ changes with the distance x from the starting point?" A generalised "formula" for the problem (work as an integral) is developed through reasoning about partial intervals (see Excerpt D) and then applied to a problem of a falling body. For doing so, the formula according to which two bodies attract each other with a force proportional to their mass is stated. Throughout the text, uncommon sense interpretations of force and distance, interval etc. are suggested by immediate use of a symbol after the words. Except for these word-symbol groups, the text employs non-specialised language, including estimation modifiers such as in "equals nearly" or "about". Some mathematical symbols are used in a non-technical way (Σ, Δx,$\rightarrow$).

Textual/Interpersonal Meaning Throughout the text, an anonymous knowledgeable author speaks to an unknowing reader, occasionally employing a general "we". Overall the text is expository, but also includes a question as introduction to the exposition, which invites reader participation. It appears logically coherent ("if", "then", "so"), even though new themes are introduced quickly. Statements only including symbolic notations are printed aligned to the centre.

Classification and Didactic Layers in the Texts

Whilst all four texts may be considered as strongly classified academic texts, two of them are concerned with the internal development of mathematics (Text B and Text C) and two include applications (Text A and Text C). Strong (internal) classification implies minimal liberty offered to the model reader insofar as there is no choice in terms of topics and aspects to be studied. But the presence or absence of numbering of sections and of theorems or equations indicate different modes of control, in addition to the level of thematic and logical coherence of the texts. A high level of coherence minimises jumping between or omitting passages with minimal choice for the model reader of how to proceed other than by reading the text in the given order with attention to detail, and recalling previous theorems or skills referred to in the text. Clearly, Text C is most coherent, and Text D least. Texts C and B have the highest density of mathematical notation, with Text B leaving more room for interpretation as the diagrams of the graphs of functions are used in the argument (lack of "rigour" in the interview with lecturers).

While Text C also includes a comment on the interpretation (in the "remark"), this is not done in a way that offers identification for a reader who is constructed as an acquirer negotiating interpretations with a fellow reader as in Text B. Hence we consider the footnote in Text B as a disembedded didactic layer which indicates a pedagogic strategy with some space for negotiation of meanings on the part of the students. Text A directly addresses the acquirer with an imperative to calculate an example (which is then worked out in the text). This is also a disembedded didactic layer, but points to a pedagogy with minimal space for interpretative activity.

Altogether, these observations lead to the following categorisation of the four texts (see Table 2), which provided a rationale for our selection of these texts to be used in the interviews.

Ranking of the Texts

In addition to these dimensions, the four texts can be ranked in terms of the strength of classification on the one hand, and the access to the generative principles of mathematical discourse provided on the other hand. Text B and Text C provide arguments for mathematical properties and establish links between topics, with Text C including a section labelled "proof". While the indirect proof in Text C operates with definitions and reference to another theorem, in Text B the theorems appear as a consequence of the exposition through one exemplar and interpretations of graphs. Hence these two texts can be ranked in terms of what is often referred to as level of abstraction and rigour (generality and discursive elaboration of the interpretation of diagrams). We interpret these features as a difference in strength of classification that constructs verticality, as Text B could be re-elaborated by means of the resources employed in Text C. Text A provides what Dowling (1998) described as *procedural elaboration* in his analysis of school textbook schemes. The calculated example does not include reference to bacteria anymore; it offers a generalizable procedure, which includes a skilled mathematical transformation. Text D also provides a procedure. It deals with a

Table 2 Categories of the texts used in the interviews

Classification	Didactic Layer	
	embedded	disembedded
strong	C	B
weak	D	A

mathematisation of "work", and derives a general "formula", which is used for calculating one example. It does not, however, elaborate the mathematical relationships in the formula and the meaning of technical symbols, which are used in a non-technical way. Hence these two texts can again be compared in terms of rigour, with Text A appearing more strongly classified than Text D. These considerations amount to a ranking of the texts (from strongly to weakly classified) in the order C-B-A-D.

Findings

In this section we present the findings from the interviews with the lecturers and the students. First we report how the group of lecturers ranked the four texts in terms of being more or less 'mathematical'. A discussion of some quantitative data of the students' ranking of the same texts is then followed by a presentation of the arguments used by the students for their rankings, organised by the operationalisation of our analytical framework as outlined in Table 1.

Lecturers' Ranking of the Texts

In a focus group interview eight lecturers who taught courses attended by the students, were *inter alia* asked to rank the four texts described above, based on their own conceptions of what it might mean for a text to be 'more or less mathematical' (see Bergsten and Jablonka 2015, for other findings from this interview). This part of the interview lasted about 10 min.

There was a rather immediate and general agreement that Text C should be ranked as most mathematical ("C definitely first"). It was also agreed by almost all lecturers that Text D is the least mathematical, while the ranking between texts A and B varied. Arguments for rankings included experiential/logical meaning (e.g. "[Text D] is a typical physical reasoning"), textual/interpersonal meaning (levels of coherence, 'didactical' functioning). Text A was ranked high by some but seen as constituting another "type of mathematics" than text C ("C is a proof and A is reasoning"). Arguments for a high ranking of Text A suggested that its didactic layer as well as its non-mathematical content were disregarded in favour of its technical mathematical parts (the particular technique for finding the value of the parameter *a*). An argument given for a lower ranking of Text A was its lack of coherence, e.g. that it "jumped back and forth". A reason put forward for ranking Text B low was that "it is written in a sloppy way". One lecturer then commented that the text is an introduction to a new topic and then "one should perhaps try to make it a little more simple", suggesting a didactical argument for 'allowing' a textbook to be less mathematical at some places. Another lecturer opposed, however, saying that by doing this "we make it easy for us and the students the first week independent of what happens the following week" and, referring to text B, "to me it is not mathematics" but more like "high school mathematics". The latter comment suggests that the lecturer recognises differences in classification between school and university mathematics, while some of the other reasons include reference to differences in pedagogy in terms of its didactic layer.

Students' Rankings of the Texts

In Table 3 the total number of students' ranks of the different texts are shown, including 'double rankings' where students in a few cases could not separate two texts as to which they saw as being 'more mathematical'.

It is clear from Table 3 that Text C was ranked as the most mathematical and D as the least mathematical by a large majority of the students. Further, the small variation of the ranking of Text C indicates a rather uniform characterisation of C as most mathematical, while the ranking of Text A is more varied than the rankings of the other texts. Similarly, the eight participating lecturers all ranked C as most mathematical and almost all ranked D as least mathematical, while there was some minor disagreement about whether Text A or B should be given rank two.

In our theoretically derived ranking we placed B before A in recognition of the difference in classificatory principles, a difference also recognised by the lecturers. Drawing on this theoretical ranking and the observations related to Table 3 above, we looked at the number of students providing the order CBAD or CBDA, as well as the number of students ranking C as most mathematical and D as least mathematical. These numbers are displayed in Table 4. The ranking order CBAD here indicates that Text C is ranked as the most mathematical and D as the least mathematical. Further, in Table 4 students are grouped according to their academic success. For doing so, the overall mathematics exams results from their first year of study are used. There were 13 (22 %) students at level L, 18 (30 %) at level M and 29 (48 %) at level H.

As can be seen from Table 4, of the most academically successful students (H) more than two thirds ranked the texts in the order CBAD/CBDA, and if also the middle achieving students (M) are included this proprtion is still around two thirds (66 %). Among the less successful students (L), however, only a little more than one third chose this ranking. Even stronger differences are found between H, M and L students ranking C as the most mathematical. A closer look at the data also shows that the rankings made by the L students are more varied (especially on texts A and C) and that only L students ranked C as least mathematical. The H and M students did rather similar rankings but with clear differences for texts A and D (rankings 3 or 4). Altogether, our data indeed indicates that recognition of differences in strength of classification of different mathematical discourses emulated in a range of pedagogic practices relates to success in the examinations.

Table 3 Students' rankings of the four texts A, B, C and D (number of students). Rank 1 refers to the "most mathematical" text, rank 4 to the "least mathematical"

Text\Rank	1	2	3	4	Median rank
A	8	8	33	25	3
B	11	40	8	3	2
C	47	11	10	2	1
D	2	4	17	40	4
	68	63	68	70	

Table 4 Number of students with given specific rankings

Exams results	Total number of students	Number and percentages of students with rank CBAD or CBDA	Number and percentages of students with C as the most mathematical	Number and percentages of students with D as least mathematical
L	13	5 (38 %)	7 (54 %)	8 (62 %)
M	18	11 (61 %)	13 (72 %)	11 (61 %)
H	29	20 (69 %)	25 (86 %)	21 (72 %)
All	60	36 (60 %)	45 (75 %)	40 (67 %)

In our presentation below of interview data relating to the arguments students provided for their rankings we include the student code, level of academic success (i.e. L, M or H) and preferred ranking of the texts (e.g. CBAD). The student code indicates programme and student number. We use the following abbreviations: C – Computer technology; E – Energy and environment; I – Industrial economy; M – Mechanical engineering, and T – Technical physics and electric engineering. A student code M9, for example, indicates student #9 in our sample of students enrolled in the Mechanical engineering programme. The notation [AB] in the ranking is used to indicate that the student did not or could not say which of A or B is more mathematical than the other.

We present a range of quotes from students across all programmes in order to increase the transparency of the analysis and represent the most typical arguments for the rankings of the texts. The subheadings of the presentation refer to modes of meaning as described in the operationalisation of the framework depicted in Table 1. The subheadings 'content' and 'representations, technical terms and symbols' refer to experiential/logical meanings; the subheadings 'format, structure and coherence' and 'accessibility' to textual/interpersonal meanings.

How Students Recognise Undergraduate Mathematics Discourse[6]

A closer look at the interview data reveals that the students differed in the type of arguments on which they based their rankings. In the following, we present some of these arguments given by students who saw Texts C or B as being more mathematical than Texts A or D. There were some L students amongst these: students C6 and E9 who both argued for the ranking CBAD, and student I1 with CBDA. We also include students who argued for Text A as more mathematical than Text B because of the more visible didactic layer in the latter, which resembles the argument in the discussion of the lecturers. All of these students appear to recognise the classificatory principles of the pedagogic discourse.

[6] We treat the discourse that is emulated in the mathematics courses as pedagogic discourse, but for the sake of simplicity do not refer to it as "pedagogic discourse in undergraduate mathematics courses" but simply as undergraduate mathematics discourse.

Content

References to the classificatory principles of academic subjects were made by some who classified Text A and Text D as less mathematical than Text B and Text C.

Strong classification was recognised in Texts B and Text C through:

> not at all applied (M5, M, [BC][AD])

> pure mathematics (T2, M, BCDA)

> these here now [texts C and B] deal more with the mathematics itself … describe things within the mathematics … this is then within pure mathematics … inner-mathematical … describes things within mathematics … more in pure maths (E3, H, CBAD)

> powerful mathematical proof (T6, H, CBAD)

> this is proof … with lots of intervals and continuous (E4, H, CBDA)

> this gives no examples from reality (T3, M, CBDA)

A few also referred to the specificity of relations between objects described in mathematics:

> greater than … smaller than … equal to (E4, H, CBDA)

> only f equals b and all such things (I9, H, CBDA)

> much palaver about greater than zero and such stuff (M7, H, CBDA)

Less mathematical texts can then be recognised by referring to something non-mathematical. This distinguished Texts A and D from Texts B and C:

> a colony of bacteria … physics formulas (C6, L, CBAD)

> something one should read in an economy book yes in a biology book when it is a bacteria colony (I1, L, CBDA)

> they give an example from reality … a real question (T3, M, CBDA)

> one applies maths … applied mathematics (T2, M, BCDA)

> it more applies mathematics (E3, H, CBAD)

> physics or what one shall call this (C9, H, CBAD)

> a problem in physics … for example here one finds nature … more biological where there I think more about physics (E3, H, CBAD)

> bacteria colonies … [Text D] I think about thermodynamics well this is also mathematics (E6, H, CB[AD])

> much about physics … here it feels more like biology … several subjects (I9, H, CBAD)

> an example that is useful in physics (T6, H, CBAD)

> does actually more describe a real situation (E4, H, CBDA)

> these are kind of physics topics … work and force … physical topics or what one shall call these … exponential growth in nature (M7, H, CBDA)

> not so much general … this physics … text (T7, H, BCAD)

Representations, Technical Terms and Symbols

Arguments for ranking Text C as most mathematical and Text B as more mathematical than the other texts included many references to different mathematical representations, technical terms and specialised mathematical symbols.

Texts B and C are recognised as more mathematical by:

> strange words and only f of a not equal to f of b and all that (E9, L CBAD)

> variables, they have a curve here where the variables are declared and shown and Greek letters … consists only of rational numbers (I1, L CBDA)

> there are many formulas in the text … there are even more formulas (E8, M, CBDA)

> here is also a graph and they express themselves more mathematically (I6, M, CBDA)

> more maths language when one talks about continuity … and there were many signs and one has taken away one's calculation (T2, M, BCDA)

> here we have a curve along the x-axis … there come more variables … a graph directly (E6, H, CBAD)

> lots of intervals and continuous (E4, H, CBDA)

> more mathematical because it has a graph (I9, H, CBDA)

> the talk as such … that this is such a general number (M7, H, CBDA)

"Less mathematical" is visible in Text A or Text D through the presence of data and calculations and lack of graphs:

> only that there are masses of numbers … the symbol delta X; they write in powers of tens (I1, L, CBAD)

> this is probably not mathematics without a graph (E8, M, CBDA)

> here are more numbers written out (I6, M, CBAD)

> calculation (E3, H, CBAD)

> numbers (C9, H, CBAD)

> they have kind of data here (C9, H, CBAD)

> these numbers … small values there (T7, H, BCAD)

More generally, more or less mathematical texts were by some differentiated by the degree of density of symbols in the text:

> [Text D] can become more, well, text [like] spoken language …[Text A] mix of formulas with text … [Texts C and B] almost much less spoken language (C9, H, CBAD)

> [Text C] easy to count the number of words that don't have to do with maths … [Text B] is more in words in fact [than Text C] (E4, H, CBDA)

Format, Structure and Coherence

A couple of students spoke more about the textual/interpersonal meanings, with reference to the function of a well-structured coherent exposition, which also establishes links to other themes. The latter points to recognition of the systematic nature, complexity in structure and generality of university mathematical discourse.

The comments about Text B and Text C included:

> it all depends on how they lay it out … a rigorous exposition … strict (C6, L, CBAD)

> one says let y be an arbitrary number … theorems refer to theorems (T3, M, CBDA)

> contains a theorem, a graph and a proof … more sectioned … with theorem and proof (E8, M, CBDA)

> very mathematical with proof (T2, M, BCDA)

> is more mathematical in itself that they apply proof and assumption (C9, H, CBAD)

> first they say something and then they prove it … with the help of certain assumptions (M8, H, CBAD)

> strict … strong proof … strong restrictions … theoretical (T6, H, CBAD)

> definitions and rules … that feels more mathematical one refers to a range of definitions and rules (I9, H, CBDA)

> one writes a theorem, which one proves that is valid for all numbers … one comes up with some form of conclusion here that appears to be true without that one has put in numbers or so (T7, H, BCAD)

Less mathematical texts were recognised by their different level of providing a coherent and general argument. Students who ranked Text D as the least mathematical mentioned, amongst other things:

> much examples … example of if we do like that so it should become like this … one does in fact not know what one is doing (C6, L, CBDA)

> several different formulas or … that one should transform them … not so many maths links in this … [should] do some more math than just put in a numeral (E3, H, CBAD)

> not so rigorous (T6, H, CBAD)

> here it says that one should discuss but not that one should prove or so … some values down there … one discusses oneself forward to things that seem reasonable (T7, H, BCAD)

The following comments were made about Text A:

> one has more [just] reasoned (T2, M, BCDA)

> this is how I solved one task (C9, H, CBAD)

> a sort of suggestion for a solution … they solve a task here (M8, H, CBAD)

> first they write as text and then they show it with numbers and symbols (E4, H, CBDA)

> more that one shall shove into and gets out some values (I9, H, CBDA)

Accessibility

A couple of students described texts B and C, and A and D, respectively, as similar in some respects. In favour of Text C as more mathematical than Text B many referred to a more didactical function of Text B. Also those two who ranked Text A over Text B gave the same reason.

> [Text B] more explaining (C6, L, CBAD)

> [Text C] a normal layman does not understand then what one talks about there … if one does not study mathematics … this does not probably not say anything (I1, L, CBDA)

> I don't know what I think about it being mathematical then one thinks it should be a little complicated (I6, M, CBDA)

> [Text B] well more like just a running text (E8, M, CBDA)

> [Text B] [reference to the footnote dialogue] this seems to be a little more fun (M3, M, CA[BD])

> [Text B] also mathematical but more understandable and so because this is more text and less only expressions and symbols … more words more text more explaining [than C] (E9, H, CBAD)

> [Text B] explains a little shows and explains … than just lining it up … if one like missed a lecture and would try to read further into it so it should somehow be such one (M8, H, CBAD)

> [Text B] not really a proof but they just explain what something means (T6, H, CBAD)

> [Text B] written more in words actually … [Text C] one must really concentrate if one should understand … one must know all these letters (E4, H, CBDA)

> [Text B] focuses not so much on the mathematics but more on the explanation … [Text A] feels more rigorous (T5, H, CABD)

Some students added critical comments about accessibility:

> math one should also be able to understand kind of so that there is nothing more mathematical just because it becomes more complicated (T2, M, CBDA)

In addition to the ranking CBAD ("describes maths best"), one student provided an alternative ranking in terms of accessibility ("teaches it best"), which was ABCD:

> this one [Text A] is very clear to read and easy to follow … there each formula stands in a separate line, then this becomes even more clear [Text C] little more difficult to follow … could have done better [Text B] a little more clear … it would be a rather negative conception of mathematical if it were only to press in as many formulas as possible [laughs] (E3, H, CBAD or ABCD)

Texts A and D, ranked as less mathematical, are seen as more accessible and the pedagogic interference of the author appears more visible, which for some is in contrast to the explanatory function of a mathematical argument.

> [Text A] guiding through all steps and so one mostly only has to learn it (C6, L, CBAD)

> [Text A] little easy language for a layperson … better for outsiders who don't study mathematics … still understandable for the most (I1, L, CBDA)

> [Text D] more of a general text (I6, M, CBDA)

> [Text D] rather easy to hang on to it (E3, H, CBAD

> [Text D] more easy to connect with … [Text A] easy to read … most easy to understand (E4, H, CBDA

> [Text A] doesn't feel like that the mathematics would be the difficult thing in this here (I9, H, CBDA

> [Text A] this is a little different language yes how one writes … for anyone (M7, H, CBDA)

A few students directly compared what they recognised in the texts with other pedagogic practices:

> [Text A] we don't talk about bacteria but we talk more about graphs (M5, M, [BC][AD])

> [Text C] this looks a little how it is done in our maths books (E8, M, CBDA)

> [Text B] this is the maths we are used to … pure analysis maths exactly the type we … looks like taken from maths books … this feels more like I recognise it (T2, M, CBDA)

> [Text B] explanation as how they show them in the maths book (C9, H, CBAD)

> [Text C] I recognise a part from the course (E6, H, CBAD)

> [Text A] such a task as one does already in compulsory school [the first 9 years in Sweden] (E4, H, CBDA)

> [Text C] is about how our teacher or lecturer goes through things … [Text D] in this way a lecturer or mathematician would not explain something … not the maths we are doing … how our physics teacher would explain things (I9, H, CBDA)

How Students Misrecognise Undergraduate Mathematics Discourse

Following our analytic framework, a ranking of Text A or Text D before Text B or Text C would indicate a misrecognition of university mathematics discourse. Our quantitative data showed that there was a higher proportion of the academically less successful students who suggested such ranking than of the successful students. In this section, we present some of the arguments given by students who suggested this ranking. We also include one less successful student who ranked Texts B and C as most mathematical, as the main arguments provided did not reflect a recognition of the classificatory principles of the pedagogic discourse.

A couple of students ranked Text A or Text D as the most mathematical. These students focussed on the textual meaning in terms of their own access. This is a different reason as the one given by the lecturers who argued that Text A should be ranked second; they appreciated the "trick" employed in the solution of the equation. Most of the students who ranked texts A or D before B or C were low achieving; one of them, however, got much higher marks on the examinations than on the diagnostic entrance test (M9).

Content

References to the experiential/logical meaning were rare amongst all of these students:

> [Text A] describes applications of these exponential functions [which makes it more mathematical than Texts B and C] (I12, L, DABC)

> yet mathematics is for me numbers and tasks …math is it is to calculate tasks to be able to apply it …(M9, H, ABCD)

Yet, the same student M9 had at the same time ranked Text A first and did not argue with the weak classification, which was only applied to Text D:

> [Text D] more a little physics about it …a relation kind of between a body and a length (M9, H, ABCD)

> a little more physics in text D …much text and numbers but few calculations (E11, M, [AC]BD)

Representations, Technical Terms and Symbols

It was also not common among these students to refer to different representations or technical terms as arguments for describing the texts. If they did so, these comments did not provide a consistent rationale for ranking the texts.

> [Texts B and C] these have more formulas (C7, L, ADBC)

> [Text C] has graphs and a little such (M4, L, BCDA)

> [Text C] lots of terms and greater or less than (E7, M, ADCB)

Format, Structure and Coherence

Student E11 made exclusively reference to the layout of the texts for ranking Text A as more mathematical than Text B.

> [Text B] here is more text …everything is wrapped up in the text (E11, M, [AC]BD)

Also student M4 focussed on the structure of the text, but also included a couple of evaluative statements about the quality of the texts without explaining these:

> [Text A] is not so mathematically set up …[Text B] is more kind of structured … mathematics is possibly dealing with being structured and show what you do it properly … [Text D] little more mussy … [Text D] is a little worse than [Text C] and text A even more worse (M4, L, BCDA)

Student C10 describes Text D as describing an experiment:

> such a task when you solve an experiment then you just don't write down a definition or so as they have done in this text [C] …it becomes another structure (C10, M, ADCB)

Some students expressed a direct recognition of texts they had used in their courses:

> [Text B] could have been taken from our algebra book as it was it explained the tasks in the same way (M1, L, [AC])

> [Text B] reminds me of the linear algebra book (M3, M, CA[BD])

> [Text C] this is from the green book I believe it looks like (C7, L, ADBC)

Accessibility

It was common in this group of students to refer to the pedagogic function of a text and its accessibility as an argument for being mathematical.

> this anyone could have written only by looking at theorems [Texts B and C] but to make people understand it is text A that appeals most to me that's what I see as most mathematical actually to be able to get someone to understand compared to

> just writing formulas straight off … this one now one can really understand [Text D] …and one does not get to grasp what about what that is what one reads [Text C] … it is more that one takes up maybe this and this [text; that is A and D] more than those two [Texts B, C]…this one I understand actually a little more [Text B] it is less mussy [than Text C] explains the way of thinking exactly how one should think how how it works simply so I rank A first (C7, L, ADBC)

In the beginning of this quote this student talked about the anonymous author in Text B and Text C, which did not appear to interfere in order to make it accessible. Also student M9 pointed at understanding as a criteria for being mathematical, linking it to the exposition of numbers over "text" and to "better describing":

> it is easier to understand the mathematics when it is written with- with numbers than with text …somehow more only text [in Text C] … [comparing Texts B and C] mathematics is actually normally not so clear until one really gets into it …[Text B] is better describing (M9, H, ABCD)

One L student ranked Text D as the most mathematical, followed by the texts A, B, and C. He named some shared features of mathematics texts, such as structure and amount of symbols:

> mathematics should feel structured … [Text A, also B] a large proportion of spoken language … don't know whether this has to do with mathematics (I12, L, DABC)

The level of structure was recognised by textual meanings that relate to the interpersonal function of the text:

> [Text D] this one I think feels clear and good …this one I like …structured and such … most academic possibly …doesn't mix so much letters and numbers but partitions it like this …so that the brain can more easily register if each stands in its own line … this [Text C] is surely mathematical but it feels very unpedagogical (I12, L, DABC)

Also student E7 talked about the structure in relation to the accessibility:

> [Text C] all is within the text so that it becomes very mussy … [Text A] is rather uncongested (E7, M, ADCB)

Similarly, student C10 concentrated on the pedagogic functioning of the texts in his arguments for ranking Text A and D over B and C, and talked about the amount of pre-knowledge of the reader:

> [Text A] assumes that one knows everything and understands what everything means … [Text C] they explain in the proof …if you know what the Greek letters mean you can follow and if you know what a function is …but here [Texts A and D] you must have knowledge from several areas then you become more mathematical (C10, M, ADCB)

Student M1 who discussed the texts A and C without saying much about the other texts, found there is:

> [in Text C] little too much greater than ..smaller than ..greater or equal to and such … that one is the best [to understand] [Text A]…[Text C] simply harder to understand … here they assume things all the time … very very much theory (M1, L, [AC])

Similarly student M14 found Text C "difficult":

> [Text C] mathematical lots of greater less equal to …perhaps a little more difficult (M14, L, A[CB]D)

Accessibility is also an issue for student E11:

> [Text C] this is a proof and it felt harder (E11, M, [AC]BD)

The reason for Text A being ranked over Text C given by student M9 includes:

> [mathematics should be about] …just problem solving with numbers (M9, M, ABCD)

Discussion

One aim of the study was to investigate the extent to which the engineering students are aware of differences in the pedagogic discourses of school and undergraduate mathematics and how they recognise and articulate these. We took each textbook as an instance of the realisation of a particular pedagogic discourse, which the students' might recognise. The differentiation in students' responses suggests that recognition of the classificatory principles of the undergraduate mathematics pedagogic discourse is linked to success in the examinations.

We did not report quantitative data about the distribution of the different rankings suggested by the students in relation to the engineering programmes in which they were enrolled because we did not find any striking patterns. One tendency in these data, however, concerns the observation that a larger proportion of the students from mechanical engineering than from the other programmes provided rankings that we describe as 'misrecognition'. More specifically, these were eight of the ten low and middle achieving students from that programme. At the same time, in this programme there was a larger group of first generation entrants in higher education than in the other programmes.[7] While we cannot make any claims about how the programme choices systematically relate to the students' family background, the observation of a difference in 'recognition rule' between more practically and more

[7] This became clear in the third interview conducted with the same students in the context of the larger project. It has to be noted though that the number of students at different achievement levels in each of the programmes was unevenly distributed in this study, which constitutes a limitation for analysis in relation to programme choice.

theoretically oriented engineering programmes indeed points to a dogmatic reading of Bernstein's theory of pedagogic discourse for theorising this phenomenon.

A pattern that emerged in the students' responses concerns similarities of dimensions they articulated, but used as indicating 'being mathematical' by the ones who recognised the specificity of the discourse ('recognition of discourse'), and indicating 'not being mathematical' by the others ('misrecognition of discourse'). Table 5 summarises these articulations in four emerging dimensions in pairs of opposing categories. In relation to our analytical tools, the first two dimensions that differentiate *Pure* versus *Applied* and *General* versus *Local* refer to the Experiential/Logical Meaning, while the two other dimensions, referred to as *Conditional* versus *Narrative* and *Non-accessible* versus *Accessible*, concern the Textual/Interpersonal Meaning (see Table 1).

In Table 5 students' articulation of recognition rules are displayed in terms of whether a particular category was used as characterising a "more mathematical" text (Ma+) or a "less mathematical" text (Ma-). The presence of empty cells in the table may be taken as suggesting some commonalities amongst the students' articulation of the criteria for what counts as legitimate mathematical activity. While there was some confusion whether, for example, a focus on 'Pure' or 'Applied' dimensions indicates undergraduate university mathematics discourse (in contrast to school mathematics), empty cells in the table suggest that some particular forms of misrecognition did not occur. For example, making generalisations ('General') was not used as an argument for concluding that a text was not mathematical. On the other hand, the presence of particular numerical calculations ('Local') was mistakenly recognised as a characteristic signature of undergraduate university mathematics discourse by some students. They might have re-recognised the pedagogic discourse of school mathematics when

Table 5 Students' articulation of recognition rules; Ma + denotes use of a category as characterising a "more mathematical" text and Ma- as a "less mathematical" text. A cell without Ma + or Ma- indicates that this type of (mis)recognition was not represented in our data

Pairs of opposing categories	Examples from interviews	Recognition of discourse	Misrecognition of discourse
Pure	within pure mathematics, not at all applied, this gives no examples from reality, taken away one's calculation, formulas	Ma+	Ma−
Applied	describes applications, numbers and tasks, to calculate tasks to be able to apply it	Ma−	Ma+
General	general number, variables, take away one's calculations, theorem, proof	Ma+	–
Local	calculations, masses of numbers, they have kind of data here	Ma−	Ma+
Conditional	they apply proof and assumption, a rigorous exposition	Ma+	–
Narrative	examples, not rigorous, we do like that so it should become like this	Ma−	–
Non-accessible	difficult, much theory, hard to understand, mathematical then one thinks it should be a little complicated	Ma+	Ma−
Accessible	easy to understand, more explaining, more words	Ma−	Ma+

alluding to this feature of the texts. The use of numerical data may indeed constitute a continuity between school and university mathematics, but there is certainly a shift in balance between numerical and algebraic calculations in the context of our study, which the students would need to recognise in order to be successful. As to the ways in which coherence in mathematical arguments is achieved ('Conditional' versus 'Narrative'), the empty cells indicate that the students recognised the importance of stating assumptions and the use of conjuncts to establish relations between propositions.

Almost all students referred to the anonymous author in plural as an active subject in their formulations: "they", perhaps the community of mathematicians or the collective of mathematics teachers who have written these texts. The students seemed to recognise a common function of the texts. Indeed, despite subtle differences in interpersonal meaning, all texts speak from a position of an anonymous author-expert to a group of like reader-apprentices, with a non-negotiable relationship between these. This experience of the authority relationship does not differentiate between the students. In contrast, the lecturers appeared to be aware of didactic layers in the texts and recognised different pedagogic strategies.

More generally, we note that the apprenticeship into undergraduate mathematics discourse in lectures and textbooks includes discursive elaboration of criteria for specific 'truth' producing procedures as well as an elaboration of established 'truths' by means of pointing to relations that expose the (more or less complex) systematic internal structure of meanings. As a mathematical argument or derivation (judged by different standards in school and at university) achieves both at the same time, the acquirer cannot make the distinction without relying on a commentary. For example, none of the students who ranked Text C over Text B (and both over A and D) alluded to the fact that what was stated in Text B could have been re-elaborated by means of the resources employed in Text C (by means of generalisation and discursive elaboration of principles for interpreting the graphs, hence with more 'rigour', as noted by the mathematicians). They certainly have seen the heading 'proof' in Text C though, while Text B only states that "We can then show the following theorem", which adumbrated the difference. A good lecturer (or textbook) might indeed be the one who provides a commentary in terms of explicit elaborations. The university students' relation to teachers' authority, however, eventually and gradually will shift from relying on comments and interpretations by the teacher to autonomous judgment. This may be reflected in the omission of a disembedded didactic layer in textbooks.

Further, the classificatory principles are not visible from entirely 'within' or 'without' the practice or discourse, but can only be accessed in elaborating the relations to other discourses. In this context we allude to Dowling's (2009) critique of Bernstein's notion of 'boundary' between discourses. The students who talked about the accessibility of the text could only be alienated by a pedagogic discourse that did not establish these relations for them, and hence favoured the texts that did so. They might have recognised the pedagogic discourse of school mathematics when alluding to 'calculations' and 'problem solving' as indicating 'mathematical', or other parts of their studies, which consist in tutored study groups for working through examples. A relationship (rather than a rupture) with school mathematics appears to have been recognised by all students who took the presence of 'graphs' as signifying mathematics. In the Swedish secondary school pre-calculus course these are indeed a dominant resource. The focus on the interpersonal meanings and their status as acquirer (as reader of the text) by some students appears to be a consequence of their alienation that impeded access to the field of the discourse. They demanded that a mathematics text should be more accessible.

The type of analysis presented in this study offers the possibility of relating it to similar studies of students' awareness of criteria for what counts as legitimate mathematical activity in school mathematics in different contexts, in order to identify continuities and discontinuities in classificatory principles that may account for the secondary-tertiary 'transition problem'.

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Appendix

Excerpt A1 (Tengstrand, 1994, p. 52)

En bakteriekoloni innehåller från början N_0 individer. Bakterierna förökar sig genom delning så att under varje minut delar sig en konstant andel p av hela kolonin. Vi tiden t = 1 har alltså pN_0 individer delat sig och kolonin består då av

$$N_0 + pN_0 = (1 + p)N_0$$

bakterier. Efter ytterligare en tidsenhet dvs då t = 2 är antalet individer

$$(1 + p)N_0 + p \cdot (1 + p)N_0 = (1 + p)^2N_0$$

och vid tiden t = n där n är ett positivt heltal så har kolonin växt till

$$(1 + p)^n N_0 = a^n N_0$$

individer. Vi har här satt 1 + p = a och a kallas tillväxtfaktorn.

Excerpt A1 (English translation)

A colony of bacteria contains in the beginning N_0 individuals. Bacteria multiply through splitting up in such a way that within each minute a constant proportion p of the whole colony splits. At the time $t = 1$ therefore pN_0 individuals have split up and the colony then consists of:

$$N_0 + pN_0 = (1 + p)N_0$$

bacteria. After one more time unit, that means when $t = 2$, the number of individuals is:

$$(1 + p)N_0 + p(1 + p)N_0 = (1 + p)^2N_0$$

and at the time $t = n$ when n is a positive whole number, the colony has grown to

$$(1 + p)^nN_0 = a^nN_0$$

individuals. We have here set $1 + p = a$ and a is called the growth factor.

Excerpt A2 (Tengstrand, 1994, p. 53)

På samma sätt visas att t ex N(1/3) = $N_0 a^{1/3}$ osv. Man kan visa att

$$N(t) = N_0 a^t$$

för ett godtyckligt t. Bakteriekolonin växer alltså exponentiellt.

Excerpt A2 (English translation)

In the same way it is shown that for example $N\left(\frac{1}{3}\right) = N_0 a^{\frac{1}{3}}$ etc. One can show that

$$N(t) = N_0 a^t$$

for an arbitrary t. Bacteria colonies therefore grow exponentially.

Excerpt B1 (Lennerstad, 2002, p. 238)

Alla potensfunktionerna x, x^2, x^3,... x^k, .. är strikt växande om vi bara tar med $x \geq 0$ – om vi väljer definitionsmängden $\mathbb{R}_+$. Då måste de vara inverterbara (varje horisontell linje har högst en skärning med kurvan). Så det finns en funktion som är invers till x^k. Denna funktion brukar betecknas som en rot: $\sqrt[k]{x}$.[92]

Excerpt B1 (English translation)

All power functions $x, x^2, x^3, \ldots x^k$,.. are strictly increasing if we only take into account $x \geq 0 -$ if we choose the domain R_+. Then they must be invertible (each horizontal line has at most one point of intersection with the curve). So there is a function that is inverse to x^k. This function usually is written as a root: $\sqrt[k]{x}$.[92]

Excerpt B2 (Lennerstad, 2002, p. 240)

Vi vet att $\frac{1}{x}$ är en avtagande funktion om vi begränsar oss till $x > 0$. Att funktionen är avtagande betyder att för alla $0 < x_1 < x_2$ gäller den omvända olikheten för motsvarande värden till $\frac{1}{x}$, nämligen $\frac{1}{x_1} > \frac{1}{x_2}$. Vi kan då visa följande sats:

Excerpt B2 (English translation)

We know that $\frac{1}{x}$ is decreasing if we limit ourselves to $x > 0$. That the function is decreasing means that for all $0 < x_1 < x_2$ the inverted inequality for concomitant values of $\frac{1}{x}$ is valid, that is, $\frac{1}{x_1} > \frac{1}{x_2}$. We can then show the following theorem:

Theorem 5.1 If f(x) is strictly increasing and positive, f(x) > 0 for all x, then $\frac{1}{f(x)}$ is strictly decreasing.

If f(x) is strictly decreasing and positive, then $\frac{1}{f(x)}$ is strictly increasing.

Excerpt B3 (Lennerstad, 2002, p. 238)

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92 Hjalmar: Vänta nu. Nu har vi två betydelser för "rot". Det betydde också lösning på en ekvation?
Inge: Ja... "roten ur x" är inte en sån rot... Men... $\sqrt{x}$ är ju en lösning till ekvationen $y = x^2$...
Hjalmar: Ja! Då är det nästan samma "rot"!
Inge: Det båda orden har samma rot... Du är språkkontrollant, va?
Hjalmar: Hehe! Ja! Det ligger i släkten! ■ D

Excerpt B3 (English translation)

92 Hjalmar: Wait a moment. Now we have two meanings of "root". That also meant solution to an equation?

Inge: Yes… "the root of x" is not such a root… But.. $\sqrt{x}$ is actually a solution of the equation $y = x^2$ …

Hjalmar: Yes! Then this is almost the same "root"!

Inge: Both these words have the same root.. You are language police, no?

Hjalmar: Hehe! Yeah! This is in the family! □ D

Excerpt C (Hyltén-Cavallius, & Sandgren, 1956, p. 184)

SATS 10: *Om funktionen f är kontinuerlig i det slutna intervallet $a \leq x \leq b$ och om $f(a) \neq f(b)$, så antar funktionen i intervallet $a < x < b$ varje värde mellan $f(a)$ och $f(b)$.* (Se fig. 6.)

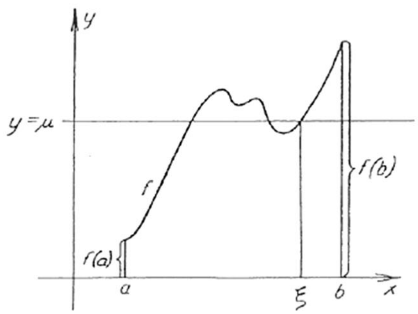

Fig. 6.

Bevis: Låt μ vara ett godtyckligt tal mellan $f(a)$ och $f(b)$, och bilda den kontinuerliga funktionen $g(x) = f(x) - \mu$. På grund av valet av μ har $g(a)$ och $g(b)$ olika tecken och vi kan antaga att $g(a) < 0$, ty annars kunde vi i stället definiera $g(x)$ som $\mu - f(x)$. Betrakta nu mängden av de x för vilka $g(x) \leq 0$. Mängden är självklart inte tom. Låt ξ vara dess övre gräns. Vi påstår, att

English translation:

Excerpt C (English translation)

THEOREM 10: If the function f is continuous in the closed interval a ≤ x ≤ b and if f(a) ≠ f(b), then on the interval a < x < b the function assumes all values between f(a) and f(b). (See Fig. 6.)

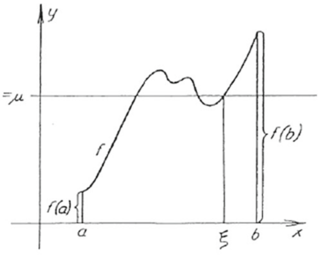

Fig. 6.

Proof: Let m be an arbitrary number between $f(a)$ and $f(b)$, and define the continuous function $g(x) = f(x) - \mu$. Due to the choice of μ, $g(a)$ and $g(b)$ have different signs and we can assume that $g(a) < 0$, because we could otherwise have defined $g(x)$ as $\mu - f(x)$. Now consider the set of those x for which $g(x) \leq 0$. This is trivially a non-empty set. Let ξ be its upper limit. We claim that

(1) $$g(\xi) = 0, \text{ i.e. } f(\xi) = \mu.$$

Excerpt D (Hellström, Morander, & Tengstrand, 1991, p. 382)

Antag att en kropp förflyttas en sträcka med längden R under påverkan av en konstant kraft K, som verkar i samma riktning som förflyttningen. Det arbete kraften utför är då K · R. Hur beräknar man arbetet om kraften K(x) varierar med avståndet x från startpunkten? Om vi låter sträckan representeras av intervallet [0, R] på tallinjen så delar vi in det i delintervall, som alla har längden Δx. Om Δx är litet så är arbetet nästan konstant lika med K(x) i varje intervall [x, x + Δx]. Totala arbetet blir då ungefär

$$\sum K(x)\,\Delta x.$$

Om Δx → 0 så går denna summa mot

$$\int_0^R K(x)\,dx$$

som är det totala arbetet K(x) utför under förflyttningen.

Excerpt D (English translation)

Assume a body is displaced along a segment with length R under influence of a constant force K, which acts in the same direction as the displacement. The work the force executes is then $K \cdot R$. How does one calculate the work when the force $K(x)$ varies with the distance x from the starting point? If we let the segment be represented by the interval [0, R] on the number line we then split that up into partial intervals, which all have the length Δx. If Δx is small, then the work is almost constant equal to $K(x)$ in each interval $[x, x+\Delta x]$. The total work then will be about

$$\sum K(x)\Delta x$$

If $\Delta x \to 0$ then this sum tends towards

$$\int_0^R K(x)\Delta x$$

which is the total work $K(x)$ does during the displacement.

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